

Section 5.2

Multiplication of Polynomials

What Are the Best Dimensions for a Box?

Many children get excited about gift boxes of all shapes and sizes, with the possible *exception* of clothing-sized boxes. (I must confess I dreaded boxes of that size.)



While completing the application exercises in this section of the textbook, we will use polynomials to model the dimensions of a box.

We will then apply the concepts of this section to model the area of the box's base and its volume.



First Steps:

- ❑ **Take comprehensive notes** from your instructor's lecture and insert your notes into this section of the *Learning Guide*. Be sure to write down all examples, definitions, and other key concepts. Additional learning resources include the *Lecture Series on DVD*, the *PowerPoints*, and Section 5.2 of your textbook which begins on page 328.
- ❑ Complete the *Concept and Vocabulary Check* on page 337 of the textbook.

Guided Practice:

- ❑ Review each of the following *Solved Problems* and complete each *Pencil Problem*.

Objective #1: Multiply monomials.

✓ *Solved Problem #1*

1. Multiply: $(6x^5y^7)(-3x^2y^4)$

$$\begin{aligned}(6x^5y^7)(-3x^2y^4) &= 6(-3)x^5 \cdot x^2 \cdot y^7 \cdot y^4 \\ &= -18x^{5+2}y^{7+4} \\ &= -18x^7y^{11}\end{aligned}$$

Pencil Problem #1

1. Multiply: $(3x^2)(5x^4)$

Objective #2: Multiply a monomial and a polynomial. **Solved Problem #2**

2. Multiply: $6x^4(2x^5 - 3x^2 + 4)$

$$\begin{aligned} 6x^4(2x^5 - 3x^2 + 4) &= 6x^4 \cdot 2x^5 - 6x^4 \cdot 3x^2 + 6x^4 \cdot 4 \\ &= 12x^9 - 18x^6 + 24x^4 \end{aligned}$$

 **Pencil Problem #2** 

2. Multiply: $5x^3(2x^5 - 4x^2 + 9)$

Objective #3: Multiply polynomials when neither is a monomial. **Solved Problem #3**

3. Multiply: $(3x + 2)(2x^2 - 2x + 1)$

$$\begin{aligned} (3x + 2)(2x^2 - 2x + 1) &= 3x(2x^2 - 2x + 1) + 2(2x^2 - 2x + 1) \\ &= 6x^3 - 6x^2 + 3x + 4x^2 - 4x + 2 \\ &= 6x^3 - 2x^2 - x + 2 \end{aligned}$$

 **Pencil Problem #3** 

3. Multiply: $(x - 3)(x^2 + 2x + 5)$

Objective #4: Use FOIL in polynomial multiplication. **Solved Problem #4**

4a. Multiply: $(x + 5)(x + 3)$

$$\begin{aligned} (x + 5)(x + 3) &= \overbrace{x \cdot x}^{\text{F}} + \overbrace{x \cdot 3}^{\text{O}} + \overbrace{5 \cdot x}^{\text{I}} + \overbrace{5 \cdot 3}^{\text{L}} \\ &= x^2 + 3x + 5x + 15 \\ &= x^2 + 8x + 15 \end{aligned}$$

 **Pencil Problem #4** 

4a. Multiply: $(y + 5)(y - 6)$

4b. Multiply: $(7x+4y)(2x-y)$

$$\begin{aligned}(7x+4y)(2x-y) &= \overbrace{7x \cdot 2x}^{\text{F}} + \overbrace{7x(-y)}^{\text{O}} + \overbrace{4y \cdot 2x}^{\text{I}} + \overbrace{4y(-y)}^{\text{L}} \\ &= 14x^2 - 7xy + 8xy - 4y^2 \\ &= 14x^2 + xy - 4y^2\end{aligned}$$

4b. Multiply: $(7xy+1)(2xy-3)$

Objective #5: Square binomials.

 **Solved Problem #5**

5a. Multiply: $(x+8)^2$

Use the special-product formula

$$(A+B)^2 = A^2 + 2AB + B^2.$$

$$\begin{aligned}(x+8)^2 &= x^2 + 2 \cdot x \cdot 8 + 8^2 \\ &= x^2 + 16x + 64\end{aligned}$$

 **Pencil Problem #5** 

5a. Multiply: $(2x+y)^2$

5b. Multiply: $(2x-6y^4)^2$

Use the special-product formula

$$(A-B)^2 = A^2 - 2AB + B^2.$$

$$\begin{aligned}(2x-6y^4)^2 &= (2x)^2 - 2 \cdot 2x \cdot 6y^4 + (6y^4)^2 \\ &= 4x^2 - 24xy^4 + 36y^8\end{aligned}$$

5b. Multiply: $(y-5)^2$

Objective #6: Multiply the sum and difference of two terms.
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 **Solved Problem #6**

6a. Multiply: $(5x + 7y)(5x - 7y)$

Use the special-product formula

$$(A + B)(A - B) = A^2 - B^2.$$

$$\begin{aligned}(5x + 7y)(5x - 7y) &= (5x)^2 - (7y)^2 \\ &= 25x^2 - 49y^2\end{aligned}$$

 **Pencil Problem #6** 

6a. Multiply: $(y^3 + 2)(y^3 - 2)$

6b. Multiply: $(3x + 2 + 5y)(3x + 2 - 5y)$

Group the first two terms and use the special-product formula $(A + B)(A - B) = A^2 - B^2$.

$$\begin{aligned}(3x + 2 + 5y)(3x + 2 - 5y) &= [(3x + 2) + 5y][(3x + 2) - 5y] \\ &= (3x + 2)^2 - (5y)^2 \\ &= (3x)^2 + 2 \cdot 3x \cdot 2 + 2^2 - (5y)^2 \\ &= 9x^2 + 12x + 4 - 25y^2\end{aligned}$$

6b. Multiply: $(x + y + 3)(x + y - 3)$

Objective #7: Find the product of functions.

<p> Solved Problem #7</p>	<p> Pencil Problem #7 </p>
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7a. Let $f(x) = x - 3$ and $g(x) = x - 7$. Find $(fg)(x)$.

$$\begin{aligned}(fg)(x) &= f(x) \cdot g(x) \\ &= (x-3)(x-7) \\ &= x^2 - 7x - 3x + 21 \\ &= x^2 - 10x + 21\end{aligned}$$

7a. Let $f(x) = x - 2$ and $g(x) = x + 6$. Find $(fg)(x)$.

7b. Let $f(x) = x - 3$ and $g(x) = x - 7$. Find $(fg)(2)$.

$$\begin{aligned}(fg)(x) &= x^2 - 10x + 21 \\ (fg)(2) &= (2)^2 - 10(2) + 21 \\ &= 4 - 20 + 21 \\ &= 5\end{aligned}$$

7b. Let $f(x) = x - 2$ and $g(x) = x + 6$. Find $(fg)(0)$.

Objective #8: Use polynomial multiplication to evaluate functions.

<p> Solved Problem #8</p>	<p> Pencil Problem #8 </p>
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8a. Given $f(x) = x^2 - 5x + 4$,
find and simplify $f(a+3)$.

$$\begin{aligned}f(x) &= x^2 - 5x + 4 \\ f(a+3) &= (a+3)^2 - 5(a+3) + 4 \\ &= a^2 + 6a + 9 - 5a - 15 + 4 \\ &= a^2 + a - 2\end{aligned}$$

8a. Given $f(x) = x^2 - 3x + 7$,
find and simplify $f(a+2)$.

8b. Given $f(x) = x^2 - 5x + 4$,
find and simplify $f(a+h) - f(a)$.

$$\begin{aligned} & f(a+h) - f(a) \\ &= ((a+h)^2 - 5(a+h) + 4) - (a^2 - 5a + 4) \\ &= (a^2 + 2ah + h^2 - 5a - 5h + 4) - (a^2 - 5a + 4) \\ &= a^2 + 2ah + h^2 - 5a - 5h + 4 - a^2 + 5a - 4 \\ &= 2ah + h^2 - 5h \end{aligned}$$

8b. Given $f(x) = x^2 - 3x + 7$,
find and simplify $f(a+h) - f(a)$.

Answers for Pencil Problems (Textbook Exercise references in parentheses):

1. $15x^6$ (5.2 #1)
 2. $10x^8 - 20x^5 + 45x^3$ (5.2 #13)
 3. $x^3 - x^2 - x - 15$ (5.2 #23)
 4a. $y^2 - y - 30$ (5.2 #37) 4b. $14x^2y^2 - 19xy - 3$ (5.2 #47)
 5a. $4x^2 + 4xy + y^2$ (5.2 #59) 5b. $y^2 - 10y + 25$ (5.2 #57)
 6a. $y^6 - 4$ (5.2 #75) 6b. $x^2 + 2xy + y^2 - 9$ (5.2 #85)
 7a. $(fg)(x) = x^2 + 4x - 12$ (5.2 #95a) 7b. $(fg)(0) = -12$ (5.2 #95c)
 8a. $a^2 + a + 5$ (5.2 #99a) 8b. $2ah + h^2 - 3h$ (5.2 #99b)

Homework:

- Review the Section 5.2 summary** on page 396 of the textbook.
- Insert your homework** into this section of the *Learning Guide*. Show all work neatly and check your answers. Strive to work through difficulties when possible, making note of any exercises where you need additional help. Remember, even if your instructor assigns homework through *MyMathLab*, you should still write out your work.