Section 3.1
Systems of Linear Equations in Two Variables

Land Lines *versus* Cell Phones!

The number of cell phone customers increases each year, while the number of land-line customers continues to decrease. Recently, the number of cell phone customers surpassed the number of land-line customers.

In this section of the textbook, we will use a system of linear equations to look at the point where the number of customers of each type was the same.

First Steps:

- ☐ **Take comprehensive notes** from your instructor's lecture and insert your notes into this section of the *Learning Guide*. Be sure to write down all examples, definitions, and other key concepts. Additional learning resources include the *Lecture Series on DVD*, the *PowerPoints*, and Section 3.1 of your textbook which begins on page 178.
- ☐ Complete the ***Concept and Vocabulary Check*** on page 189 of the textbook.

Guided Practice:

- ☐ Review each of the following ***Solved Problems*** and complete each ***Pencil Problem***.

Objective #1: Determine whether an ordered pair is a solution of a system of linear equations.

✔ *Solved Problem #1*

1. Determine if the ordered pair $(-7,-2)$ is a solution of the system: $\begin{cases} 2x+5y=-24 \\ 3x-5y=14 \end{cases}$

To determine if $(-7,-2)$ is a solution to the system, replace x with -7 and y with -2 in both equations.

$$\begin{aligned} 2x+5y &= -24 \\ 2(-7)+5(-2) &= -24 \\ -24 &= -24, \text{ true} \end{aligned}$$

$$\begin{aligned} 3x-5y &= 14 \\ 3(-7)-5(-2) &= 14 \\ -11 &= 14, \text{ false} \end{aligned}$$

The ordered pair does not satisfy both equations, so it is not a solution to the system.

✎ *Pencil Problem #1* ✎

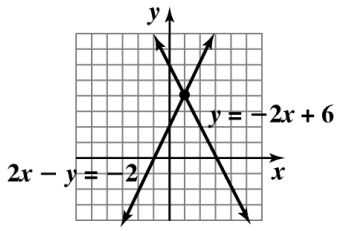
1. Determine if the ordered pair $(5,-3)$ is a solution of the system: $\begin{cases} y=2x-13 \\ 4x+9y=-7 \end{cases}$

Objective #2: Solve systems of linear equations by graphing.

✔ *Solved Problem #2*

2. Solve by graphing: $\begin{cases} y = -2x + 6 \\ 2x - y = -2 \end{cases}$

Graph both equations.



The intersection is $(1,4)$.

The solution set is $\{(1,4)\}$.

✎ *Pencil Problem #2*

2. Solve by graphing: $\begin{cases} 3x - 2y = 6 \\ x - 4y = -8 \end{cases}$

Objective #3: Solve systems of linear equations by substitution.

✔ *Solved Problem #3*

3. Solve by the substitution method: $\begin{cases} 3x + 2y = 4 \\ 2x + y = 1 \end{cases}$

Solve $2x + y = 1$ for y.

$$2x + y = 1$$
$$y = -2x + 1$$

Substitute: $3x + 2y = 4$

$$3x + 2(\overbrace{-2x + 1}^{y}) = 4$$
$$3x - 4x + 2 = 4$$
$$-x + 2 = 4$$
$$-x = 2$$
$$x = -2$$

Find y.

$$y = -2x + 1$$
$$y = -2(-2) + 1$$
$$y = 5$$

The solution is $(-2,5)$.

The solution set is $\{(-2,5)\}$.

✎ *Pencil Problem #3*

3. Solve by the substitution method: $\begin{cases} x + y = 6 \\ y = 2x \end{cases}$

Objective #4: Solve systems of linear equations by addition.

✔ *Solved Problem #4*

4. Solve by the addition method: $\begin{cases} 4x-7y=-16 \\ 2x+5y=9 \end{cases}$

Multiply the second equation by –2.

$$\begin{array}{r} 4x-7y=-16 \\ -4x-10y=-18 \\ \hline -17y=-34 \\ y=2 \end{array}$$

Back-substitute to find x:

$$\begin{aligned} 2x+5y&=9 \\ 2x+5(2)&=9 \\ 2x+10&=9 \\ 2x&=-1 \\ x&=-\tfrac{1}{2} \end{aligned}$$

The solution is $\left(-\frac{1}{2},2\right)$.

The solution set is $\left\{\left(-\frac{1}{2},2\right)\right\}$.

✎ *Pencil Problem #4*

4. Solve by the addition method: $\begin{cases} 6x-y=-5 \\ 4x-2y=6 \end{cases}$

Objective #5: Select the most efficient method for solving a system of linear equations.

✔ *Solved Problem #5*

5. True or false: The addition method gives exact solutions for systems of equations.

True

✎ *Pencil Problem #5*

5. True or false: The substitution method is a good choice for solving a system of equations if one of the equations has a variable on one side by itself.

Objective #6: Identify systems that do not have exactly one ordered-pair solution.

✔ *Solved Problem #6*

6a. Solve the system: $\begin{cases} 5x-2y=4 \\ -10x+4y=7 \end{cases}$

Multiply the first equation by 2, and then add the equations.

$$\begin{array}{r} 10x-4y=8 \\ -10x+4y=7 \\ \hline 0=15 \end{array}$$

Since there are no pairs (x,y) for which 0 will equal 15, the system is inconsistent and has no solution.
The solution set is $\varnothing$ or $\{\ \}$.

✎ *Pencil Problem #6*

6a. Solve the system: $\begin{cases} x-2y=4 \\ 2x-4y=5 \end{cases}$

6b. Solve the system: $\begin{cases} x = 4y - 8 \\ 5x - 20y = -40 \end{cases}$

Substitute $4y - 8$ for x in the second equation.

$$5x - 20y = -40$$
$$5(\overbrace{4y - 8}^{x}) - 20y = -40$$
$$20y - 40 - 20y = -40$$
$$-40 = -40$$

Since $-40 = -40$ for all values of x and y, the system is dependent.

The solution set is $\{(x, y) | x = 4y - 8\}$ or $\{(x, y) | 5x - 20y = -40\}$.

6b. Solve the system: $\begin{cases} x = 3y - 1 \\ 2x - 6y = -2 \end{cases}$

Answers for Pencil Problems *(Textbook Exercise references in parentheses)*:

1. The ordered is a solution to the system. *(3.1 #5)*

2. $\{(4,3)\}$ *(3.1 #11)*

3. $\{(2,4)\}$ *(3.1 #25)*

4. $\{(-2,-7)\}$ *(3.1 #49)*

5. true *(3.1 #59-81)*

6a. $\varnothing$ or $\{\ \}$ *(3.1 #33)*

6b. $\{(x, y) | x = 3y - 1\}$ or $\{(x, y) | 2x - 6y = -2\}$ *(3.1 #67)*

Homework:

- ☐ **Review the Section 3.1 summary** which begins on page 243 of the textbook.
- ☐ **Insert your homework** into this section of the *Learning Guide*. Show all work neatly and check your answers. Strive to work through difficulties when possible, making note of any exercises where you need additional help. Remember, even if your instructor assigns homework through *MyMathLab*, you should still write out your work.