SUPPLEMENT TO §1.5

1. The area of a triangle is given by the formula

$$A = \frac{1}{2}bh,$$

where A is the area, b is the length of the base, and h is the height of the triangle.

Suppose the area of a triangle is 55.6 square inches and the base is 7 inches long. Find the height of the triangle. Round to the nearest tenths if necessary.

2. In order to convert a temperature from degrees Fahrenheit to degrees Celsius you can use the formula

$$C = \frac{5}{9}(F - 32),$$

where F is the temperature in degrees Fahrenheit and C is the temperature in degrees Celsius.

Suppose a friend traveling in Russia tells you that the thermostat where they are at says that it is $31°C$. What is the temperature in Fahrenheit? Round to the nearest hundredths if needed.

3. The circumference of a circle and its radius are related by the formula

$$C = 2\pi r,$$

where C is the circumference of the circle and r is the radius of the circle.

Suppose the circumference of a circle is 5.6 cm. What is the radius?

SUPPLEMENT TO §1.6

RULES OF EXPONENTS

i.	$x^a \cdot x^b = x^{a+b}$	When we multiply like bases raised to powers, we add the powers.
ii.	$(x^a)^b = x^{a \cdot b}$	When we raise a base-with-a-power to a power, we multiply the powers together.
iii.	$(xy)^a = x^a y^a$	We apply the exponent to each factor inside the parentheses.

Note that one of the most common mistakes is to confuse rules *i* and *ii*. To avoid this mistake, notice that if there are **two** factors with like bases, you should use rule *i*.

EXAMPLE: Simplify the following expressions using the rules of exponents.

a. $-2t^3 \cdot 4t^5$

b. $5\left(v^4\right)^2$

c. $4(3u)^2$

d. x^3y^2

Solutions:

a.
$$
\begin{aligned}
-2t^3 \cdot 4t^5 &= -8t^3 \cdot t^5 && \text{Multiply the -2 and the 4 together.}\\
&= -8t^{3+5} && \text{Add the two exponents together using rule } i.\\
&= -8t^8
\end{aligned}
$$

b.
$$5\left(v^4\right)^2 = 5 \cdot v^{4\cdot 2}$$
Apply rule *ii*; note that the power “2” doesn't affect the coefficient “5” since we apply exponents before multiplication.
$$= 5v^8$$

c.
$$
\begin{aligned}
4(3u)^2 &= 4\left(3^2u^2\right) && \text{Apply rule } iii.\\
&= 4(9u^2) && \text{(simplify)}\\
&= 36u^2
\end{aligned}
$$
Multiply the 4 and 9 together; note that we don't distribute 4 to 9 and u^2 since there is no addition or subtraction.

d. x^3y^2 cannot be simplified further since the bases are different variables.

■

EXERCISES:

1. True of False. If a statement is false, change one side of the equation to make it true.

   a. $x^2 + x^3 = x^5$

   b. $3^2 \cdot 3^4 = 3^6$

   c. $3^2 \cdot 2^3 = 6^5$

   d. $x^2 x^3 = 2x^5$

   e. $(5^2)^3 = 5^5$

   f. $x^3 + x^3 = 2x^3$

SUPPLEMENT TO §2.1

1. Functions f and h are defined below. Use them to answer the given questions.

$f(x) = 3x - 7$

x	$y = h(x)$
-7	-3
0	64
1	64
3	-4
5	0
10	3

a. Evaluate $f(-3)$

b. Solve $f(x) = 11$

c. Evaluate $h(3)$

d. Evaluate $h(-7)$

e. Evaluate $h(-3)$

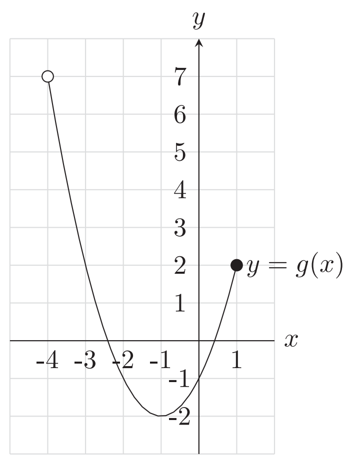
f. If $h(x) = 0$, then what does x have to equal?

g. State the domain of h.

h. State the range of h.

SUPPLEMENT TO §2.2

1. Function g is defined below. Use it to answer the given questions.



a. Evaluate $g(0)$

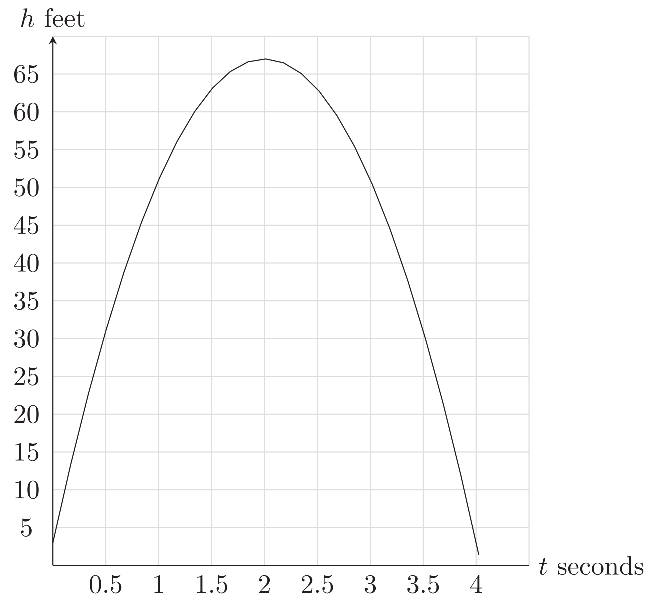
b. Evaluate $g(-3)$

c. If $g(x) = 2$, then what does x have to equal?

d. State the domain of g.

e. State the range of g.

2. A baseball batter hits a ball into the air. The height (in feet) of the baseball t seconds after the batter hits the ball is given by the function f in the following graph. Use the graph to answer the following questions.



a. Estimate $f(3)$ and interpret.

b. If $f(t) = 20$, then $t \approx$ ______________ .
Interpret.

c. When will the ball be at a height of 50 feet?

d. What are the domain and range of this function? Interpret each based on this application.

3. Let the functions f, g, and h be defined as follows:

$$f(x) = x^2 - 1,\ g(x) = 3x - 1, \text{ and } h(x) = 2x - 6.$$

Use these to answer the following questions.

a. Find $f(-4)$.

b. Is the "-4" in $f(-4)$ an input or an output?

c. Is "$f(-4)$" an input or an output?

d. If $h(3) = 0$, what point do you know is on the graph of $y = h(x)$?

e. If $f(-3) = 8$, what point do you know is on the graph of $y = f(x)$?

f. Determine which function must have the point (2,3) on its graph.

g. Determine which function must have the point $(-2, -7)$ on its graph.

h. Solve $g(x) = 7$. State a conclusion using set notation in a complete sentence.

i. Solve $h(x) = 0$. State a conclusion using set notation in a complete sentence.

j. For what value of x will $f(x) = 8$? Try and find the two solutions, even if you don't quite know how to do the algebra. State a conclusion using set notation in a complete sentence.

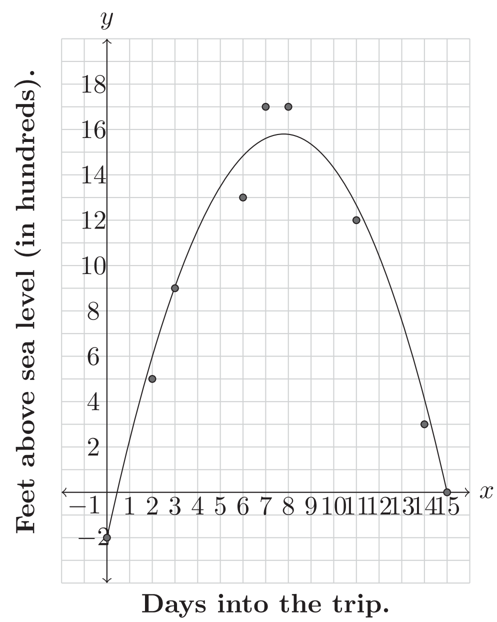
k. Fix and explain what is wrong with the underlined part of each of the following.

   i. $\underline{f(x)} = (3)^2 - 1$

   ii. $g(2) = \underline{3x - 1}$

   iii. $h(a) = \underline{2x - 6}$

4. The following graph of $y = S(d)$ documents a person's approximate height above sea level, S, during a mountain climb d days into the trip. Each big dot represents a journal entry with the day and their elevation d days into the trip. However, notice every journal entry point is *not* on the parabola since the parabola is only an approximation to their actual elevations. Use this information and the given graph to perform the following exercises.



a. Evaluate $S(4)$ and interpret it in context of the problem. Approximate to the nearest tenth if necessary.

b. Estimate the solutions to $S(x) = 15$ and interpret them in context of the problem. Approximate to the nearest tenth if necessary.

c. State the domain of the function S in interval notation. What does the domain represent in context of the problem?

d. State the range of the function S in interval notation. Approximate to the nearest tenth if necessary. What does the range represent in context of the problem?

e. Is the graph of the parabola a reasonable way to describe the hiker's journey?

SUPPLEMENT TO §2.4

1. State whether the rate is positive, negative, or zero for the given scenario.

   a. The rate at which a child's height changes with respect to time.

   b. The rate at which an elderly person's height changes with respect to time.

   c. The rate at which a middle aged person's height changes with respect to time.

   d. The rate at which a pond's water level changes during a drought with respect to time.

   e. The rate at which a pond's water level changes during the rainy season with respect to time.

   f. The rate at which a healthy person's heart beats while they are at rest with respect to time.

2. Begin by stating which variable is the independent vairable and which is the dependent variable. Then interpret the slope of each of the following formulas as a rate of change in practical terms. Make sure you include the unit of the slope in your interpretation.

   a. The temperature in Mathville on November 5, 2013 is given by the formula

   $$T = -2.1t + 54$$

   where T represents the temperature in degrees Fahrenheit and t represents the time (in minutes) since $9:00$am.

   b. The number of calories a runner burns is given by the formula

   $$C = 15t$$

   where C represents the number of calories burned and t represents the time (in minutes) spent running.

c. The average price of a new laptop computer is given by the formula

$$A = -\frac{50}{3}t + 1200$$

where A represents the average price, in dollars, of a new laptop computer and t represent the number of months since December 2006.

d. The number of dollars a professional blogger is paid is given by the formula

$$A = 25p$$

where p represents the number of posts they create.

3. Elijah, Logan, and Savannah each go out for separate walks. The following graph, table, and formula describe the number of kilometers, D, that Elijah, Logan, and Savannah have walked in t hours respectively.

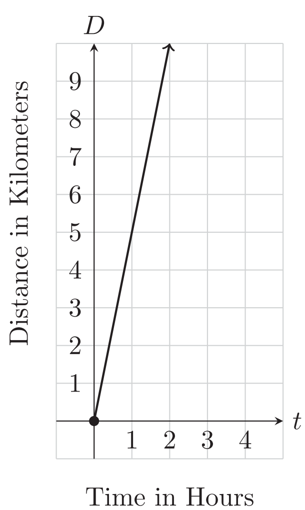


Figure 1: Elijah's distance walked.

t (hours)	D (km)
0	0
2	9
4	18
6	27

Table 1: Logan's distance walked.

The distance Savannah walked is given by the formula

$$D = \frac{13}{4}t.$$

Given this information, who walks fastest and who walks slowest? How do you know?

4. For the following equations, state what the slope and y-intercept are and then use that slope and y-intercept to graph the solutions to the equation in the space provided. Label the y-intercept and label the graph with its corresponding equation. You will need to choose an appropriate scale for the grid on the coordinate plane.

   a. $y = 6x + 30$

   b. $y = -50x + 250$

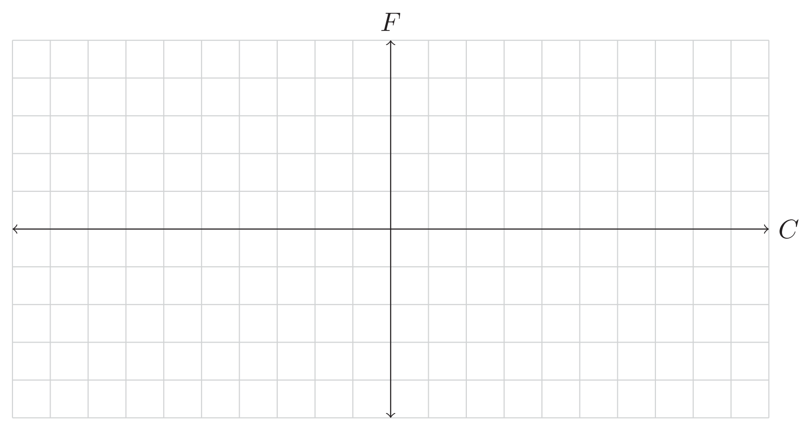
   c. $y = \frac{1}{12}x - 4$

   d. $y = -\frac{25}{4}x + 25$

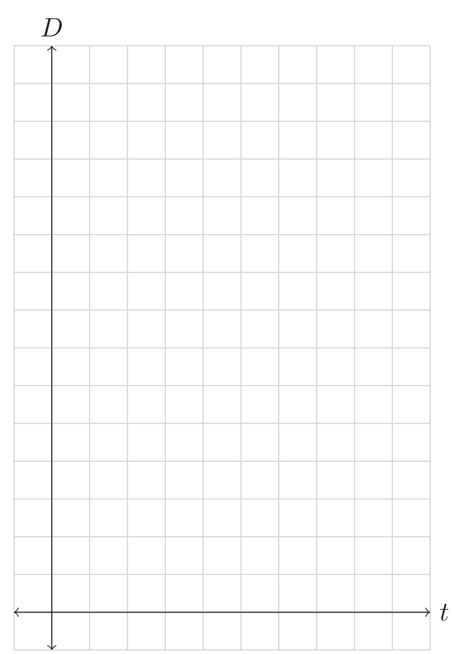
5. The linear equation

$$F = \frac{9}{5}C + 32$$

models the temperature in degrees Fahrenheit, F, given the temperature in degrees Celsius, C. **Begin by stating which variable is the independent variable and which is the dependent variable.** Then graph the equation in the provided coordinate plane. You will need to set up an appropriate scale on the coordinate plane for your graph to fit properly. Label the C- and F-intercepts and the equation of the line.



6. You're leaving Eugene from a Ducks game and driving back home to Portland. Eugene is about 104 miles from Portland and the traffic keeps you driving an average of 52 miles per hour. Model an equation which gives the distance, D, you are from home t hours after leaving Eugene and then graph that equation in the provided coordinate plane. You will need to set up an appropriate scale on the coordinate plane for your graph to fit properly. Label the t- and D-intercepts and the equation of the line.



SUPPLEMENT TO §2.5

1. Determine the equations for the lines graphed in the following figures.

a.

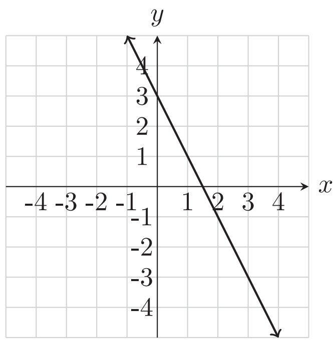


Figure 1

b.

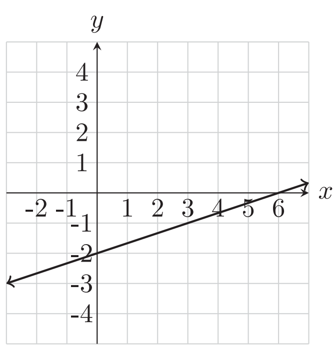


Figure 2

c.

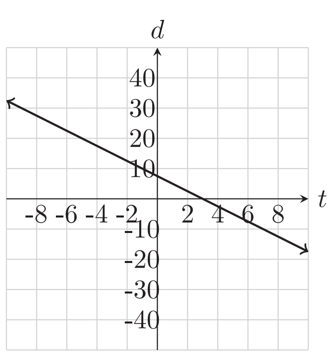


Figure 3

d.

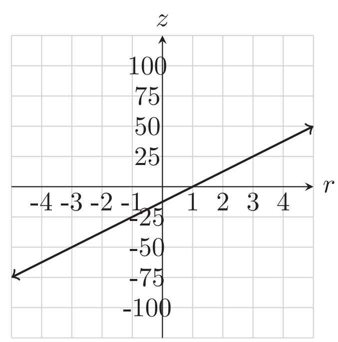


Figure 4

e.

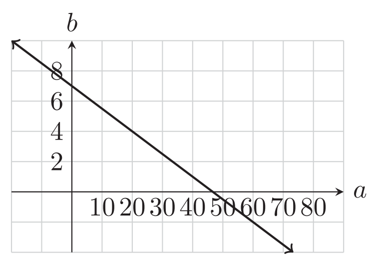


Figure 5

f.

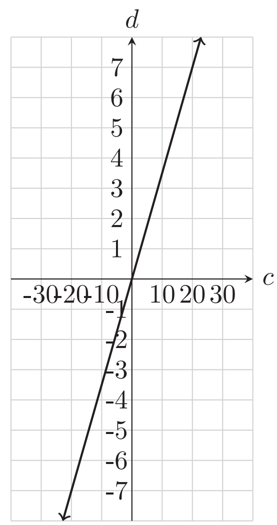


Figure 6

2. The population of a suburb of Portland was 12,500 in 1990. The population of the suburb has been increasing at an average rate of 750 people per year.

   a. Write a linear equation that gives the town population as dependent on the year using P to represent the population of the Portland suburb t years since 1990.

   b. Use the linear equation found in part (a) to predict the suburb's population in 2005. Show all of your work and then state a conclusion using a complete sentence.

3. A storage tank at a production factory holds 250 gallons of a liquid chemical at the beginning of a day. Over the course of the day the chemical is used for production purposes at a constant rate. Two hours after the production begins there are 210 gallons remaining.

   a. Determine the linear equation which models the amount of chemical remaining in the tank as dependent on the number of hours of production time during this day. Be sure to define any variable you use.

   b. Use the linear equation you found in part (a) to determine the number of hours of production which will result in the tank being half full.

4. The life span of an insect can be modified by the temperature of the environment. Assume that the relationship between temperature of the environment, in degrees Celsius, and life span of the fruit fly, in days, is linear. Suppose a population of fruit flies has a life span of 80 days at a temperature of 10 degrees and a life span of 50 days at a temperature of 20 degrees.

   a. Write a linear relationship between the temperature and the life span with temperature as the independent variable and life span as the dependent variable.

   b. What is the life span at a temperature of 25 degrees?

   c. At what temperature is the life span 92 days?

5. Savannah worked 12 hours one week and earned \$137.40. The next week she worked 17 hours and earned \$194.65.

   a. Write a linear equation that gives Savannah's weekly wages depending on the number of hours she worked that week.

   b. If Savannah works 15 hours in a week, how much does she make?

   c. If Savannah earns \$240.45 in a week, how many hours does she work?

SUPPLEMENT TO §7.1

If we want to estimate $\sqrt{10}$, we need to find the nearest integers below and above the 10 that are perfect squares (so the square root can be found). The nearest perfect square integers to 10 are 9 and 16. So, since 10 is between 9 and 16 (i.e., $9 < 10 < 16$) we know that $\sqrt{10}$ will be between $\sqrt{9}$ and $\sqrt{16}$ (i.e., $\sqrt{9} < \sqrt{10} < \sqrt{16}$). Since we can simplify $\sqrt{9}$ and the $\sqrt{16}$ to 3 and 4 respectively it must be that $\sqrt{10}$ is between 3 and 4 (i.e., $3 < \sqrt{10} < 4$).

1. Use your calculator to estimate $\sqrt{10}$. Is it in fact between 3 and 4?

2. Fill-in the blanks with integers. Verify using your calculator.

   a. $\sqrt{19}$: Since _______ $< 19 <$ _______ , we know that _______ $< \sqrt{19} <$ _______ . Without a calculator, I estimate that $\sqrt{19} \approx$ _______ .

   Now check your reasoning with a calculator. What is the approximation of $\sqrt{19}$ to the nearest hundredth according to your calculator?

   b. $\sqrt{3.2}$: Since _______ $< 3.2 <$ _______ , we know that _______ $< \sqrt{3.2} <$ _______ .

   Without a calculator, I estimate that $\sqrt{3.2} \approx$ _______ .

   Now check your reasoning with a calculator. What is the approximation of $\sqrt{3.2}$ to the nearest hundredth according to your calculator?

SUPPLEMENT TO §7.3

1. Write the simplified exact form of the following radical expressions or state that the expression is already simplified. Check your work by using your calculator to approximate the original and simplified expressions.

   a. $\sqrt{12}$

   b. $\sqrt{75}$

   c. $\sqrt{30}$

   d. $\sqrt{18}$

   e. $\sqrt{50}$

   f. $\sqrt{200}$

2. Write the simplified exact form of the following expressions. If there are two simplifications contained in the expression, show both.

   a. $\dfrac{3-\sqrt{180}}{12}$

   b. $\dfrac{4-8\sqrt{6}}{4}$

a. $\dfrac{5 \pm 3}{-2}$

c. $\dfrac{6 \pm \sqrt{20}}{2}$

b. $\dfrac{-3 \pm \sqrt{36}}{2}$

SUPPLEMENT TO §8.1

1. The stopping distance of a car is proportional to the square of its speed before slamming on the brakes. That means that the function

$$D(v) = kv^2$$

gives the stopping distance, D, in feet, where v is the speed of the car before braking, in miles per hour. Further, k is a number that depends on how good the tires are and the road conditions. For a car with good tires on a good road, $k \approx 0.05$, so

$$D(v) = 0.05v^2.$$

a. Find $D(60)$ and interpret your answer.

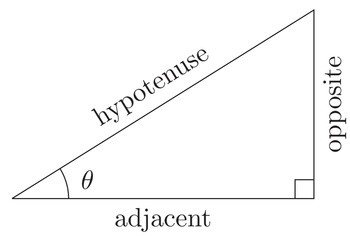
b. Find $D(70)$ and interpret your answer.

c. Using your answers above, what is the ratio $\dfrac{D(70)}{D(60)}$ and what does it mean in context of the story?

d. A neighbor's basketball rolls out into the road 30 feet in front of you. There is no room to swerve to avoid it! What is the maximum speed you could be going and still not hit the ball if you slam on the brakes?

Right Triangle Trigonometry:

Consider a right triangle, one of whose acute angles (an angle whose measure is less than 90°) is θ (the Greek letter theta). The three sides of the triangle are the called the hypotenuse (across from the right angle), the side opposite θ, and the side adjacent to θ.



Ratios of a right triangle's three sides are used to define the six trigonometric functions: sine, cosine, tangent, cosecant, secant, and cotangent. These six functions are abbreviated sin, cos, tan, csc, sec, and cot, respectively.

Right Triangle Definition of Trigonometric Functions

Let θ be an acute angle of a right triangle. The six trigonometric functions of θ are defined as follows.

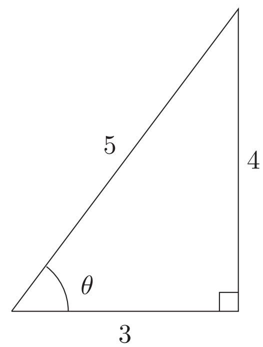
$$\sin(\theta) = \frac{\text{opp}}{\text{hyp}} \qquad \cos(\theta) = \frac{\text{adj}}{\text{hyp}} \qquad \tan(\theta) = \frac{\text{opp}}{\text{adj}}$$

$$\csc(\theta) = \frac{\text{hyp}}{\text{opp}} \qquad \sec(\theta) = \frac{\text{hyp}}{\text{adj}} \qquad \cot(\theta) = \frac{\text{adj}}{\text{opp}}$$

The abbreviations "opp," "adj," and "hyp" stand for length of the "opposite," "adjacent," and "hypotenuse" respectively, as seen in the diagram of the right triangle above. Note that the ratios in the second row are the reciprocals of the ratios in the first row. That is:

$$\csc(\theta) = \frac{1}{\sin(\theta)} \qquad \sec(\theta) = \frac{1}{\cos(\theta)} \qquad \cot(\theta) = \frac{1}{\tan(\theta)}$$

EXAMPLE: Evaluate the six trigonometric functions of the angle θ shown in the given right triangle.



Solutions: Using adj = 3, opp = 4, and hyp = 5, you can write the following:

$$\sin(\theta) = \frac{\text{opp}}{\text{hyp}} = \frac{4}{5} \qquad \cos(\theta) = \frac{\text{adj}}{\text{hyp}} = \frac{3}{5} \qquad \tan(\theta) = \frac{\text{opp}}{\text{adj}} = \frac{4}{3}$$

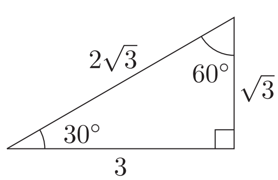
$$\csc(\theta) = \frac{\text{hyp}}{\text{opp}} = \frac{5}{4} \qquad \sec(\theta) = \frac{\text{hyp}}{\text{adj}} = \frac{5}{3} \qquad \cot(\theta) = \frac{\text{adj}}{\text{opp}} = \frac{3}{4}$$

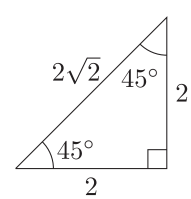
■

Some people like to use the mnemonic device SOH-CAH-TOA to remember the relationships. The SOH reminds us that sine outputs the opposite over the hypotenuse, the CAH reminds us that cosine outputs the adjacent over the hypotenuse, and the TOA reminds us that tangent outputs the opposite over the adjacent.

EXERCISES:

2. The angles 30°, 45°, and 60° occur frequently in trigonometry. Use the given triangles to fill out Table 1, giving both the exact form and approximation (rounded to the hundredths).

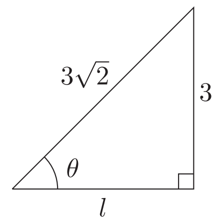




θ	$\sin(\theta)$	$\cos(\theta)$	$\tan(\theta)$	$\csc(\theta)$	$\sec(\theta)$	$\cot(\theta)$
30°						
60°						
45°						

Table 1

EXAMPLE: Find the output of sine, cosine, and tangent for the angle θ shown.



Solutions: We can use the Pythagorean Theorem to find the length of the side adjacent to θ:

$$\begin{aligned} 3^2 + l^2 &= \left(3\sqrt{2}\right)^2 \\ 9 + l^2 &= 9 \cdot 2 \\ 9 + l^2 &= 18 \\ l^2 &= 9 \\ l &= \pm\sqrt{9} \\ l &= \pm 3 \end{aligned}$$

Since l represents a length, we choose the positive value, so $l = 3$. We now use opp$= 3$, adj$= 3$, and hyp$= 3\sqrt{2}$ to evaluate the sine, cosine, and tangent of θ:

$$\begin{aligned} \sin(\theta) &= \frac{\text{opp}}{\text{hyp}} \\ &= \frac{3}{3\sqrt{2}} \\ &= \frac{1}{\sqrt{2}} \\ &= \frac{1}{\sqrt{s}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \\ &= \frac{\sqrt{2}}{2} \end{aligned}$$

$$\begin{aligned} \cos(\theta) &= \frac{\text{adj}}{\text{hyp}} \\ &= \frac{3}{3\sqrt{2}} \\ &= \frac{1}{\sqrt{2}} \\ &= \frac{1}{\sqrt{s}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \\ &= \frac{\sqrt{2}}{2} \end{aligned}$$

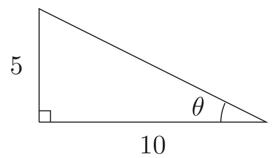
$$\begin{aligned} \tan(\theta) &= \frac{\text{opp}}{\text{adj}} \\ &= \frac{3}{3} \\ &= 1 \end{aligned}$$

■

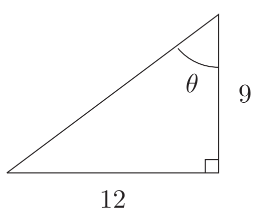
EXERCISES:

3. Find the outputs of the sine, cosine, and tangent functions given the angle θ from the given figure.

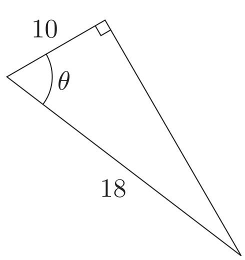
a.



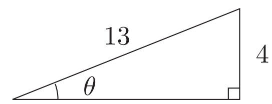
b.



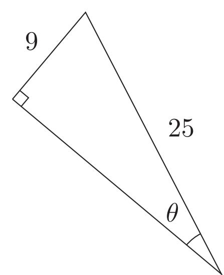
c.



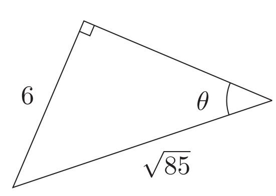
d.



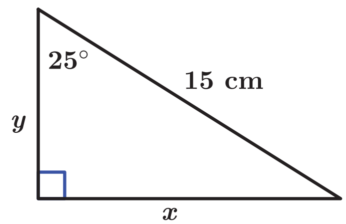
e.



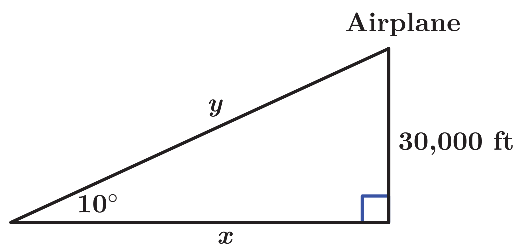
f.



4. Find the lengths of the sides of the triangle (x and y) accurate to 3 decimal places.



5. An airplane is flying at an altitude of 30,000 feet. When the airplane is at a 10° angle of elevation from the airport, it will start to receive radar signals from the airport's landing system. How far away, measured along the ground as x, is the plane from the airport when it receives the radar signals? How far away, measured by line of sight as y, is the plane from the airport when it receives the radar signals? Answer both questions accurate to the nearest hundred feet.



SUPPLEMENT TO §8.3

1. A company's profit can be modeled by the function $P(t) = t^2 - 6t + 17$ which outputs the profit (in thousands of dollars) given a number of years, t, since 2005.

   a. Find and interpret $P(3)$.

   b. Find and interpret $P(0)$.

2. Determine whether the equation/function is linear, quadratic, or another kind of equation/function.

   a. $5x^2 + 2y = 2$

   b. $5x + 2y = 2x$

   c. $y = \sqrt{2}x + 1$

   d. $f(x) = x$

   e. $x = -1$

   f. $\pi x\sqrt{7}y = 3^2$

   g. $y = 4(x-1) + 2(x-3)$

   h. $y = \dfrac{3}{x} + 4x$

   i. $h(r) = r^3 + r^2$

   j. $g(x) = (x-1)(x+5)$

   k. $y = \dfrac{4}{5}$

   l. $A(r) = \pi r^2$

3. If the graph of a function f given by $y = f(x)$ is symmetric about the line $x = 5$ and the point $(1, -9)$ is on the graph, what other point must be on the graph of f? Explain your reasoning.

4. If the graph of a quadratic function g given by $y = g(x)$ has the points $(-3, 6)$ and $(7, 6)$ on it, what is the x-value of the vertex? Explain your reasoning.

5. If a parabola has vertex $(-6, 17)$, use the information given and symmetry to fill in the missing values in the following table.

x	-2		2	
y	18	18	21	21

6. An artist, Michael, sells 100 prints over one year for \$10 each. Michael, who is an amateur mathematician and statistician, did some research and found out that for each price increase of \$2 per print, the sales drop by 5 prints per year. The revenue, R, from selling the prints is given by

$$R(x) = (100 - 5x)(2x + 10)$$

where $100 - 5x$ is the number of prints sold, $2x + 10$ is the cost of each print, and x is the number of \$2 price increases he has done.

a. At what price should Michael sell his prints to maximize the revenue? How many prints will he sell at this price?

b. Find the horizontal intercepts of $y = R(x)$ and interpret their meaning in the context of the problem.

7. During an infrequent rainstorm, the L.A. River collects all the runoff rainwater from the storm and channels it out to sea. The depth of water in the L.A. River, D, in feet, is a function of the time t, in hours, since the storm ended. The function is given by $D(t) = -t^2 + 2t + 8$.

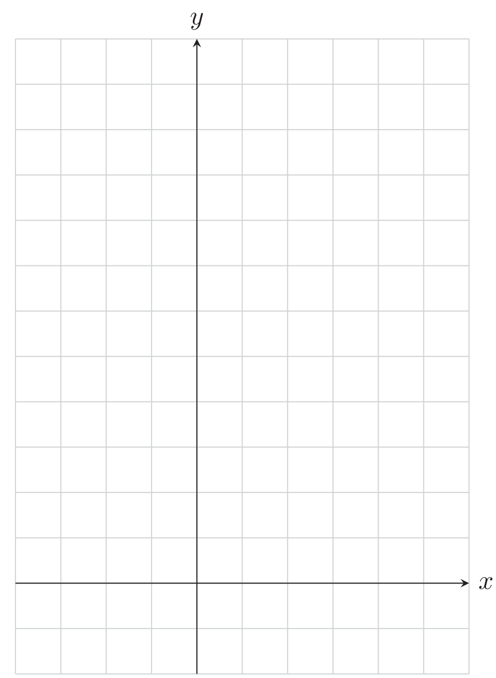
   a. Evaluate and interpret $D(3)$.

   b. Find and interpret the vertical-intercept.

   c. Find and interpret the horizontal-intercepts.

   d. At what time will the water be deepest? What is that maximum depth?

   e. Graph $y = D(t)$ on its domain.



   f. Realistically, what is the range of D and what does it mean?

8. A trebuchet is a French catapult originally used to launch large projectiles long distances. North of England a man named Hew Kennedy built a full-size one to hurl random objects. Seriously, look it up online after you finish this homework. It's pretty fantastic. Suppose that the height of a piano off the ground, h, in feet, is a function of the horizontal distance along the ground in the direction it is thrown, x, also in feet. Given that $h = f(x) = -0.002x^2 + 0.6x + 60$, answer the following questions.

   a. Find the vertex. Explain what the vertex means in context of the situation.

   b. If I want to crush a Volkswagen bug with the piano, how far from the trebuchet should I park it?

9. A young child takes a shot at a 10 foot high basketball hoop. The function

$$h(t) = -16t^2 + 16t + 4$$

models the height of the ball, in feet, t seconds after the kid takes his shot. How long will it take for the ball to reach 10 feet in height?

ANSWERS TO SUPPLEMENT §1.5**:**

1. The height of the triangle is approximately 15.9 inches.

2. The temperature is approximately 88°F.

3. The radius of the circle is approximately 0.9 cm.

ANSWERS TO SUPPLEMENT §1.6**:**

1. a. $x^2 + x^3 = x^5$ is *false.*
   There is no way to combine $x^2 + x^3$ because $x^2 + x^3 = x \cdot x + x \cdot x \cdot x$ while $x^5 = x \cdot x \cdot x \cdot x \cdot x$.

   b. $3^2 \cdot 3^4 = 3^6$ is *true.*

   c. $3^2 \cdot 2^3 = 6^5$ is *false.*
   We can add the exponents only when the bases are the same. By order of operations $3^2 \cdot 2^3 = 9 \cdot 8 = 72$ while $6^5 = 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 = 7776$.

   d. $x^2x^3 = 2x^5$ is *false.*
   $x^2 \cdot x^3 = x \cdot x \cdot x \cdot x \cdot x = x^5$ by the definition of exponents.

   e. $\left(5^2\right)^3 = 5^5$ is *false.*
   $\left(5^2\right)^3 = 5^2 \cdot 5^2 \cdot 5^2 = 5^6$ by the definition of exponents.

   f. $x^3 + x^3 = 2x^3$ is *true.*

ANSWERS TO SUPPLEMENT §2.1**:**

1.
   a. $f(-3) = -16$
   b. $x = 6$. That is, the solution set is $\{6\}$.
   c. $h(3) = -4$
   d. $h(-7) = -3$
   e. $h(-3)$ is undefined.
   f. If $h(x) = 0$, then $x = 5$.
   g. The domain of h is $\{-7, 0, 1, 3, 5, 10\}$.
   h. The range of h is $\{-4, -3, 0, 3, 64\}$.

ANSWERS TO SUPPLEMENT §2.2:

1. a. $g(0) = -1$

   b. $g(-3) = 2$

   c. If $g(x) = 2$ then $x = 1$ or $x = -3$.

   d. The domain of g is $(-4, 1] = \{x| -4 < x \leq 1\}$.

   e. The range of g is $[-2, 7) = \{y| -2 \leq y < 7\}$.

2. a. $f(3) \approx 51$ meaning that 3 seconds after the ball is hit it is 51 feet high.

   b. $f(t) = 20$ when t is approximately 0.25 and 3.75 meaning that the ball is 20 feet up 0.25 and 3.75 seconds after being hit.

   c. The ball will be at a height of 50 feet approximately 0.9 and 3.1 seconds after being hit.

   d. The domain is approximately $[0, 4.1]$ meaning that this function, as described by the graph, takes time inputs from zero to 4.1 seconds. The range is approximately $[0, 67]$ meaning that this function, as described by the graph, outputs heights of the ball ranging from 0 to 67 feet above the ground.

3. a. $f(-4) = 15$

   b. The -4 is an input.

   c. $f(-4)$ is an output.

   d. The point $(3, 0)$ is on the graph of $y = h(x)$.

   e. The point $(-3, 8)$ is on the graph of $y = f(x)$.

   f. The graph of $y = f(x)$ has the point $(2, 3)$ on it.

   g. The graph of $y = g(x)$ has the point $(-2, -7)$ on it.

   h. The solution set is $\left\{\frac{8}{3}\right\}$.

   i. The solution set is $\{3\}$.

   j. The solution set is $\{-3, 3\}$.

   k. i. You should write $f(3) = (3)^2 - 1$ to show that $(3)^2 - 1$ is the output of f *when you input 3* into the function f.

      ii. When you input 2 into the function g you need to show that you've done so in the expression as well. That is, write $g(2) = 3(2) - 1$.

      iii. When you input a into the function h you need to show that you've done so in the expression as well. That is, write $h(a) = 2(a) - 6$.

4. a. $S(4) \approx 11.5$. When the hiker is 4 days into the climb, she is 1150 feet above sea level.

   b. x is approximately 6.1 and 9.2 when $S(x) = 15$. The hiker is 1500 feet above sea level twice. Once on the way up, approximately 6.1 days into the journey, and again when going down when approximately 9.2 days into the journey.

   c. The domain is $[0, 15]$ which represents the entire time, in days, that the hiker was on the trail.

   d. The range is approximately $[-2, 15.9]$ which represents the all of the elevations (in hundreds of feet) the hiker reached during her trip. Note that she started 200 feet below sea level.

   e. Answers will differ. The parabola does come close to most of the journal entries given by the points. However, it misses the fact that the hiker spent some time at 1700 feet above sea level (likely enjoying the view!).

ANSWERS TO SUPPLEMENT §2.4:

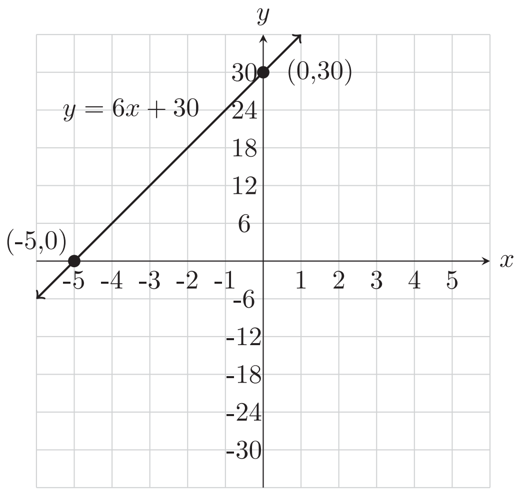
1.
   a. Positive
   b. Negative
   c. Zero
   d. Negative
   e. Positive
   f. Zero

2.
   a. The independent variable is t and the dependent variable is T. The temperature in Mathville was decreasing at a rate of $2.1\ \frac{°\text{F}}{\text{minute}}$.
   b. The independent variable is t and the dependent variable is A. The number of calories the runner burns increases at a rate of $15\ \frac{\text{calories}}{\text{minute}}$.
   c. The independent variable is t and the dependent variable is C. The average price of a new laptop computer is decreasing at a rate of $\frac{50}{3}\frac{\text{dollars}}{\text{month}}$.
   d. The independent variable is p and the dependent variable is A. The amount a professional blogger is paid increases at a rate of $25\ \frac{\text{dollars}}{\text{post}}$.

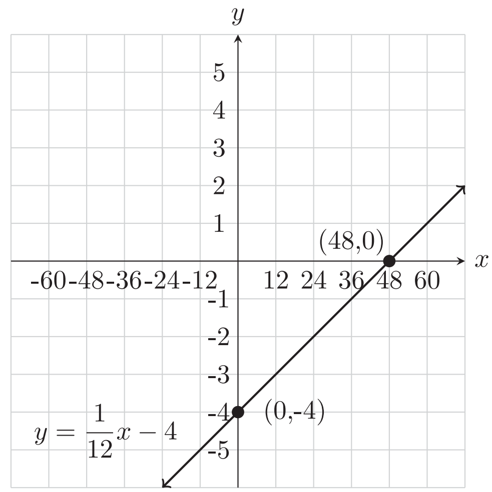
3. Elijah is walking at a rate of 5 kilometers per hour, Logan at a rate of 4.5 kilometers per hour, and Savannah at a rate of 3.25 kilometers per hours, thus Elijah is walking the fastest and Savannah the slowest.

4. The scale you chose for your graphs may be different than the scales shown in these solutions and thus your graphs may appear different. Double check the scales and your points to ensure you are graphing properly if you notice a difference.

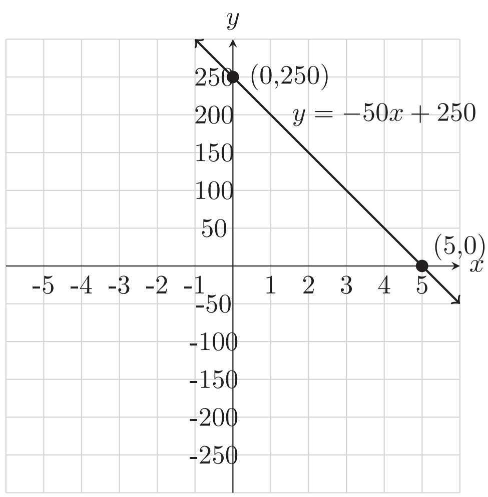
a.



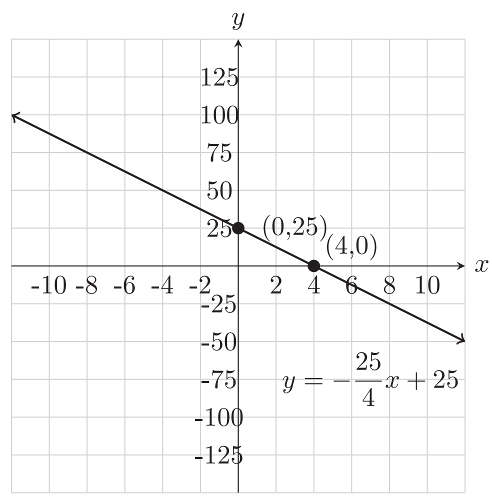
c.



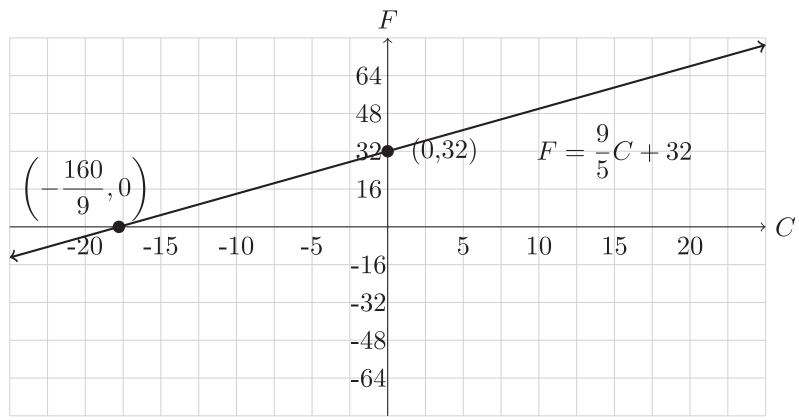
b.



d.

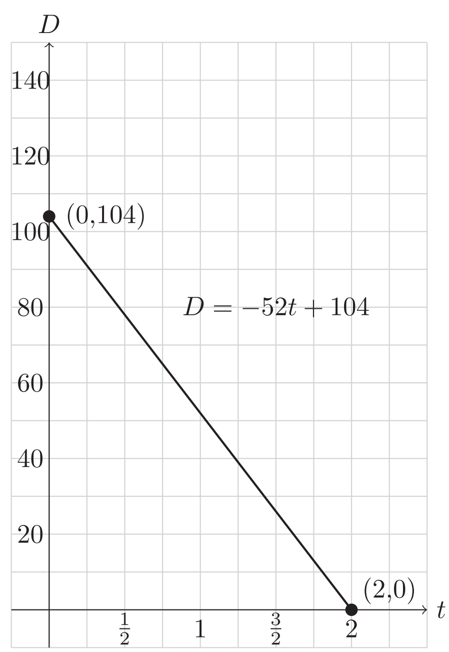


5. The independent variable is C and the dependent variable is F. The F-intercept is $(0, 32)$ and the slope is $m = \frac{9}{5}$. The scale you chose for your graph may be different than the scales shown in this and thus your graph may appear different. Double check the scales and your points to ensure you are graphing properly if you notice a difference.



6. The D-intercept is $(0, 104)$ since you are 104 miles from Portland when you start driving. You're getting closer to Portland so your distance is decreasing at a rate of 52 miles per hour thus the slope is -52. Hence the equation which models your distance from Portland given the number of hours you've been driving is

$$D = -52t + 104.$$



The scale you chose for your graph may be different than the scales shown in this and thus your graph may appear different. Double check the scales and your points to ensure you are graphing properly if you notice a difference.

ANSWERS TO SUPPLEMENT §2.5**:**

1. a. $y = -2x + 3$

   b. $y = \frac{1}{3}x - 2$

   c. $d = -\frac{5}{2}t + \frac{15}{2}$

   d. $z = \frac{25}{2}r - \frac{25}{2}$

   e. $b = -\frac{3}{20}a + 7$

   f. $d = \frac{7}{20}c$

2. a. $P = 750t + 12,500$

   b. According to the model, the population of the suburb will be 23,750 in 2005.

3. a. Let t represent the number of hours of production and let A represent the amount of chemical (in gallons) remaining in the tank.

   $$A = -20t + 250$$

   b. The tank will be half empty after 6 hours and 15 minutes of production.

4. a. Let L represent the life span of the fruit fly (in days) at a temperature of T in degrees Celsius. Then the linear model which describes the fruit fly's lifespan in terms of the environment's temperature in degrees Celsius is given by

   $$L = -3T + 110.$$

   b. The life span is 35 days when the temperature is 25 degrees Celsius.

   c. The temperature is 6 degrees Celsius when the life span is 92 days.

5. a. Let W represent Savannah's weekly pay, in dollars, when she works h hours. Then the linear model of Savannah's wages in terms of hours is given by

   $$W = 11.45h.$$

   b. Savannah earns \$171.75 if she works 15 hours in a week.

   c. Savannah worked 21 hours if she earned \$240.45 for the week.

ANSWERS TO SUPPLEMENT §7.1**:**

1. $\sqrt{10} \approx 3.16$. Yes it is in fact between 3 and 4.

2. a. Since $16 < 19 < 25$, we know that $\sqrt{16} < \sqrt{19} < \sqrt{25}$. Without a calculator, I estimate that $\sqrt{19} \approx 4.2$. According to the calculator $\sqrt{19} \approx 4.36$ which shows my original approximation was a little low.

   b. Since $1 < 3.2 < 4$, we know that $\sqrt{1} < \sqrt{3.2} < \sqrt{4}$. Without a calculator, I estimate that $\sqrt{3.2} \approx 1.8$. According to the calculator $\sqrt{3.2} \approx 1.79$ which shows that may original estimation was just barely too high.

ANSWERS TO SUPPLEMENT §7.3**:**

1. (a) $\sqrt{12} = 2\sqrt{3} \approx 3.5$
   (b) $\sqrt{75} = 5\sqrt{3} \approx 8.7$
   (c) $\sqrt{30} \approx 5.5$ but cannot be simplified.
   (d) $\sqrt{18} = 3\sqrt{2} \approx 4.2$
   (e) $\sqrt{50} = 5\sqrt{2} \approx 7.1$
   (f) $\sqrt{200} = 10\sqrt{2} \approx 14.1$

2. a. The equivalent simplified expression is $\dfrac{1-2\sqrt{5}}{4}$ or $\dfrac{1}{4} - \dfrac{\sqrt{5}}{2}$
   b. The equivalent simplified expression is $1 - 2\sqrt{6}$
   c. The equivalent simplified expression is -4 (for the "$+$") and -1 (for the "$-$").
   d. The equivalent simplified expression is $\dfrac{3}{2}$ (for the "$+$") and $-\dfrac{9}{2}$ (for the "$-$").
   e. The equivalent simplified expression is $3 + \sqrt{5}$ (for the "$+$") and $3 - \sqrt{5}$ (for the "$-$").

ANSWERS TO SUPPLEMENT §8.1**:**

1. a. $D(60) = 180$. This means that a car going 60 miles per hour will take approximately 180 feet to stop if the brakes are slammed.

   b. $D(70) = 245$. This means that a car going 70 miles per hour will take approximately 245 feet to stop if the brakes are slammed.

   c. The ratio $\dfrac{D(70)}{D(60)} = \dfrac{49}{36}$ which means that your stopping distance is seven-fifths longer when you drive 70 miles per hour than when you drive 60 miles per hour.

   d. The maximum speed you can be driving to be able to still stop in 30 feet is $10\sqrt{6}$ or approximately 24.5 miles per hour.

2.

θ	$\sin(\theta)$	$\cos(\theta)$	$\tan(\theta)$	$\csc(\theta)$	$\sec(\theta)$	$\cot(\theta)$
30°	$\frac{1}{2} = .50$	$\frac{3}{2\sqrt{3}} \approx 0.87$	$\frac{\sqrt{3}}{3} \approx 0.58$	2	$\frac{2\sqrt{3}}{3} \approx 1.15$	$\frac{3}{\sqrt{3}} \approx 1.73$
60°	$\frac{3}{2\sqrt{3}} \approx 0.87$	$\frac{1}{2} = .5$	$\frac{3}{\sqrt{3}} \approx 1.73$	$\frac{2\sqrt{3}}{3} \approx 1.15$	2	$\frac{\sqrt{3}}{3} \approx 0.58$
45°	$\frac{1}{\sqrt{2}} \approx 0.71$	$\frac{1}{\sqrt{2}} \approx 0.71$	1	$\sqrt{2} \approx 1.41$	$\sqrt{2} \approx 1.41$	1

Table 1

3. a. $\sin(\theta) = \dfrac{\sqrt{5}}{5}$, $\cos(\theta) = \dfrac{2\sqrt{5}}{5}$, $\tan(\theta) = \dfrac{1}{2}$

   b. $\sin(\theta) = \dfrac{4}{5}$, $\cos(\theta) = \dfrac{3}{5}$, $\tan(\theta) = \dfrac{4}{3}$

   c. $\sin(\theta) = \dfrac{2\sqrt{14}}{9}$, $\cos(\theta) = \dfrac{5}{9}$, $\tan(\theta) = \dfrac{2\sqrt{14}}{5}$

   d. $\sin(\theta) = \dfrac{4}{13}$, $\cos(\theta) = \dfrac{3\sqrt{17}}{13}$, $\tan(\theta) = \dfrac{4\sqrt{17}}{51}$

   e. $\sin(\theta) = \dfrac{9}{25}$, $\cos(\theta) = \dfrac{4\sqrt{34}}{25}$, $\tan(\theta) = \dfrac{9\sqrt{34}}{136}$

   f. $\sin(\theta) = \dfrac{6\sqrt{85}}{85}$, $\cos(\theta) = \dfrac{7\sqrt{85}}{85}$, $\tan(\theta) = \dfrac{6}{7}$

4. The base length, x, is $15\sin(25°) \approx 6.339$ centimeters. The height length, y, is $15\cos(25°) \approx 13.595$ centimeters.

5. The **ground** distance of the plane from airport, x, is $\dfrac{30,000}{\tan(10°)} \approx 170,100$ feet and the **line of sight** distance of the plane from the airport, y, is $\dfrac{30,000}{\sin(10°)} \approx 172,800$ feet when the plane starts receiving radar signals.

ANSWERS TO SUPPLEMENT §8.3

1. a. $P(3) = 8$ meaning that three years after 2005 (2008) the profit of this business was $8,000.

   b. $P(0) = 17$ meaning that in 2005 the profit of this business was $17,000.

2. a. This is a quadratic equation.
   b. This is a linear equation.
   c. This is a linear equation.
   d. This is a linear function.
   e. This is a linear equation.
   f. This is neither a linear nor a quadratic equation.
   g. This is a linear equation.
   h. This is neither a linear nor a quadratic equation.
   i. This is neither a linear nor a quadratic function.
   j. This is a quadratic function.
   k. This is a linear equation.
   l. This is a quadratic function.

3. If the graph of a function is symmetric about the line $x = 5$ and the point $(1, -9)$ is on the graph then the point $(9, -9)$ must also be on the graph since $(1, -9)$ is 4 units from the axis of symmetry so you need to go another 4 units to the right of the axis of symmetry to find the point which is a reflection of $(1, -9)$ across the line $x = 5$.

4. If the graph of a quadratic function has the points $(-3, 6)$ and $(7, 6)$ on it, then the axis of symmetry must be exactly halfway between these two points since they share the same y-value. Thus the x-value of the axis of symmetry is 2. Since the axis of symmetry is a vertical line through the vertex, it follows that the x-value of the vertex is 2.

5.

x	-2	-10	2	-14
y	18	18	21	21

6. a. The vertex of the parabola is $(7.5, 1562.50)$ so the revenue can reach a maximum of \$1562.50 when Michael makes 7.5 \$2 price increases. This means that the cost of of each print will be \$25 (evaluate $2x + 10$ with $x = 7.5$) and he will need to sell 62.5 prints per year (evaluate $100 - 5x$ with $x = 7.5$). Since it is not possible to sell a half of a print, Michael won't be able to earn the maximum possible revenue in one year, but he can aim to sell an average of 62.5 prints per year over several years. Let's take a look at what happens if he sells 62 prints or 63 prints per year:

   If Michael sells 62 prints per year, the revenue is given by $R(7.6)$ where 7.6 is found by solving $100 - 5x = 62$. So, the revenue for selling 62 prints per year is \$1562.40 and the price per unit is \$25.20. If Michael sells 63 prints per year, the revenue is given by $R(7.4)$ where 7.4 is found by solving $100 - 5x = 63$. So, the revenue for selling 63 prints per year is \$1562.40 and the price per print is \$24.80. Can you explain why the revenue is the same for selling 62 and 63 prints?

   b. The horizontal intercepts of $y = R(x)$ are $(20, 0)$ and $(-5, 0)$.

   The horizontal intercept (20,0) means that Michael has increased the price of his print by 2 dollars 20 times and he has a revenue of \$0. This means that no one feels Michael's prints are worth buying at \$50 per print (evaluate $2x + 10$ with $x = 20$) – poor Michael! The horizontal intercept $(-5, 0)$ means that Michael has increased the price of his print by \$2, "-5 times" (i.e., he has decreased the price by \$2, 5 times) and he has a revenue of \$0 because he would be giving his prints away for free (evaluate $2x + 10$ with $x = -5$).

7. For $D(t) = -t^2 + 2t + 8$:

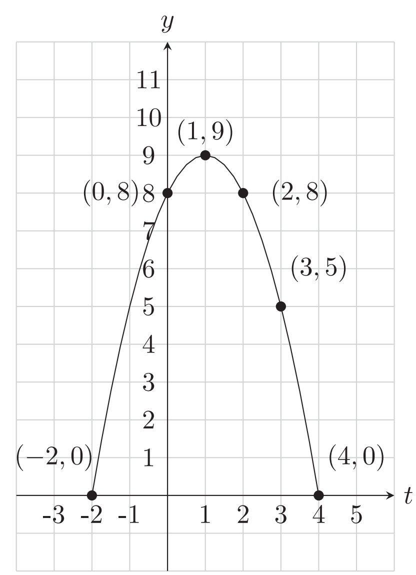
   a. $D(3) = 5$ represents that 3 hours after the storm ends the L.A. River is 5 feet deep.

   b. The y-intercept is $(0, 8)$ meaning that at the end of the storm the river is 8 feet deep.

   c. The t-intercepts are $(-2, 0)$ and $(4, 0)$. These represent that 2 hours before the storm ends and 4 hours after it ends the river is at 0 feet in depth.

   d. We find the maximum of $D(t)$ which occurs at the vertex $(1, 9)$ meaning the water reaches a maximum depth of 9 feet 1 hour after the storm ends.

   e.



Graph of $y = D(t)$.

   f. The range of D is $[0, 9]$ since the L.A. river can have a depth of anywhere between and including 0 to 9 feet of water.

8. For $f(x) = -0.002x^2 + 0.6x + 60$:

   a. The vertex is $(150, 105)$. This represents the maximum height which the piano will reach of 105 feet and telling that it occurs 150 feet horizontally from the launch point.

   b. If you want to crush a Volkswagen with the piano you should park it about 379.13 feet in front of the trebuchet.

9. After attempting to solve the equation $h(t) = 0$ you end up with a complex solution. This implies that, during this particular shot, the ball does not reach 10 feet in height.