

Section 3.2

Problem Solving and Business Applications

Using Systems of Equations

Glittering Gold!

For thousands of years, gold has been considered one of Earth's most precious metals.

One hundred percent pure gold is 24-karat gold, which is too soft to be made into jewelry.

In the United States, most gold jewelry is 14-karat gold, approximately 58% gold.

18-karat gold is 75% gold

and

12-karat gold is 50% gold.

In the Exercise Set of this section of the textbook, you will use systems of equations to determine how much 18-karat gold must be mixed with 12-karat gold to make a 14-karat gold piece of jewelry.



First Steps:

- Take comprehensive notes** from your instructor's lecture and insert your notes into this section of the *Learning Guide*. Be sure to write down all examples, definitions, and other key concepts. Additional learning resources include the *Lecture Series on DVD*, the *PowerPoints*, and Section 3.2 of your textbook which begins on page 194.
- Complete the *Concept and Vocabulary Check* on page 204 of the textbook.

Guided Practice:

- Review each of the following *Solved Problems* and complete each *Pencil Problem*.

Objective #1: Solve problems using systems of equations.

✓ *Solved Problem #1*

- 1a.** The number of calories in one portion of hamburger and fries and two portions of fettuccine Alfredo is 4240. The number of calories in two portions of hamburger and fries and one portion of fettuccine Alfredo is 3980. Find the number of calories in each of these dishes.

Let x = the number of calories in hamburger and fries.

Let y = the number of calories in fettuccine Alfredo.

The system of equations
$$\begin{cases} x + 2y = 4240 \\ 2x + y = 3980 \end{cases}$$

can be solved by substitution.

✎ *Pencil Problem #1*

- 1a.** One week a computer store sold a total of 36 computers and external hard drives. The revenue from these sales was \$27,710. If computers sold for \$1180 per unit and hard drives for \$125 per unit, how many of each did the store sell?

Solve for x in terms of y : $x + 2y = 4240$

$$x = -2y + 4240$$

Substitute this value into the other equation.

$$2(\overbrace{-2y + 4240}^x) + y = 3980$$

$$-4y + 8480 + y = 3980$$

$$-3y + 8480 = 3980$$

$$-3y = -4500$$

$$y = 1500$$

Back-substitute to find x .

$$x = -2y + 4240$$

$$x = -2(1500) + 4240$$

$$x = 1240$$

There are 1240 calories in hamburger and fries and 1500 calories in fettuccine Alfredo.

1b. You inherited \$5000 with the stipulation that for the first year the money had to be invested in two funds paying 9% and 11% annual interest. How much did you invest at each rate if the total interest earned for the year was \$487?

Let x = the amount invested at 9%.

Let y = the amount invested at 11%.

The system of equations $\begin{cases} x + y = 5000 \\ 0.09x + 0.11y = 487 \end{cases}$

can be solved by substitution.

Solving for y in terms of x gives $y = -x + 5000$, which can be substituted into the other equation.

$$0.09x + 0.11(\overbrace{-x + 5000}^y) = 487$$

$$0.09x - 0.11x + 550 = 487$$

$$-0.02x + 550 = 487$$

$$-0.02x = -63$$

$$x = 3150$$

Back-substitute to find y .

$$y = -x + 5000$$

$$y = -(3150) + 5000$$

$$y = 1850$$

There was \$3150 invested at 9% and \$1850 invested at 11%.

1b. You invested \$7000 in two accounts paying 6% and 8% annual interest. If the total interest earned for the year was \$520, how much was invested at each rate?

- 1c.** With the current, a motorboat can travel 84 miles in 2 hours. Against the current, the same trip takes 3 hours. Find the average rate of the boat in still water and the average rate of the current.

Let x = the rate of the motorboat in still water.
Let y = the rate of the current.

	Rate	× Time	= Distance
Trip with the Current	$x + y$	2	$2(x + y)$
Trip against the Current	$x - y$	3	$3(x - y)$

This gives, $2(x + y) = 84$
 $3(x - y) = 84$

This system simplifies to: $x + y = 42$
 $x - y = 28$

This system can be solved by addition.

$$\begin{array}{r} x + y = 42 \\ x - y = 28 \\ \hline 2x = 70 \\ x = 35 \end{array}$$

Back-substitute to find y .

$$\begin{array}{r} x + y = 42 \\ 35 + y = 42 \\ y = 7 \end{array}$$

The rate of the motorboat in still water is 35 miles per hour and the rate of the current is 7 miles per hour.

- 1c.** When a small plane flies with the wind, it can travel 800 miles in 5 hours. When the plane flies in the opposite direction, against the wind, it takes 8 hours to fly the same distance. Find the rate of the plane in still air and the rate of the wind.

Objective #2:

Use functions to model revenue, cost, and profit, and perform a break-even analysis.

✓ Solved Problem #2

- 2.** A company that manufactures running shoes has a fixed cost of \$300,000. Additionally, it costs \$30 to produce each pair of shoes. The shoes are sold at \$80 per pair.
- 2a.** Write the cost function, C , of producing x pairs of running shoes.

$$C(x) = \overbrace{300,000}^{\text{fixed costs}} + \overbrace{30x}^{\text{\$30 per pair}}$$

Pencil Problem #2

- 2.** A company that manufactures small canoes has a fixed cost of \$18,000. Additionally, it costs \$20 to produce each canoe. The selling price is \$80 per canoe.
- 2a.** Write the cost function, C , of producing x canoes.

2b. Write the revenue function, R , from the sale of x pairs of running shoes.

$$R(x) = \overbrace{80x}^{\text{\$80 per pair}}$$

2c. Determine the break-even point. Describe what this means.

$$\text{The system is } \begin{cases} y = 300,000 + 30x \\ y = 80x \end{cases}$$

The break-even point is where $R(x) = C(x)$.

$$\begin{aligned} R(x) &= C(x) \\ 80x &= 300,000 + 30x \\ 50x &= 300,000 \\ x &= 6000 \end{aligned}$$

Back-substitute to find y : $y = 80x$

$$\begin{aligned} y &= 80(6000) \\ y &= 480,000 \end{aligned}$$

The break-even point is (6000, 480,000).

This means the company will break even when it produces and sells 6000 pairs of shoes. At this level, both revenue and costs are \$480,000.

2b. Write the revenue function, R , from the sale of x canoes.

2c. Determine the break-even point. Describe what this means.

Answers for Pencil Problems (Textbook Exercise references in parentheses):

1a. 22 computers and 14 hard drives (3.2 #11) **1b.** \$2000 at 6% and \$5000 at 8% (3.2 #13)

1c. The plane's rate in still air is 130 miles per hour and the rate of the wind is 30 miles per hour. (3.2 #27)

2a. $C(x) = 18,000 + 20x$ (3.2 #47a) **2b.** $R(x) = 80x$ (3.2 #47b)

2c. Break-even point: (300, 24,000). Which means when 300 canoes are produced the company will break-even with cost and revenue at \$24,000. (3.2 #47c)

Homework:

- Review the Section 3.2 summary** on page 244 of the textbook.
- Insert your homework** into this section of the *Learning Guide*. Show all work neatly and check your answers. Strive to work through difficulties when possible, making note of any exercises where you need additional help. Remember, even if your instructor assigns homework through *MyMathLab*, you should still write out your work.