Section 8.3
Quadratic Functions and Their Graphs

Heads UP!!!

Many sports involve objects that are thrown, kicked, or hit, and then proceed with no additional force of their own. Such objects are called projectiles.

In this section of your textbook, you will learn to use graphs of quadratic functions to gain a visual understanding of various projectile sports.

<u>First Steps</u>:

- ☐ **Take comprehensive notes** from your instructor's lecture and insert your notes into this section of the *Learning Guide*. Be sure to write down all examples, definitions, and other key concepts. Additional learning resources include the *Lecture Series on DVD*, the *PowerPoints*, and Section 8.3 of your textbook which begins on page 611.
- ☐ Complete the ***Concept and Vocabulary Check*** on page 625 of the textbook.

<u>Guided Practice</u>:

- ☐ Review each of the following ***Solved Problems*** and complete each ***Pencil Problem***.

Objective #1: Recognize characteristics of parabolas.

✔ *Solved Problem #1*

1. True or false: The *vertex* of a parabola is also called the *turning point*.

True

✎ *Pencil Problem #1* ✐

1. True or false: The *vertex* of a parabola is always the minimum point of the parabola.

Objective #2: Graph parabolas in the form $f(x)=a(x-h)^2+k$.

✔ *Solved Problem #2*

2. Graph the quadratic function: $f(x)=-(x-1)^2+4$

Since $a=-1$ is negative, the parabola opens downward.
The vertex of the parabola is $(h,k)=(1,4)$.

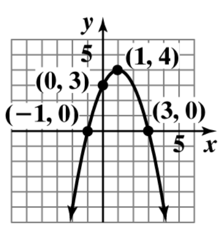
Replace $f(x)$ with 0 to find x–intercepts.

$$0=-(x-1)^2+4$$
$$(x-1)^2=4$$
$$x-1=\pm\sqrt{4}$$
$$x-1=\pm 2$$
$$x-1=2 \quad \text{or} \quad x-1=-2$$
$$x=3 \qquad\qquad x=-1$$

The x–intercepts are -1 and 3.

Set $x=0$ and solve for y to obtain the y–intercept.

$$y=-(0-1)^2+4=3$$



$f(x)=-(x-1)^2+4$

✎ *Pencil Problem #2* ✎

2. Graph the quadratic function: $f(x)=(x-4)^2-1$

Objective #3: Graph parabolas in the form $f(x)=ax^2+bx+c$.

✔ *Solved Problem #3*

3a. Find the vertex for the parabola whose equation is $f(x)=2x^2+8x-1$.

The x–coordinate of the vertex of the parabola is $-\frac{b}{2a}=-\frac{8}{2(2)}=-\frac{8}{4}=-2$, and the y–coordinate of the vertex of the parabola is

$$f\left(-\frac{b}{2a}\right)=f(-2)$$
$$=2(-2)^2+8(-2)-1$$
$$=-9.$$

The vertex is $(-2,-9)$.

✎ *Pencil Problem #3* ✎

3a. Find the vertex for the parabola whose equation is $f(x)=x^2+2x-3$.

3b. Graph the quadratic function $f(x) = -x^2 + 4x + 1$. Use the graph to identify the function's domain and its range.

Since $a = -1$ is negative, the parabola opens downward.

The x–coordinate of the vertex of the parabola is

$$-\frac{b}{2a} = -\frac{4}{2(-1)} = -\frac{4}{-2} = 2.$$

The y–coordinate of the vertex of the parabola is

$$f\left(-\frac{b}{2a}\right) = f(2)$$
$$= -(2)^2 + 4(2) + 1$$
$$= 5.$$

The vertex is $(2, 5)$.

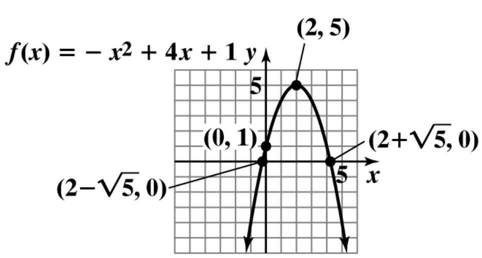
Replace $f(x)$ with 0 to find x–intercepts.

$$0 = -x^2 + 4x + 1$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-4 \pm \sqrt{4^2 - 4(-1)(1)}}{2(-1)}$$
$$x = 2 \pm \sqrt{5}$$
$$x \approx -0.2 \text{ or } x \approx 4.2$$

The x–intercepts are -0.2 and 4.2.

Set $x = 0$ and solve for y to obtain the y–intercept.

$$y = -0^2 + 4 \cdot 0 + 1 = 1$$



Domain: $(-\infty, \infty)$

Range: $(-\infty, 5]$

3b. Graph the quadratic function $f(x) = x^2 + 3x - 10$. Use the graph to identify the function's range.

Objective #4: Determine a quadratic function's minimum or maximum value.

✔ *Solved Problem #4*

4. Consider the quadratic function $f(x) = 4x^2 - 16x + 1000$.

4a. Determine, without graphing, whether the function has a minimum value or a maximum value.

Because $a > 0$, the function has a minimum value.

4b. Find the minimum or maximum value and determine where it occurs.

The minimum value occurs at $-\frac{b}{2a} = -\frac{-16}{2(4)} = 2$.

The minimum of $f(x)$ is $f(2) = 4 \cdot 2^2 - 16 \cdot 2 + 1000$
$= 984$.

4c. Identify the function's domain and its range.

Like all quadratic functions, the domain is $(-\infty, \infty)$.

Because the minimum is 984, the range includes all real numbers at or above 984. The range is $[984, \infty)$.

✎ *Pencil Problem #4* ✐

4. Consider the quadratic function $f(x) = -4x^2 + 8x - 3$.

4a. Determine, without graphing, whether the function has a minimum value or a maximum value.

4b. Find the minimum or maximum value and determine where it occurs.

4c. Identify the function's domain and its range.

Objective #5: Solve problems involving a quadratic function's minimum or maximum value.

✔ *Solved Problem #5*

5a. When an athlete whose event is the shot put releases the shot at an angle of 65°, its path can be modeled by the function $g(x) = -0.04x^2 + 2.1x + 6.1$ in which x is the shot's horizontal distance, in feet, and $g(x)$ is its height, in feet. What is the maximum height of this shot's path?

Because $a < 0$, the function has a maximum value that occurs at $-\frac{b}{2a} = -\frac{2.1}{2(-0.04)} = -\frac{2.1}{-0.08} = 26.25$.

The maximum height is given by
$g(26.25) = -0.04(26.25)^2 + 2.1(26.25) + 6.1 \approx 33.7$

The maximum height of about 33.7 feet occurs at a horizontal distance of 26.25 feet.

✎ *Pencil Problem #5* ✐

5a. You have 50 yards of fencing to enclose a rectangular region. Find the dimensions of the rectangle that maximize the enclosed area. What is the maximum area?

5b. Among all pairs of numbers whose difference is 8, find a pair whose product is as small as possible. What is the minimum product?

Let the two numbers be represented by x and y, and let the product be represented by P.

We must minimize $P = xy$.

Because the difference of the two numbers is 8, then $x - y = 8$.

Solve for y in terms of x.

$$x - y = 8$$
$$-y = -x + 8$$
$$y = x - 8$$

Write P as a function of x.

$$P = xy$$
$$P(x) = x(x-8)$$
$$P(x) = x^2 - 8x$$

Because $a > 0$, the function has a minimum value that occurs at

$$x = -\frac{b}{2a} = -\frac{-8}{2(1)} = 4.$$

Substitute to find the other number.

$$y = x - 8$$
$$y = 4 - 8 = -4$$

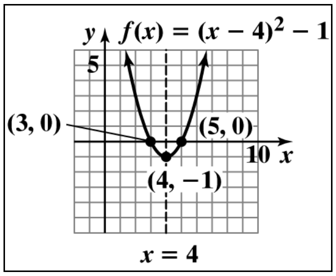
The two numbers are 4 and -4.

The minimum product is $P = xy = (4)(-4) = -16$.

5b. Among all pairs of numbers whose sum is 16, find a pair whose product is as large as possible. What is the maximum product?

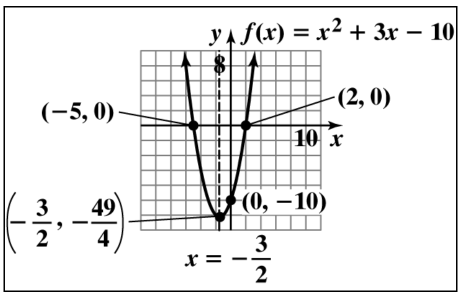
Answers for Pencil Problems *(Textbook Exercise references in parentheses)*:

1. false *(8.3 #41)*

2. 

(8.3 #17)

3a. $(-1,-4)$ *(8.3 #27)*

3b. 

Range: $\left[-\frac{49}{4},\infty\right)$ *(8.3 #29)*

4a. maximum *(8.3 #41a)*

4b. The maximum is 1 at $x=1$. *(8.3 #41b)*

4c. Domain: $(-\infty,\infty)$; Range: $(-\infty,1]$ *(8.3 #41c)*

5a. The maximum area is 156.25 square yards when the dimensions are 12.5 yards by 12.5 yards. *(8.3 #65)*

5b. The maximum product is 64 when the numbers are 8 and 8. *(8.3 #59)*

Homework:

- ☐ **Review the Section 8.3 summary** on page 655 of the textbook.

- ☐ **Insert your homework** into this section of the *Learning Guide*. Show all work neatly and check your answers. Strive to work through difficulties when possible, making note of any exercises where you need additional help. Remember, even if your instructor assigns homework through *MyMathLab*, you should still write out your work.

