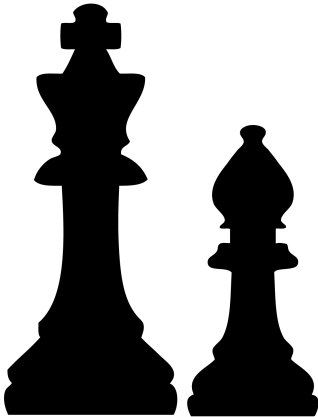


Section 5.7

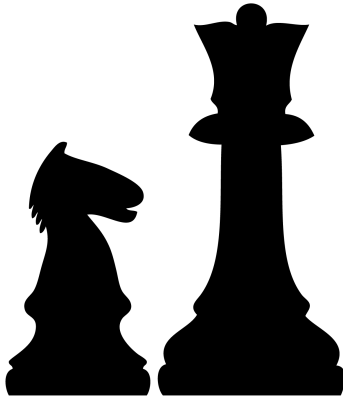
Polynomial Equations and Their Applications



Checkmate !

A *round-robin* tournament is one in which each participant plays every other participant once.

In this section of the textbook, you will solve a quadratic equation in order to determine the number of games necessary in a round-robin chess tournament.



First Steps:

- Take comprehensive notes** from your instructor's lecture and insert your notes into this section of the *Learning Guide*. Be sure to write down all examples, definitions, and other key concepts. Additional learning resources include the *Lecture Series on DVD*, the *PowerPoints*, and Section 5.7 of your textbook which begins on page 381.
- Complete the *Concept and Vocabulary Check* on page 390 of the textbook.

Guided Practice:

- Review each of the following *Solved Problems* and complete each *Pencil Problem*.

Objective #1: Solve quadratic equations by factoring.

✓ **Solved Problem #1**

1a. Solve: $2x^2 - 9x = 5$

$$\begin{aligned} 2x^2 - 9x &= 5 \\ 2x^2 - 9x - 5 &= 0 \\ (2x+1)(x-5) &= 0 \end{aligned}$$

Apply the zero-product principle.

$$\begin{aligned} 2x+1=0 & \quad \text{or} \quad x-5=0 \\ x=-\frac{1}{2} & \quad \quad \quad x=5 \end{aligned}$$

The solution set is $\{-\frac{1}{2}, 5\}$.

 **Pencil Problem #1** 

1a. Solve: $x^2 + 6x = 7$

1b. Solve: $3x^2 = 2x$

$$3x^2 = 2x$$

$$3x^2 - 2x = 0$$

$$x(3x - 2) = 0$$

Apply the zero-product principle.

$$x = 0 \quad \text{or} \quad 3x - 2 = 0$$

$$x = \frac{2}{3}$$

The solution set is $\left\{0, \frac{2}{3}\right\}$.

1b. Solve: $x^2 = 8x$

1c. Solve: $(x-2)(x+3) = 6$

Although the left side of the equation is factored, we cannot use the zero-product principle at this moment because the right side of the equation is not 0.

Begin by multiplying the factors on the left side of the equation. Then subtract 6 from both sides to obtain 0 on the right side.

$$(x-2)(x+3) = 6$$

$$x^2 + x - 6 = 6$$

$$x^2 + x - 12 = 0$$

$$(x+4)(x-3) = 0$$

Apply the zero-product principle.

$$x+4 = 0 \quad \text{or} \quad x-3 = 0$$

$$x = -4 \quad \quad \quad x = 3$$

The solution set is $\{-4, 3\}$.

1c. Solve: $(x-3)(x+8) = -30$

Objective #2: Solve higher-degree polynomial equations by factoring.

 **Solved Problem #2**

2. Solve by factoring: $2x^3 + 3x^2 = 8x + 12$

First, move all terms to one side and obtain zero on the other side.

$$2x^3 + 3x^2 = 8x + 12$$

$$2x^3 + 3x^2 - 8x - 12 = 0$$

Factor by grouping.

$$2x^3 + 3x^2 - 8x - 12 = 0$$

$$(2x^3 + 3x^2) + (-8x - 12) = 0$$

$$x^2(2x + 3) - 4(2x + 3) = 0$$

$$(2x + 3)(x^2 - 4) = 0$$

Continue to factor completely by factoring $x^2 - 4$ as the difference of two squares.

$$(2x + 3)(x^2 - 4) = 0$$

$$(2x + 3)(x + 2)(x - 2) = 0$$

Apply the zero-product principle.

$$2x + 3 = 0 \quad \text{or} \quad x + 2 = 0 \quad \text{or} \quad x - 2 = 0$$

$$x = -\frac{3}{2} \quad x = -2 \quad x = 2$$

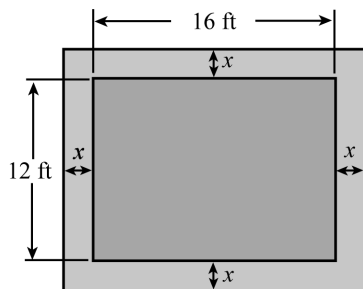
The solution set is $\left\{-2, -\frac{3}{2}, 2\right\}$.

 **Pencil Problem #2**

2. Solve by factoring: $x^3 + 4x^2 - 25x - 100 = 0$

Objective #3: Solve problems using polynomial equations.**✓ Solved Problem #3**

3. A rectangular garden measures 16 feet by 12 feet. A path of uniform width is to be added so as to surround the entire garden. The landscape artist doing the work wants the garden and path to cover an area of 320 square feet.



How wide should the path be?

Let x = the width of the path.

$$(12 + 2x)(16 + 2x) = 320$$

$$192 + 24x + 32x + 4x^2 = 320$$

$$4x^2 + 56x + 192 = 320$$

$$4x^2 + 56x - 128 = 0$$

$$4(x^2 + 14x - 32) = 0$$

$$4(x + 16)(x - 2) = 0$$

$$x + 16 = 0 \quad \text{or} \quad x - 2 = 0$$

$$x = -16 \quad \quad \quad x = 2$$

Reject -16 because width cannot be negative.
The width of the path is 2 feet.

✎ Pencil Problem #3

3. The length of a rectangular sign is 3 feet longer than the width. If the sign's area is 54 square feet, find its length and width.

Answers for Pencil Problems (Textbook Exercise references in parentheses):

1a. $\{-7, 1\}$ (5.7 #3) 1b. $\{0, 8\}$ (5.7 #11) 1c. $\{-3, -2\}$ (5.7 #27) 2. $\{-5, -4, 5\}$ (5.7 #37)

3. The length is 9 feet and the width is 6 feet. (5.7 #71)

Homework:

- Review the Section 5.7 summary** on page 398 of the textbook.
- Insert your homework** into this section of the *Learning Guide*. Show all work neatly and check your answers. Strive to work through difficulties when possible, making note of any exercises where you need additional help. Remember, even if your instructor assigns homework through *MyMathLab*, you should still write out your work.