SUPPLEMENT TO §2.1

1. Determine whether one quantity is a function of another within real life contexts by applying the definition of a function. Give your interpretation of the situation if you feel the description is vague.

    a. Is height a function of age?

    b. Is age a function of height?

    c. Is name a function of G# (i.e, PCC ID number)?

    d. Is G# a function of name?

    e. Is the cost per person a function of the number of people sharing a $20 pizza?

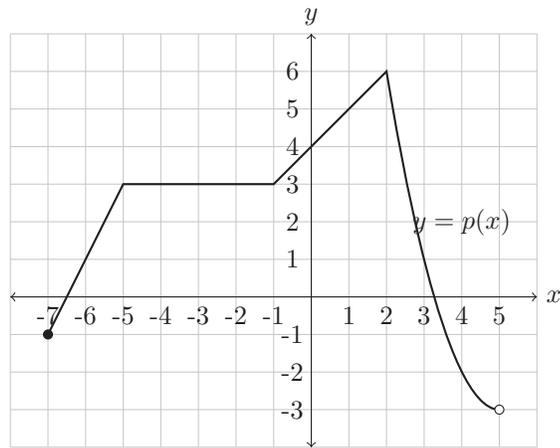2. The value of a computer, in dollars, is given by $c(t) = 600 - 150t$, where t is the number of years since the computer was purchased.

    a. Find and interpret the vertical intercept of the function c.

    b. Find and interpret the horizontal intercept of the function c.

    c. Find the domain and range of c. State it using interval notation.

3. The function $y = p(x)$ is graphed in the below figure. Use it to determine the following values.

a. Find $p(0)$.

b. Find $p(1.5)$.

c. Find $p(5)$.

d. Find $p(-7)$.

e. Find $p(2)$.

f. Solve $p(x) = -2$.

g. Solve $p(x) = 1$.

h. Solve $p(x) = -4$.

i. Solve $p(x) = 3$.

j. Solve $p(x) = 0$.

k. State the domain of p in interval notaion.

l. State the range of p in interval notation.

m. Find all x for which $p(x) > 1$.

4. A group of rafters take a 15 day rafting trip down the Colorado River through the Grand Canyon. The distance, in miles, $K(t)$, that the group has floated is a function of the number of days traveled, t. The table below gives some data about the trip.

| t	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$K(t)$	0	20	35	45	58	75	100	125	135	145	158	175	211	230	258	277

a. Find and interpret $K(8)$.

b. Find and interpret $K(35)$.

c. Solve $K(t) = 125$ and interpret your answer.

d. Find the average rate of change between $t = 11$ and $t = 13$. Explain your answer

e. Find and interpret the vertical intercept of the function K.

f. What is the domain of the function K? Explain your answer.

g. What is the range of the function K? Explain your answer.

5. The historic Arlene Schnitzer Concert Hall originally opened in 1928 as the Portland Publix Theatre. The Theatre seated 2776 people and it cost 60 cents per ticket. Let $r(x)$ be the revenue, in dollars, from selling x tickets for an event at the Theatre.

    a. Find the algebraic formula for $r(x)$.

    b. Find the domain and range of r. Explain your answer.

6. Consider the linear function $f(x) = 4 - 0.5x$.

    a. Find and simplify the expressions for $y = f(x + 6)$ and $y = f(x) + 6$.
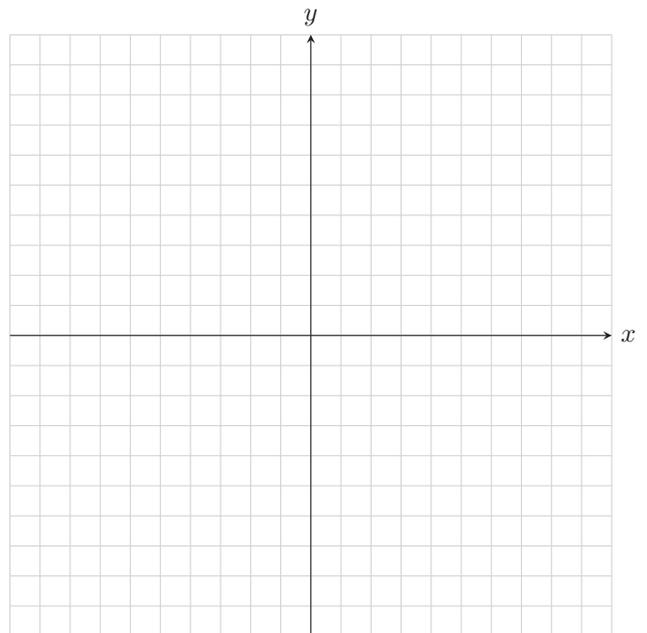
    b. Graph $y = f(x)$, $y = f(x + 6)$, and $y = f(x) + 6$ on the same set of coordinate axes.

7. If $p(x) = x^2$, find and simplify expressions for $p(x + 2)$ and $p(x) + 2$. Are the resulting expressions equivalent?

8. Let $j(x) = 4 - t + 2t^2$. Find and simplify the following expressions.

    a. $j(3t)$ c. $3j(t)$

    b. $j(3 - t)$ d. $3 - j(t)$

9. Suppse that $y = g(x)$ is a linear function and that $g(0) = -150$ and $g(50) = 2000$. Find the algebraic formula for $g(x)$.
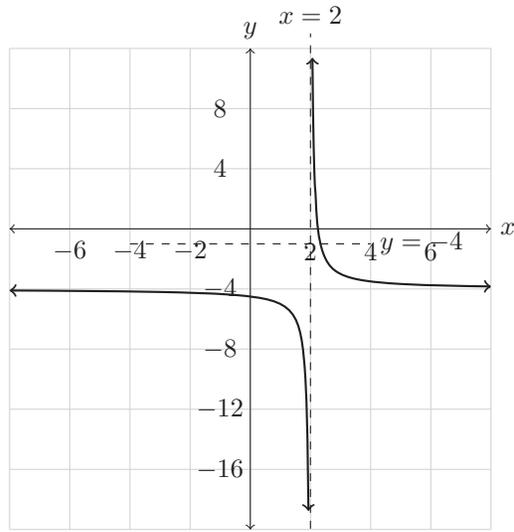
10. Suppose that $y = h(x)$ is a linear function and that $h(-5) = 2$ and $h(5) = 20$. Find an algebraic formula for $h(x)$.

11. Suppose that $y = j(t)$ is a linear function and that $j(-6) = 5$ and $j(10) = -1$. Find an algebraic formula for $j(t)$.

SUPPLEMENT TO §6.1
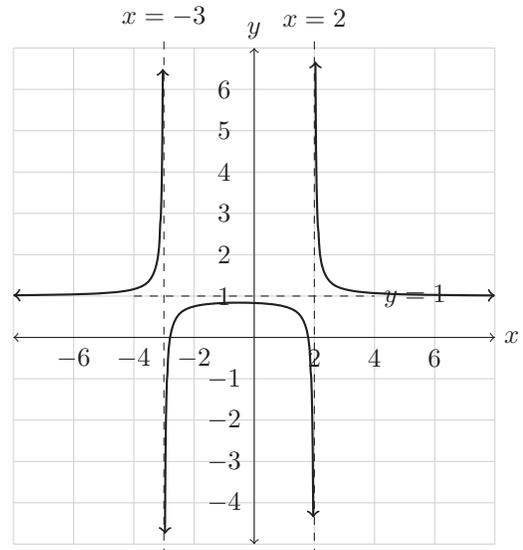
1. State the domain and range of the functions shown below using interval notation and set notation.

a.

Graph of $y = k(x)$.

b.

Graph of $y = j(x)$.

2. Let $f(x) = \dfrac{2x + 1}{5 - x}$. Simplify the following expressions.

a. $f(x) + 2$

b. $3f(x)$

c. $f(x+2)$

d. $f(3x)$

SUPPLEMENT TO §6.2

When we simplify the algebraic formula for a function we can lose information about the domain. We need to communicate hidden domain restrictions after we cancel factors in the denominator.

Example: Simplify the algebraic formula for $p(x) = \dfrac{3x + 3}{4x^2 - 4x - 8}$.

$$
\begin{aligned}
p(x) &= \frac{3x + 3}{4x^2 - 4x - 8} \\
&= \frac{3(x + 1)}{4(x^2 - x - 2)} \\
&= \frac{3(x + 1)}{4(x + 1)(x - 2)} \\
&= \frac{3\cancel{(x + 1)}}{4\cancel{(x + 1)}(x - 2)} \\
&= \frac{3}{4(x - 2)}, \quad x \neq -1
\end{aligned}
$$

(We write "$x \neq -1$" since the previous expression is not defined when $x = -1$ while this final expression is defined when $x = -1$. Thus they are only truly equal when $x \neq -1$.)

1. Simplify the expressions for the following rational functions. State any domain restriction necessary so that the expressions truly are equivalent.

   a. $f(x) = \dfrac{2x^2 + 6x}{3x + 9}$

   b. $T(x) = \dfrac{x^2 + 6x + 9}{x^2 - 9}$

c. $g(x) = \dfrac{2x^2 - x - 6}{x^2 + x - 6}$

e. $d(x) = \dfrac{x^2 - 16}{12 - 3x}$

d. $L(x) = \dfrac{x^3 - 4x^2 - 5x}{2x^2 - 13x + 15}$

f. $A(x) = \dfrac{12x^2 - 23x + 10}{12x^2 - 23x + 10}$

SUPPLEMENT TO §6.3

1. Let $S(x) = \dfrac{x+1}{x-3}$ and $R(x) = \dfrac{2}{x+2}$.

    a. If $a(x) = S(x) + R(x)$, find a simplified expression for $a(x)$ and state the domain of a using set notation.

    c. If $c(x) = S(x) \cdot R(x)$, find a simplified expression for $c(x)$ and state the domain of c using set notation.

    b. If $b(x) = S(x) - R(x)$, find a simplified expression for $b(x)$ and state the domain of b using set notation.

    d. If $d(x) = \dfrac{S(x)}{R(x)}$, find a simplified expression for $d(x)$ and state the domain of d using set notation.

SUPPLEMENT TO §7.1

1. Let $Y(x) = \sqrt{x-5} + 3$.

    a. Determine algebraically the domain of Y. State your conclusion using interval notation.

    b. Graph $y = Y(x)$ on your calculator and use it to determine the range of Y. State your conclusion using interval notation.

2. Let $Z(x) = \sqrt{7-2x} - 5$.

    a. Determine algebraically the domain of Z. State your conclusion using interval notation.

    b. Graph $y = Z(x)$ on your calculator and use it to determine the range of Z. State your conclusion using interval notation.

3. Let $U(t) = 2 - \sqrt{3+2t}$.

    a. Determine algebraically the domain of U. State your conclusion using interval notation.

    b. Graph $y = U(t)$ on your calculator and use it to determine the range of U. State your conclusion using interval notation.

4. Let $V(x) = \sqrt[3]{x+2}$.

    a. Determine algebraically the domain of V. State your conclusion using interval notation.

    b. Graph $y = V(x)$ on your calculator and use it to determine the range of V. State your conclusion using interval notation.

5. Let $T(k) = 3 - \sqrt[4]{3k-7}$.

    a. Determine algebraically the domain of T. State your conclusion using interval notation.

    b. Graph $y = T(k)$ on your calculator and use it to determine the range of T. State your conclusion using interval notation.

SUPPLEMENT TO §7.6

1. Let $f(x) = \sqrt{8 - 4x}$

    a. Evaluate $f(-2)$

    c. Solve $f(x) = 2$ and state your conclusion using set notation in a complete sentence.

    b. State the domain of f using set notation and interval notation.

2.   a. Solve the equation $1 + \sqrt{x} = \sqrt{3x - 3}$. State your conclusion using set notation in a complete sentence. Be sure to check the solutions you find to determine if there are any extraneous solutions.

    b. Solve the equation $1 + \sqrt{x} = \sqrt{3x - 3}$ by defining a function to represent each side of the equation and then using your calculator to graph both functions and find their intersection. Sketch the graphs in the provided space.
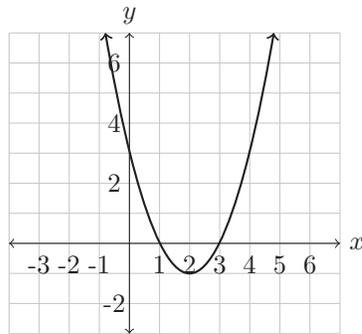

    c. What is the connection between what you found in part (a) in regards to extraneous solutions and the graphs in part (b)?

SUPPLEMENT TO §8.2

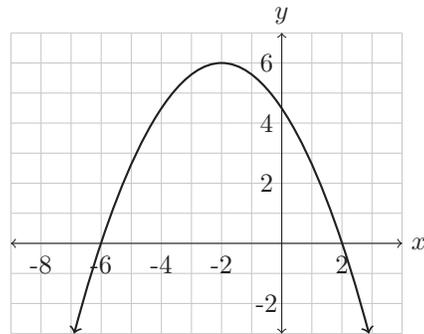1. Use interval notation to express domain and range of the quadratic functions graphed below.

a.

The graph of $y = m(x)$.

b.

The graph of $y = n(x)$.

2. Graph the quadratic function $f(x) = -0.4x^2 + 5x + 15$ on your graphing calculator. Be sure to find a viewing window that allows you to see the vertex and all intercepts.

    a. Use either the min or max key to estimate the coordinates of the vertex of $y = f(x)$.

    d. Use the trace key and substitute $x = 0$ to find the vertical intercept of $y = f(x)$.

    b. Use function notation to state the vertex as an input and output in the function f.

    e. Use interval notation to express the domain and range of f.

    c. Use the calculator to determine the horizontal intercepts of $y = f(x)$.

SUPPLEMENT TO §8.3

1. The graph of $y = w(x)$ is given in the figure to the right. Use it to do the following exercises.
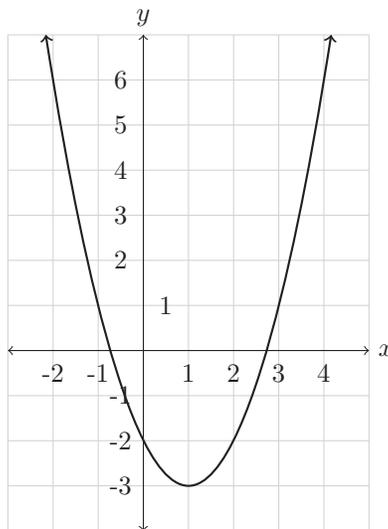
    a. Solve $w(x) < 1$.

    b. Solve $w(x) \geq -2$.

    c. Solve $-2 < w(x) \leq 6$.

    d. Solve $w(x) < -3$.

    e. Determine the domain and range of w. State your answer using interval notation.
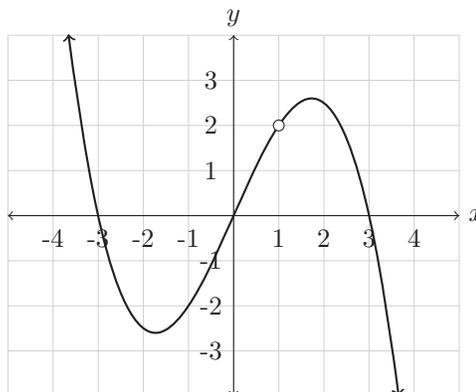
2. Use the graph of $y = f(x)$ to answer the following.
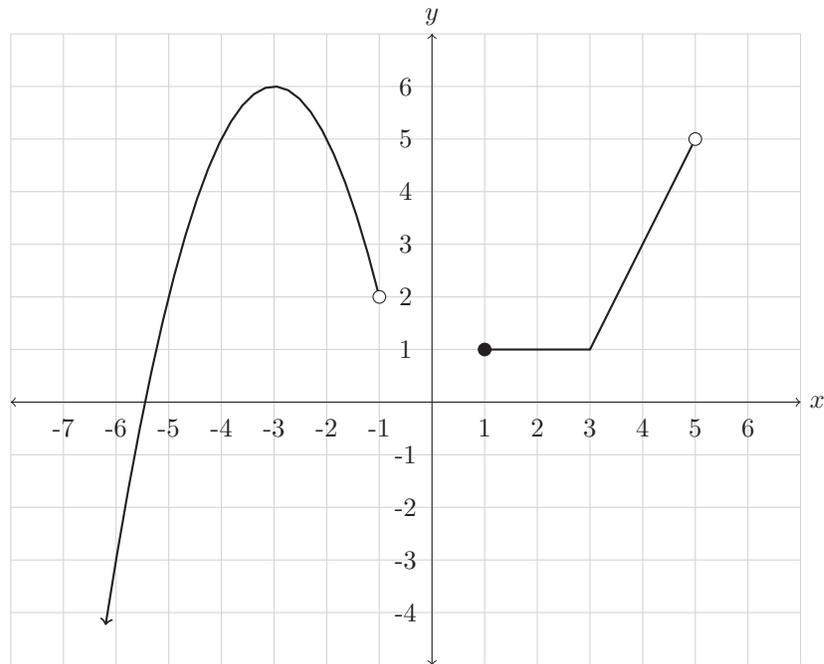
    a. Evaluate $f(-1)$.

    b. Solve $f(x) = 0$.

    c. Solve $f(x) > 0$.

    d. Estimate solutions to $f(x) = 1$.

    e. What are the domain and range of f. Use interval notation.

3. Use the graph of $y = m(x)$ to answer the following questions.

a. Evaluate $m(-5)$.

b. Evaluate $m(3)$.

c. Solve $m(x) = 2$.

d. Solve $m(x) = 6$.

e. Solve $m(x) > 0$.

f. Evaluate $m(-1)$.

g. Solve $m(x) = -3$.

h. Solve $m(x) = 3$.

i. Solve $m(x) \leq -3$.

j. Solve $m(x) > 5$.

k. State the domain and range of m using interval notation.

4. The function $f(t) = -t^2 + 2t + 3$ models the depth of water in feet in a large drainage ditch, where t is measured in hours and $t = 0$ corresponds to the moment that a summer storm has ended.

a. Evaluate and interpret $f(2)$ in the context of the real world function.

b. Write $f(t)$ in vertex form by completing the square. State the meaning of the vertex as a maximum or minimum in context of the situation.

c. Using the vertex form of $f(t)$ you found in part (b), solve the equation $f(t) = 0$ using the square root method.

d. What is the domain of and range of f in context of the situation? Write your answer in interval notation and explain your answer using a complete sentence.

e. At what time(s) will the water in the ditch be 1 foot deep? Round your solutions to three decimal places. Interpret your solutions in the context of the problem.

f. Make a graph the parabola $y = f(t)$ **on its implied domain** without using your calculator. Scale and label your axes.
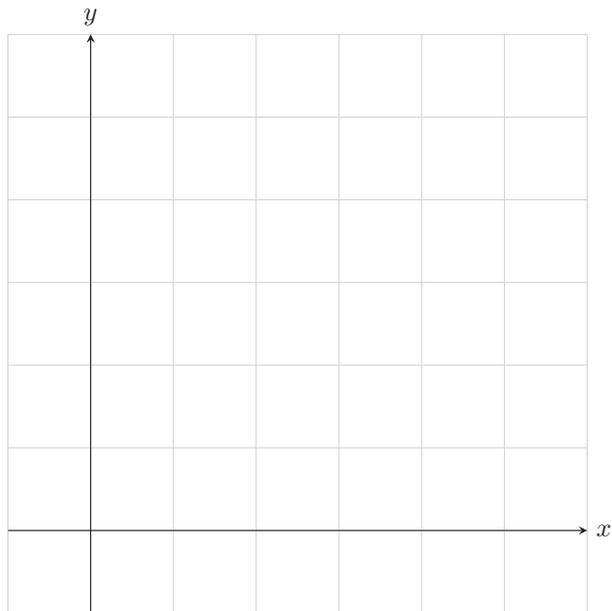
5. A television is launched with a trebuchet. Suppose that the function $h(d) = -\dfrac{1}{100}d^2 + \dfrac{6}{5}d + 28$ models the television's height in feet above ground when its horizontal distance from the trebuchet is d feet.

a. Find and interpret the vertical intercept.

b. Find and interpret the horizontal intercept(s).

c. Find and interpret the vertex.

d. What are the horizontal distances at which the TV is 40 feet above the ground?

e. What are the horizontal distances at which the TV is 75 feet above the ground?

f. A six foot tall pole is positioned 130 feet from the trebuchet. How high above the pole is the TV as it passes over?

g. What is the domain based on the context of the problem? Explain your reasoning.

h. What is the range based on the context of the problem? Explain your reasoning.

ANSWERS TO SUPPLEMENT §2.1**:**

1. a. Answers may vary. You could answer, "Yes," since, at a particular time, you can only be one height. Or you could answer, "No," since, during a given year, you can grow and thus be more than one height at that age.

    b. No. You can be the same height for many years.

    c. Yes. Each G# is associated with only one name.

    d. No. There can be more than one person with the same name attending PCC, so there can be more than one G# associated with a particular name.

    e. Yes, if you split the cost evenly. The cost per person, c, can be a function of the number of people splitting the pizza, n, given by $c = f(n) = \dfrac{2}{n}$. For each positive integer number, n, the cost c will be unique.

2. a. The point $(0, 600)$ is the vertical intercept. It means that when the computer is brand new, it is worth \$600.

    b. The point $(4, 0)$ is the horizontal intercept. It means that when the computer is 4 years old, it is worth nothing.

    c. The domain is $[0, 4]$. The range is $[0, 600]$.

3. a. $p(0) = 4$

    b. $p(1.5) = 5.5$

    c. $p(5)$ is undefined.

    d. $p(-7) = -1$

    e. $p(2) = 6$

    f. The solution set is $\{4\}$.

    g. The solutions set is $\{-6, 3\}$.

    h. The are no solutions. The solution set is $\{\}$.

    i. The solution set is $\{x| -5 \le x \le -1 \text{ or } x \approx 2.5\}$.

    j. The solution set is $\{x|x = -6.5 \text{ or } x \approx 3.2\}$.

    k. The domain is $[-7, 5)$.

    l. The range is $(-3, 6]$.

    m. $p(x) > 1$ for all x in the set $\{x| -6 < x < 3\}$.

4. a. $K(8) = 135$. After 8 days of rafting, the group has floated 135 miles down the Colorado River.

    b. $K(35)$ is undefined. The rafting trip was finished after 15 days, so the group was no longer rafting 35 days after starting the trip.

    c. The solution to $K(t) = 125$ is $t = 7$ since the group has floated down 125 miles of the Colorado River 7 days after starting the trip.

    d. The average rate of change is 27.5 miles per day. This means that each day between days 11 and 13 the rafting group traveled an average of 27.5 miles.

    e. The vertical intercept is $(0,0)$, so the vertical and horizontal intercepts are the same. This point means that after 0 days of rafting, the group had floated 0 miles down the Colorado River.

    f. The domain is $[0, 15]$. The input values for this function represent the number of days of the rafting trip and the trip was 15 days long, so all time periods between and including 0 and 15 days must be in the domain.

    g. The range is $[0, 277]$. The rafters floated all 277 miles of the Grand Canyon so at some moment during the 15 day trip, the rafters had floated each unique distance between and including 0 and 277 miles, so all of these values must be in the range.

5. a. $r(x) = 0.6x$

    b. Domain: $\{0, 1, 2, ..., 2775, 2776\}$
       Range:
       $\{0.00, 0.60, 1.20, 1.80, ..., 1664.40, 1665.00, 1665.60\}$.

6. a. $y = f(x + 6) = 1 - 0.5x$ and
       $y = f(x) + 6 = 10 - 0.5x$

    b. Check your graph by graphing these functions on your graphing calculator.

7. $p(x + 2) = x^2 + 4x + 4$ while $p(x) + 2 = x^2 + 2$ which are not equivalent.

8. a. $j(3t) = 18t^2 - 3t + 4$

    b. $j(3 - t) = 2t^2 - 11t + 19$

    c. $3j(t) = 6t^2 - 3t + 12$

    d. $3 - j(t) = -2t^2 + t - 1$

9. $g(x) = 43x - 150$

10. $h(x) = \dfrac{9}{5}x + 11$

11. $j(x) = -\dfrac{3}{8}x + \dfrac{11}{4}$

ANSWERS TO SUPPLEMENT §6.1

1. a.   Domain $= \{x|x \neq 2\}$
$= (-\infty, 2) \cup (2, \infty)$
Range $= \{y|y \neq -4\}$
$= (-\infty, -4) \cup (-4, \infty)$

    b.   Domain $= \{x|x \neq -3 \text{ and } x \neq 2\}$
$= (-\infty, -3) \cup (-3, 2) \cup (2, \infty)$
Range $= \{y|y \neq 1\}$
$= (-\infty, 1) \cup (1, \infty)$

2. a. $f(x) + 2 = \dfrac{11}{5-x}$

    c. $f(x+2) = \dfrac{2x+5}{3-x}$

    b. $3f(x) = \dfrac{6x+3}{5-x}$

    d. $f(3x) = \dfrac{6x+1}{5-3x}$

ANSWERS TO SUPPLEMENT §6.2

1.

    a. $f(x) = \dfrac{2x}{3}$, $D = \{x | x \neq -3\}$

    b. $T(x) = \dfrac{x+3}{x-3}$, $D = \{x | x \neq -3\}$

    c. $g(x) = \dfrac{2x+3}{x+3}$, $D = \{x | x \neq 2\}$

    d. $L(x) = \dfrac{x(x+1)}{2x-3}$, $D = \{x | x \neq 5\}$

    e. $d(x) = -\dfrac{x+4}{3}$, $D = \{x | x \neq 4\}$

    f. $A(x) = 1$, $D = \left\{x \middle| x \neq \dfrac{5}{4},\ x \neq \dfrac{2}{3}\right\}$

ANSWERS TO SUPPLEMENT §6.3:

1.

a. $a(x) = \dfrac{x^2 + 5x - 4}{(x+2)(x-3)}$,

   $D = \{x | x \neq -2 \text{ and } x \neq 3\}$

b. $b(x) = \dfrac{x^2 + x + 8}{(x+2)(x-3)}$,

   $D = \{x | x \neq -2 \text{ and } x \neq 3\}$

c. $c(x) = \dfrac{2(x+1)}{(x+2)(x-3)}$,

   $D = \{x | x \neq -2 \text{ and } x \neq 3\}$

d. $d(x) = \dfrac{(x+1)(x+2)}{2(x-3)}$,

   $D = \{x | x \neq 3 \text{ and } x \neq -2\}$
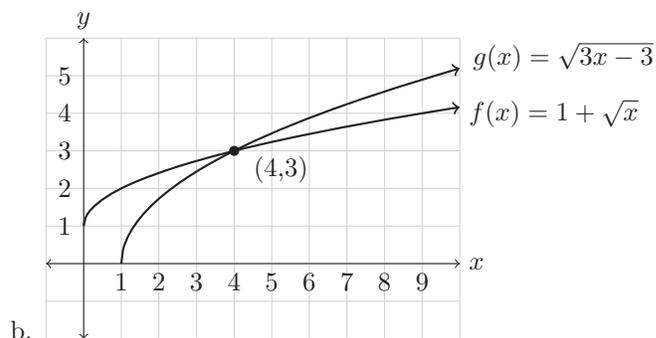
ANSWERS TO SUPPLEMENT §7.1

1.   a. The domain of Y is $[5, \infty)$.

      b. The range of Y is $[3, \infty)$.

2.   a. The domain of Z is $\left(-\infty, \dfrac{7}{2}\right]$.

      b. The range of Y is $[-5, \infty)$.

3.   a. The domain of U is $\left[-\dfrac{3}{2}, \infty\right)$.

      b. The range of U is $(-\infty, 2]$.

4.   a. The domain of V is $(-\infty, \infty)$.

      b. The range of V is $(-\infty, \infty)$.

5.   a. The domain of T is $\left[\dfrac{7}{3}, \infty\right)$.

      b. The range of T is $(-\infty, 3]$.

ANSWERS TO SUPPLEMENT §7.6

1.   a. $f(-2) = 4$ c. The solution set is $\{1\}$.

      b. The domain of f is $\{x | x \le 2\} = (-\infty, 2]$.

2.   a. The solution set is $\{4\}$. While it appears as though x could also equal 1, checking this as a solution in the original equation reveals that it is not actually a solution.

      b.

      The solution set is $\{4\}$.

      c. We see that the numbers which end up not being solutions to the problem when checked also do not show up as solutions when solving graphically.

ANSWERS TO SUPPLEMENT §8.2

1. a. The domain of m is $(-\infty, \infty)$ and the range is $[-1, \infty)$.

    b. The domain of n is $(-\infty, \infty)$ and the range is $(-\infty, 6]$.

2. a. The vertex is $(6.25, 30.625)$.

    d. The vertical intercept is $(0, 15)$.

    b. $f(6.25) = 30.625$

    e. The domain of f is $(-\infty, \infty)$ and the range is $(-\infty, 30.625]$.

    c. The horizontal intercepts are $(-2.5, 0)$ and $(15, 0)$.

ANSWERS TO SUPPLEMENT §8.3

1. a. The solutions are in the interval $(-1, 3)$.

   b. The solutions are in the interval $(-\infty, 0] \cup [2, \infty)$.

   c. The solutions are in the interval $[-2, 0) \cup (2, 4]$.

   d. There are no solutions. The solution set is $\{\}$.

   e. The domain is $(-\infty, \infty)$ and the range is $[-3, \infty)$.

2. a. $f(-1) = -2$

   b. The solution set is $\{-3, 0, 3\}$.

   c. The solution set is $(-\infty, -3) \cup (0, 1) \cup (1, 3)$.

   d. The solution set is $\{x | x \approx -3.2 \text{ or } x \approx 0.5 \text{ or } x \approx 2.8\}$.

   e. The domain is $(-\infty, 1) \cup (1, \infty)$ and the range is $(-\infty, \infty)$.

3. a. $m(-5) = 2$

   b. $m(3) = 1$

   c. The solution set is $\{-5, 3.5\}$.

   d. The solution set is $\{-3\}$.

   e. The interval of solutions is $(-5.5, -1) \cup [1, 5)$.

   f. $m(-1)$ is not defined.

   g. The solution set is $\{-6\}$.

   h. The solution set is $\{x | x \approx -4.8 \text{ or } x \approx -1.3 \text{ or } x = 4\}$.

   i. The interval of solutions is $(-\infty, -6]$.

   j. The interval of solutions is $(-4, -2)$.

   k. The domain is $(-\infty, -1) \cup [1, 5)$. and the range is $(-\infty, 6]$.

4.   a. $f(2) = 3$. Two hours after the storm ended, the water in the ditch was 3 feet deep.

   b. $f(t) = -(t-1)^2 + 4$. The vertex is $(1, 4)$. One hour after the storm ended, the water in the ditch reached its maximum depth of 4 feet.

   c. The solution set is $\{-1, 3\}$.

   d. The domain is $[-1, 3]$. Since one hour before the storm ended the ditch was empty and 3 hours after the storm ended, the ditch was empty again, having drained completely. The range is $[0, 4]$ since the minimum water depth is 0 feet and the maximum water depth is 4 feet deep.

   e. The water in the ditch will be 1 foot deep at approximately 0.732 hours before the storm ended and approximately 2.732 hours after the storm ended.

   f.
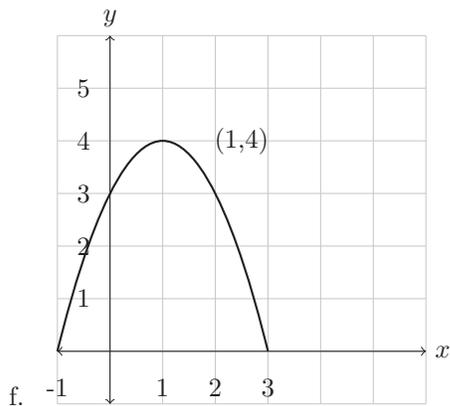
5. a. The vertical intercept is $(0, 28)$. The TV is 28 feet up in the air when at the point of launch.

   b. The horizontal intercepts are $(-20, 0)$ and $(140, 0)$. Only the second point makes sense in context and it represents the TV being 140 feet from the trebuchet when it hits the ground.

   c. The vertex is $(60, 64)$. The TV reaches its maximum height of 64 feet when it is 60 feet horizontally from the trebuchet.

   d. The TV is 40 feet above the ground when it is approximately 11.01 feet and approximately 108.99 feet horizontally away from the trebuchet.

   e. The TV never reaches 75 feet above the ground.

   f. The TV is 9 feet above the pole when it passes over it.

   g. The domain is $[0, 140]$ since the TV traveled from 0 feet to 140 feet where it hit the ground.

   h. The range is $[0, 64]$ since the TV's heights went from 28 feet to 64 feet and then back down all the way to 0 feet.