Chapter 6: Rational Expressions and Functions

Section 6.1: Intro

Rational? what is a rational number?

Any number that can be written as a fraction

$\frac{a}{b}$, $b \neq 0$

$\frac{\sqrt{2}}{\pi}$ irrational

rational – decimal repeats or terminates

$.\overline{3}$ $\frac{1}{3}$

$.25$ $\frac{1}{4}$

A rational expression is ~~made of~~ a fraction of expressions (polynomials)

ex: $\frac{1}{x}$ yes, this is rational

$x \neq 0$

Domain: $\{x \mid x \neq 0\}$

$\frac{2x^2+3}{x-1}$ rational

Domain: $\{x \mid x \neq 1\}$

$x - 1 \neq 0$

$+1 \quad +1$

$x \neq 1$

$\frac{\sqrt{x}}{|x|}$ ← irrational not rational

28. $f(x) = \frac{2x^2+x+5}{2x^2-x-15}$ yes, it is rational

$2x^2 - x - 15 \neq 0$

Set the denominator $\neq 0$

$2x^2 - x - 15 \neq 0$ factor

$ac = -30$

$1 \cdot 30$
$2 \cdot 15$
$3 \cdot 10$
$5 \cdot -6$

$2x^2 + 5x - 6x - 15 \neq 0$

$x(2x+5) - 3(2x+5) \neq 0$

$(2x+5)(x-3) \neq 0$

$2x+5 \neq 0$ or $x-3 \neq 0$

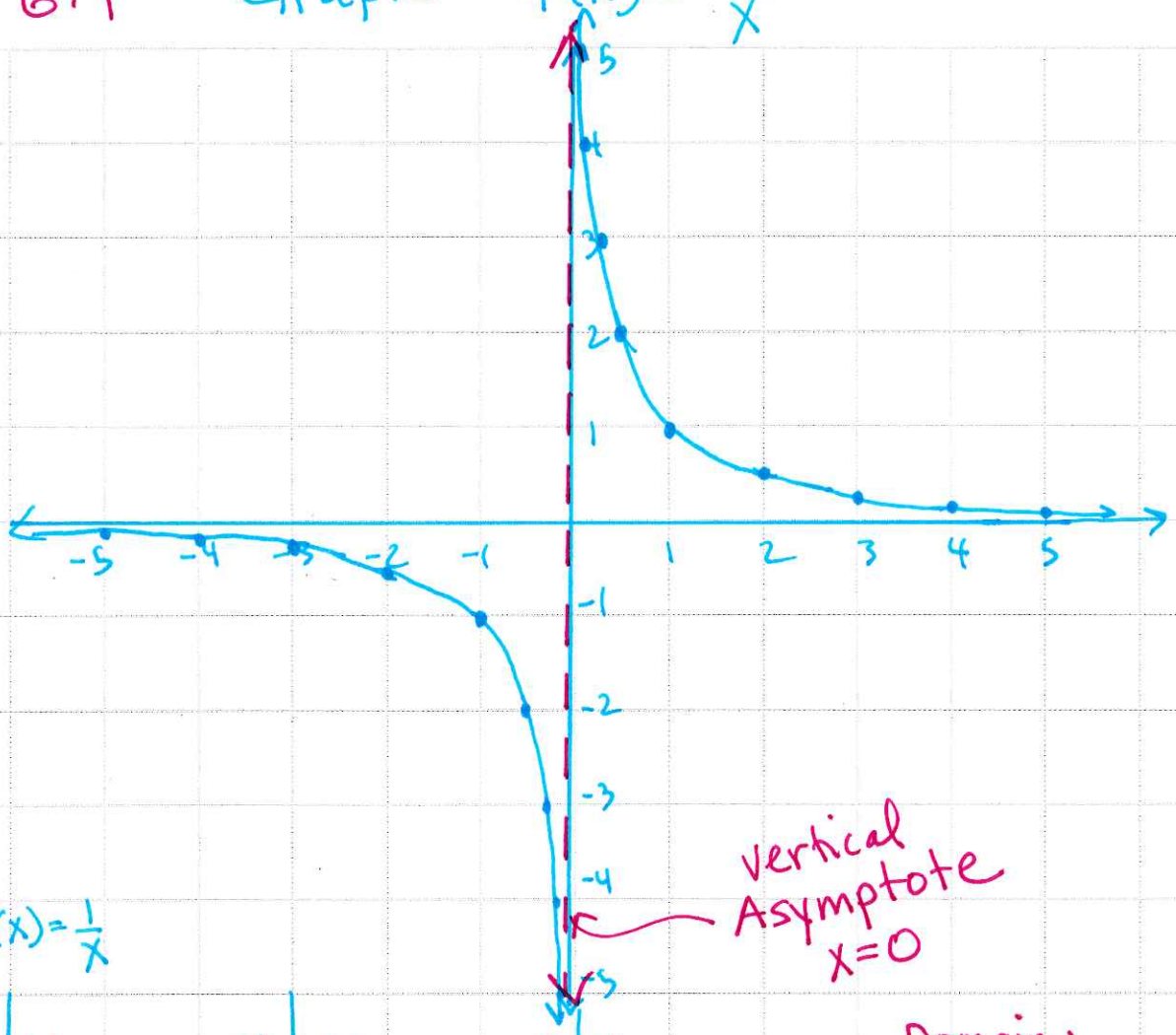
$-5 \quad -5 \qquad +3 \quad +3$

$\frac{2x}{2} \neq \frac{-5}{2} \qquad x \neq 3$

$x \neq -\frac{5}{2}$

Domain: $\{x \mid x \neq -\frac{5}{2}$ or $\overbrace{3}^{x \neq 3}\}$

6.1 Graph $f(x) = \frac{1}{x}$



$f(x) = \frac{1}{x}$

x	y
1	$\frac{1}{1} = 1$
2	$\frac{1}{2} = \frac{1}{2}$
3	$\frac{1}{3}$
4	$\frac{1}{4}$
5	$\frac{1}{5}$

x	y
-1	-1
-2	$-\frac{1}{2}$
-3	$-\frac{1}{3}$
-4	$-\frac{1}{4}$
-5	$-\frac{1}{5}$

x	y
0	$\frac{1}{0}$ undefined
$\frac{1}{2}$	$\frac{1}{\frac{1}{2}} = 2$ $1 \cdot \frac{2}{1} = 2$
$\frac{1}{3}$	$\frac{1}{\frac{1}{3}} = 3$
$\frac{1}{4}$	$\frac{1}{\frac{1}{4}} = 4$
$\frac{1}{5}$	$\frac{1}{\frac{1}{5}} = 5$

Domain:
$(-\infty, 0) \cup (0, \infty)$
$\{x \mid x \neq 0\}$

Range:
$(-\infty, 0) \cup (0, \infty)$
$\{y \mid y \neq 0\}$

6.1 continued

32. Graph a rational function

$f(x) = \frac{1}{x+3}$ Find the asymptote first

$x+3 \neq 0$

$-3 \quad -3$

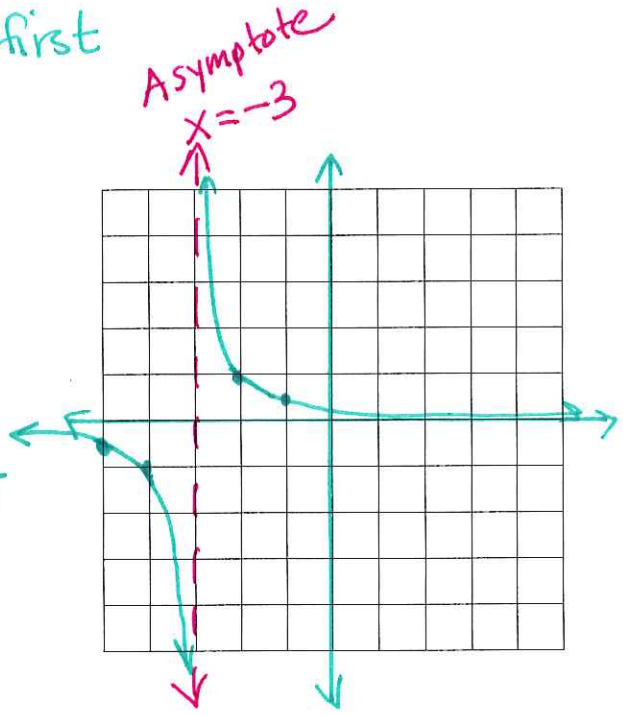
$x \neq -3$

x	$f(x)$
-5	$-\frac{1}{2}$
-4	-1
-3 (Asymptote →)	— undefined
-2	1
-1	$\frac{1}{2}$

$f(-5) = \frac{1}{-5+3} = \frac{1}{-2} = -\frac{1}{2}$

$f(-4) = \frac{1}{-4+3} = \frac{1}{-1} = -1$

$f(-2) = \frac{1}{-2+3} = \frac{1}{1}$



$f(-1) = \frac{1}{-1+3} = \frac{1}{2}$

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42. $f(x) = \frac{4}{4-x^2}$ use the table function on your calculator to plot points

Denominator $4 - x^2 \neq 0$

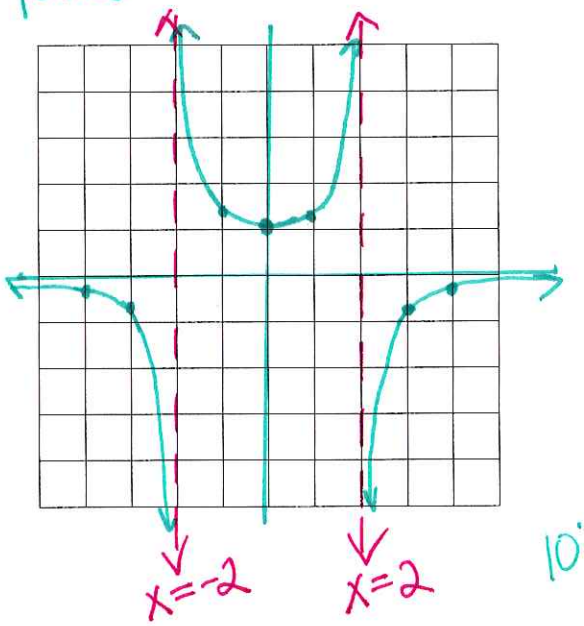
$+x^2 \quad +x^2$

$\pm\sqrt{4} \neq \sqrt{x^2}$

$\pm 2 \neq x$

$x \neq \pm 2$

$y_1 = 4/(4 - x\hat{}2)$



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Domain: $\{x \mid x \neq -2 \text{ or } x \neq 2\}$

Solving Rational Equations

62. $\frac{x+3}{4x+1} = \frac{3}{5}$ cross-multiply

$5(x+3) = 3(4x+1)$

$5x+15 = 12x+3$

$-5x \qquad -5x$

$15 = 7x+3$

$-3 \qquad -3$

$\frac{12}{7} = \frac{7x}{7}$

$x = \frac{12}{7}$ $\left\{\frac{12}{7}\right\}$

72. $\frac{6x}{3x-2} = \frac{4}{4-6x}$

$6x(4-6x) = 4(3x-2)$

$24x - 36x^2 = 12x - 8$

$-24x \quad +36x^2 \quad +36x^2 \quad -24x$

make a positive

$\frac{0}{4} = \frac{36x^2}{4} - \frac{12x}{4} - \frac{8}{4}$

$0 = 9x^2 - 3x - 2$

$ac = -18$

$1 \cdot 18$

$2 \cdot 9$

$3 \cdot 6$

$$0 = 9x^2 + 3x - 6x - 2$$

$$0 = 3x(3x+1) - 2(3x+1)$$

$$0 = (3x+1)(3x-2)$$

$3x+1=0$ or $3x-2=0$

$-1 \quad -1 \qquad +2 \quad +2$

$\frac{3x}{3} = \frac{-1}{3}$ or $\frac{3x}{3} = \frac{2}{3}$

$x = -\frac{1}{3}$ ~~$x = \frac{2}{3}$~~

Check the domain

$$\frac{6x}{3x-2} = \frac{4}{4-6x}$$

$3x-2 \neq 0$ or $4-6x \neq 0$

$+2 \quad +2 \qquad \frac{4}{6} \neq \frac{6x}{6}$

$\frac{3x}{3} \neq \frac{2}{3} \qquad \frac{2}{3} \neq x$

$x \neq \frac{2}{3}$

$\frac{2}{3}$ would make the denominator zero.

~~EXAM~~

$\{-\frac{1}{3}\}$

Solve Graphically or Numerically

$y_1 = \frac{6x}{3x-2}$

$y_2 = \frac{4}{4-6x}$