

Section 5.1

Introduction to Polynomials and Polynomial Functions

For a Limited Time Only!

Throughout the textbook so far, you have seen a wide variety of ways that mathematical models can be used to describe the world around us.



In this section, we will look at several such models that work well for a while, but eventually breakdown.

For example, a function that models the weight of a child based on the child's age in months, might work well for a 1-year-old (12 months). But that same function is likely to lead to unreliable results if you attempt to use it to calculate the weight of a 70-year-old (840 months).



First Steps:

- Take comprehensive notes** from your instructor's lecture and insert your notes into this section of the *Learning Guide*. Be sure to write down all examples, definitions, and other key concepts. Additional learning resources include the *Lecture Series on DVD*, the *PowerPoints*, and Section 5.1 of your textbook which begins on page 314.
- Complete the *Concept and Vocabulary Check* on page 323 of the textbook.

Guided Practice:

- Review each of the following *Solved Problems* and complete each *Pencil Problem*.

Objective #1: Use the vocabulary of polynomials.

✓ *Solved Problem #1*

1. Determine the coefficient of each term, the degree of each term, the degree of the polynomial, the leading term, and the leading coefficient of the polynomial $8x^4y^5 - 7x^3y^2 - x^2y - 5x + 11$.

The degree of the polynomial is 9, the leading term is $8x^4y^5$, and the leading coefficient is 8.

Term	Coefficient	Degree
$8x^4y^5$	8	$4 + 5 = 9$
$-7x^3y^2$	-7	$3 + 2 = 5$
$-x^2y$	-1	$2 + 1 = 3$
$-5x$	-5	1
11	11	0

✎ *Pencil Problem #1* ✎

1. Determine the coefficient of each term, the degree of each term, the degree of the polynomial, the leading term, and the leading coefficient of the polynomial $x^3y^2 - 5x^2y^7 + 6y^2 - 3$.

Objective #2: Evaluate polynomial functions. **Solved Problem #2**

2. Find
- $f(2)$
- for the polynomial function

$$f(x) = 4x^3 - 3x^2 - 5x + 6.$$

$$f(x) = 4x^3 - 3x^2 - 5x + 6$$

$$f(2) = 4(2)^3 - 3(2)^2 - 5(2) + 6$$

$$= 4(8) - 3(4) - 5(2) + 6$$

$$= 32 - 12 - 10 + 6$$

$$= 16$$

 **Pencil Problem #2** 

2. Find
- $g(3)$
- for the polynomial function

$$g(x) = 2x^3 - x^2 + 4x - 1.$$

Objective #3: Determine end behavior. **Solved Problem #3**

- 3a. Use the Leading Coefficient Test to determine the end behavior of the graph of
- $f(x) = x^4 - 4x^2$
- .

The leading coefficient is positive and the degree is even. Therefore the graph rises to the left and to the right.

 **Pencil Problem #3** 

- 3a. Use the Leading Coefficient Test to determine the end behavior of the graph of
- $f(x) = -x^4 + x^2$
- .

- 3b. The polynomial function

$f(x) = -0.27x^3 + 9.2x^2 - 102.9x + 400$ models the ratio of students to computers in U.S. public schools x years after 1980.

Use end behavior to determine whether this function could be an appropriate model for computers in the classroom well into the twenty-first century. Explain your answer.

This model would not be appropriate over long time periods. Because the leading coefficient is negative, the graph falls to the right. At some point the ratio would be negative, which is not possible.

- 3b. A herd of 100 elk is introduced to a small island. The number of elk,
- $f(x)$
- , after
- x
- years is modeled by the polynomial function

$$f(x) = -x^4 + 21x^2 + 100.$$

Use the Leading Coefficient Test to determine the graph's end behavior to the right. What does this mean about what will eventually happen to the elk population?

Objective #4: Add polynomials.

<p style="text-align: center;"> Solved Problem #4</p> <p>4a. Add:</p> $(-7x^3 + 4x^2 + 3) + (4x^3 + 6x^2 - 13)$ $(-7x^3 + 4x^2 + 3) + (4x^3 + 6x^2 - 13)$ $= -7x^3 + 4x^2 + 3 + 4x^3 + 6x^2 - 13$ $= \underbrace{-7x^3 + 4x^3}_{-3x^3} + \underbrace{4x^2 + 6x^2}_{+10x^2} + \underbrace{3 - 13}_{-10}$ $= -3x^3 + 10x^2 - 10$	<p style="text-align: center;"> Pencil Problem #4</p> <p>4a. Add:</p> $(-6x^3 + 5x^2 - 8x + 9) + (17x^3 + 2x^2 - 4x - 13)$
<p>4b. Add by aligning vertically:</p> $(7xy^3 - 5xy^2 - 3y) + (2xy^3 + 8xy^2 - 12y - 9)$ $7xy^3 - 5xy^2 - 3y$ $\underline{2xy^3 + 8xy^2 - 12y - 9}$ $9xy^3 + 3xy^2 - 15y - 9$	<p>4b. Add by aligning vertically:</p> $(7x^2y - 5xy) + (2x^2y - xy)$

Objective #5: Subtract polynomials.
--

<p style="text-align: center;"> Solved Problem #5</p> <p>5a. Subtract:</p> $(14x^3 - 5x^2 + x - 9) - (4x^3 - 3x^2 - 7x + 1)$ $(14x^3 - 5x^2 + x - 9) - (4x^3 - 3x^2 - 7x + 1)$ $= (14x^3 - 5x^2 + x - 9) + (-4x^3 + 3x^2 + 7x - 1)$ $= 14x^3 - 5x^2 + x - 9 - 4x^3 + 3x^2 + 7x - 1$ $= 14x^3 - 4x^3 - 5x^2 + 3x^2 + x + 7x - 9 - 1$ $= 10x^3 - 2x^2 + 8x - 10$	<p style="text-align: center;"> Pencil Problem #5</p> <p>5a. Subtract:</p> $(17x^3 - 5x^2 + 4x - 3) - (5x^3 - 9x^2 - 8x + 11)$
---	---

5b. Subtract $-7x^2y^5 - 4xy^3 + 2$
from $6x^2y^5 - 2xy^3 - 8$.

$$\begin{aligned} & (6x^2y^5 - 2xy^3 - 8) - (-7x^2y^5 - 4xy^3 + 2) \\ &= 6x^2y^5 - 2xy^3 - 8 + 7x^2y^5 + 4xy^3 - 2 \\ &= 6x^2y^5 + 7x^2y^5 - 2xy^3 + 4xy^3 - 8 - 2 \\ &= 13x^2y^5 + 2xy^3 - 10 \end{aligned}$$

5b. Subtract $-5a^2b^4 - 8ab^2 - ab$
from $3a^2b^4 - 5ab^2 + 7ab$.

Answers for Pencil Problems (Textbook Exercise references in parentheses):

- 1.** The coefficient of x^3y^2 is 1 and the degree is 5. The coefficient of $-5x^2y^7$ is -5 and the degree is 9.
The coefficient of $6y^2$ is 6 and the degree is 2. The coefficient of -3 is -3 and the degree is 0.
The degree of the polynomial is 9. The leading term is $-5x^2y^7$ and the leading coefficient is -5 . (5.1 #7)
- 2.** 56 (5.1 #15)
- 3a.** the graph falls to the left and to the right (5.1 #25)
- 3b.** The polynomial will fall to the right. The elk population will die out over time. (5.1 #71)
- 4a.** $11x^3 + 7x^2 - 12x - 4$ (5.1 #29)
- 4b.** $9x^2y - 6xy$ (5.1 #33)
- 5a.** $12x^3 + 4x^2 + 12x - 14$ (5.1 #41)
- 5b.** $8a^2b^4 + 3ab^2 + 8ab$ (5.1 #51)

Homework:

- Review the Section 5.1 summary** on page 395 of the textbook.
- Insert your homework** into this section of the *Learning Guide*. Show all work neatly and check your answers. Strive to work through difficulties when possible, making note of any exercises where you need additional help. Remember, even if your instructor assigns homework through *MyMathLab*, you should still write out your work.