

8.1 Quadratic Functions and Their Graphs

Graphs of Quadratic Functions Basic Transformations of Graphs More About Graphing Quadratic Functions (Optional) Min-Max Applications
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Key Terms

Use the vocabulary terms listed below to complete the statements in exercises 1-5. Note that some terms may be used more than once or not at all.

wider
vertex
upward

narrower
downward
x-coordinate

axis of symmetry
y-coordinate
quadratic function

1. A(n) _____ can be written in the form $f(x) = ax^2 + bx + c$, where a , b , and c are constants with $a \neq 0$.
2. The _____ is the lowest (or highest) point on the graph of a parabola that opens upward (or downward).
3. The _____ is a vertical line about which a parabola is symmetric.
4. The _____ of the vertex of the graph of $y = ax^2 + bx + c$, $a \neq 0$, is given by $x = -\frac{b}{2a}$.
5. The graph of $y = ax^2$ is a parabola with the following characteristics.
 - a. The _____ is $(0,0)$ and the _____ is given by $x = 0$.
 - b. It opens _____ if $a > 0$ and opens _____ if $a < 0$.
 - c. It is _____ than the graph of $y = x^2$ if $0 < |a| < 1$.
 - d. It is _____ than the graph of $y = x^2$ if $|a| > 1$.

Graphs of Quadratic Functions

Exercises 1-6: For each problem, do the following.

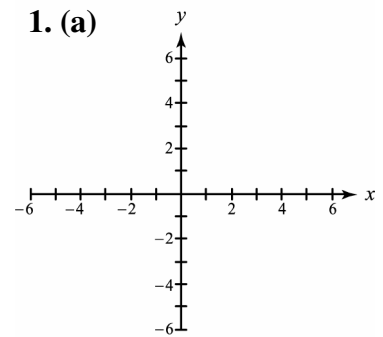
(a) Graph each quadratic function.

(b) Identify the vertex.

(c) Identify the axis of symmetry.

(d) State where the graph is increasing and where it is decreasing.

1. $f(x) = x^2 - 2$

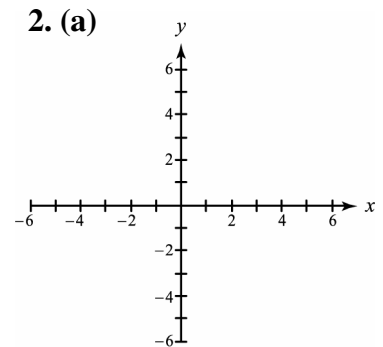


(b) _____

(c) _____

(d) _____

2. $f(x) = -x^2 + 3$

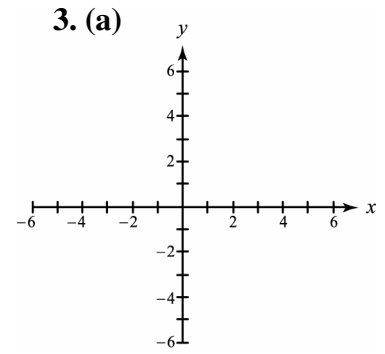


(b) _____

(c) _____

(d) _____

3. $f(x) = (x-1)^2$

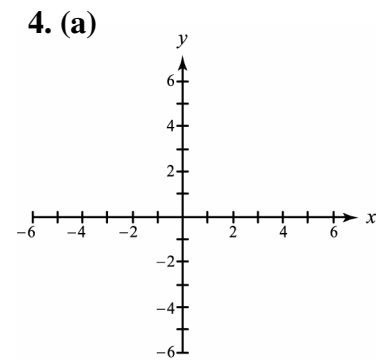


(b) _____

(c) _____

(d) _____

4. $f(x) = -(x+4)^2$



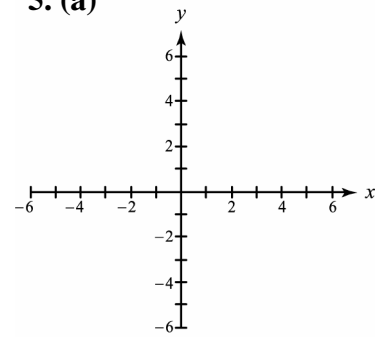
(b) _____

(c) _____

(d) _____

5. $f(x) = x^2 - 4x + 3$

5. (a)



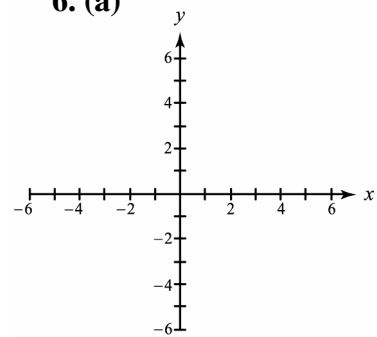
(b) _____

(c) _____

(d) _____

6. $f(x) = -x^2 - 2x$

6. (a)



(b) _____

(c) _____

(d) _____

Exercises 7-12: Find the vertex for the graph of the function.

7. $f(x) = x^2 - 2x + 3$

7. _____

8. $f(x) = 2x^2 + 4x - 1$

8. _____

9. $f(x) = -\frac{1}{2}x^2 + 2x + 3$

9. _____

10. $f(x) = 4 - 4x + x^2$

10. _____

11. $f(x) = 2x^2 - 5$

11. _____

12. $f(x) = \frac{1}{3}x^2 - 2x + 4$

12. _____

Exercises 13-16: For each problem, do the following.

(a) Find the maximum y -value on the graph of the function.

(b) State where the graph of f is increasing and where it is decreasing.

13. $f(x) = -x^2 - 4x + 5$

13. (a) _____

(b) _____

14. $f(x) = 8x - x^2$

14. (a) _____

(b) _____

15. $f(x) = -x^2 - 3x - 2$

15. (a) _____

(b) _____

16. $f(x) = 5 - x^2$

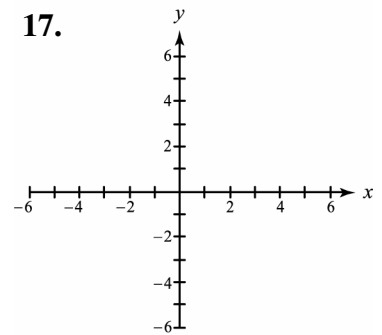
16. (a) _____

(b) _____

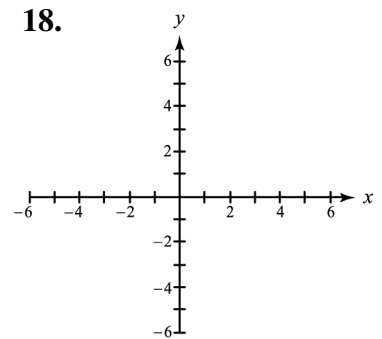
Basic Transformations of Graphs

Exercises 17-20: Graph each function on the same coordinate axes as $y = x^2$.

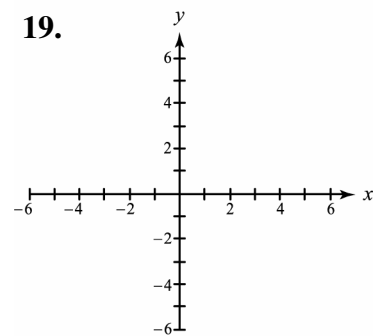
17. $f(x) = 3x^2$



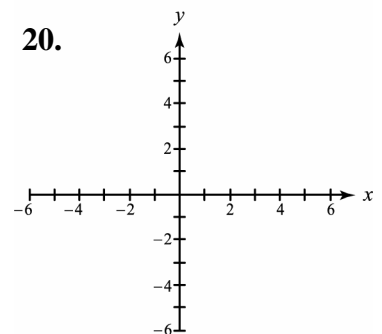
18. $f(x) = \frac{1}{2}x^2$



19. $f(x) = -x^2$



20. $f(x) = -2x^2$



More About Graphing Quadratic Functions (Optional)**Exercises 21-24:** For each problem, answer the following.

- (a) Does the graph of f open upward or downward?
 (b) Is the graph of f wider, narrower, or the same as the graph of $y = x^2$?
 (c) Find the axis of symmetry and the vertex.
 (d) Find the y -intercept and any x -intercepts.
 (e) Sketch a graph of f .

21. $f(x) = \frac{1}{2}x^2 - 2x - \frac{5}{2}$

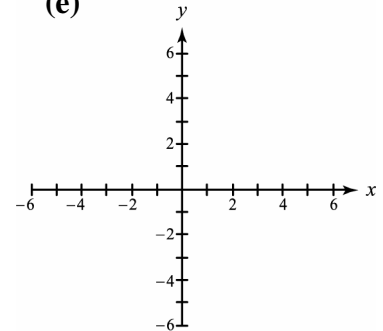
21. (a) _____

(b) _____

(c) _____

(d) _____

(e)



22. $f(x) = -x^2 - 2x + 3$

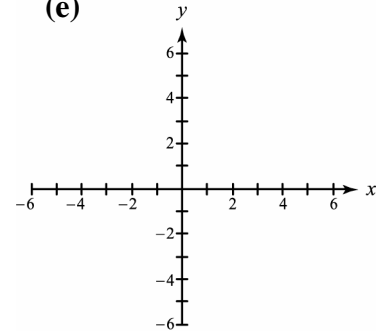
22. (a) _____

(b) _____

(c) _____

(d) _____

(e)



23. $f(x) = 4x - x^2$

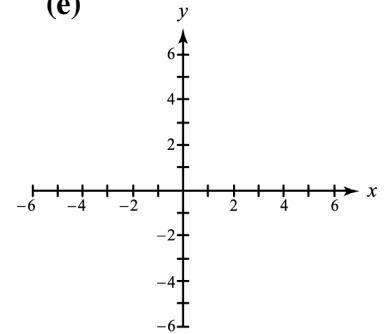
23. (a) _____

(b) _____

(c) _____

(d) _____

(e)



24. $f(x) = 2x^2 + 8x + 8$

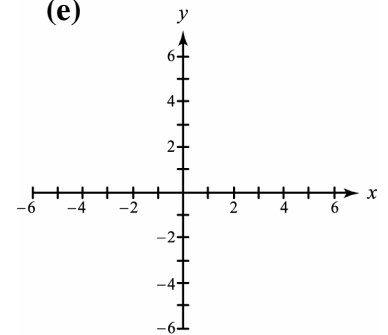
24. (a) _____

(b) _____

(c) _____

(d) _____

(e)



Min-Max Applications

- 25.** A baseball is hit into the air and its height h in feet after t seconds can be calculated by $h(t) = -16t^2 + 96t + 4$.
- (a) What is the height of the baseball when it is hit? **25. (a)** _____
- (b) Determine the maximum height of the baseball. **(b)** _____
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- 26.** A golf ball is hit into the air, and its height h in feet after t seconds is given by $h(t) = -16t^2 + 112t$.
- (a) What is the height of the golf ball when it is hit? **26. (a)** _____
- (b) After how many seconds does the golf ball reach its maximum height? **(b)** _____
- (c) Determine the maximum height of the golf ball. **(c)** _____
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- 27.** A printing company gives discounts for bulk sales of shirts. The regular price of a shirt is \$30, and the price decreases by \$0.50 for each shirt sold.
- (a) Write a formula $f(x)$ that gives the total price for x shirts. **27. (a)** _____
- (b) Determine how many shirts should be sold to maximize the revenue. **(b)** _____

- 28.** A hotel is considering giving the following group discount on room rates. The regular price for a room is \$112, but for each room rented, the price decreases by \$4.

(a) Write a formula $f(x)$ that gives the total revenue from renting x rooms. **28. (a)**_____

(b) Determine how many room should be rented to maximize the revenue. **(b)**_____

(c) What is the maximum revenue? **(c)**_____

- 29.** A rectangular pen requires 80 feet of fencing.

(a) Write a formula $f(x)$ that gives the area of the pen if one side of the pen has length x . **29. (a)**_____

(b) Determine the dimensions of the pen that give the largest area. **(b)**_____

- 30.** The annual percentage increase C in the cost of health insurance premiums between 1992 and 2000 is modeled by the formula $C(t) = 0.65t^2 - 7.8t + 24.4$, where $t = 2$ corresponds to 1992.

(a) Determine the year when the percentage increase in cost was a minimum. **30. (a)**_____

(b) What was the minimum percentage? **(b)**_____