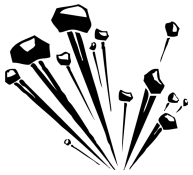
Section 5.3
Greatest Common Factors and Factoring by Grouping

KABOOM!!!!



An explosion caused debris to rise vertically with an initial velocity of 72 feet per second. It is possible to calculate the height of the debris at any given time after the explosion.

In this section of the textbook, we will look at how a polynomial can be used to model this situation, and we will apply the concept of factoring to such polynomials.

First Steps:

- ☐ **Take comprehensive notes** from your instructor's lecture and insert your notes into this section of the *Learning Guide*. Be sure to write down all examples, definitions, and other key concepts. Additional learning resources include the *Lecture Series on DVD*, the *PowerPoints*, and Section 5.3 of your textbook which begins on page 341.
- ☐ Complete the ***Concept and Vocabulary Check*** which begins on page 346 of the textbook.

Guided Practice:

- ☐ Review each of the following ***Solved Problems*** and complete each ***Pencil Problem***.

Objective #1: Factor out the greatest common factor of a polynomial.

✔ *Solved Problem #1*

1a. Factor: $20x^2 + 30x$

Factor out the GCF.

$$20x^2 + 30x = \overbrace{10x}^{\text{GCF}} \cdot 2x + \overbrace{10x}^{\text{GCF}} \cdot 3$$
$$= 10x(2x + 3)$$

Pencil Problem #1

1a. Factor: $y^2 - 4y$

1b. Factor: $9x^4 + 21x^2$

Factor out the GCF.

$$9x^4 + 21x^2 = \overbrace{3x^2}^{\text{GCF}} \cdot 3x^2 + \overbrace{3x^2}^{\text{GCF}} \cdot 7$$
$$= 3x^2(3x^2 + 7)$$

1b. Factor: $x^3 + 5x^2$

1c. Factor: $15x^3y^2 - 25x^4y^3$

Factor out the GCF.

$$15x^3y^2 - 25x^4y^3 = \overbrace{5x^3y^2}^{\text{GCF}} \cdot 3 - \overbrace{5x^3y^2}^{\text{GCF}} \cdot 5xy$$
$$= 5x^3y^2(3 - 5xy)$$

1c. Factor: $4x^2y^3 + 6xy$

1d. Factor: $16x^4y^5 - 8x^3y^4 + 4x^2y^3$

Factor out the GCF.

$$16x^4y^5 - 8x^3y^4 + 4x^2y^3$$
$$= \overbrace{4x^2y^3}^{\text{GCF}} \cdot 4x^2y^2 - \overbrace{4x^2y^3}^{\text{GCF}} \cdot 2xy + \overbrace{4x^2y^3}^{\text{GCF}} \cdot 1$$
$$= 4x^2y^3(4x^2y^2 - 2xy + 1)$$

1d. Factor: $32x^4 + 2x^3 + 8x^2$

Objective #2: Factor out a common factor with a negative coefficient.

✔ *Solved Problem #2*

2. Factor out a common factor with a negative coefficient: $-2x^3 + 10x^2 - 6x$

Factor out the GCF.

$$-2x^3 + 10x^2 - 6x = \overbrace{-2x}^{\text{GCF}} \cdot x^2 \overbrace{-2x}^{\text{GCF}} \cdot (-5x) \overbrace{-2x}^{\text{GCF}} \cdot 3$$
$$= -2x(x^2 - 5x + 3)$$

Pencil Problem #2

2. Factor out a common factor with a negative coefficient: $-5y^2 + 40x$

Objective #3: Factor by grouping.

✔ *Solved Problem #3*

3a. Factor: $x^3 - 4x^2 + 5x - 20$

$$x^3 - 4x^2 + 5x - 20 = \overbrace{x^3 - 4x^2}^{\text{common factor is } x^2} + \overbrace{5x - 20}^{\text{common factor is } 5}$$
$$= x^2(x-4) + 5(x-4)$$
$$= (x-4)(x^2+5)$$

✎ *Pencil Problem #3* ✎

3a. Factor: $x^2 + 7x - 4x - 28$

3b. Factor: $4x^2 + 20x - 3xy - 15y$

$$4x^2 + 20x - 3xy - 15y = \overbrace{4x^2 + 20x}^{\text{common factor is } 4x} + \overbrace{-3xy - 15y}^{\text{common factor is } -3y}$$
$$= 4x(x+5) - 3y(x+5)$$
$$= (x+5)(4x-3y)$$

3b. Factor: $10x^2 - 12xy + 35xy - 42y^2$

Answers for Pencil Problems *(Textbook Exercise references in parentheses)*:

1a. $y(y-4)$ *(5.3 #3)*

1b. $x^2(x+5)$ *(5.3 #5)*

1c. $2xy(2xy^2+3)$ *(5.3 #11)*

1d. $2x^2(16x^2+x+4)$ *(5.3 #9)*

2. $-5(y^2-8x)$ *(5.3 # 29)*

3a. $(x+7)(x-4)$ *(5.3 # 47)*

3b. $(5x-6y)(2x+7y)$ *(5.3 # 55)*

Homework:

- ☐ **Review the Section 5.3 summary** which begins on page 396 of the textbook.
- ☐ **Insert your homework** into this section of the *Learning Guide*. Show all work neatly and check your answers. Strive to work through difficulties when possible, making note of any exercises where you need additional help. Remember, even if your instructor assigns homework through *MyMathLab*, you should still write out your work.

