Section 2.4
Linear Functions and Slope

Read for Life!

Is there a relationship between literacy and child mortality?

As the percentage of adult females who are literate increases, does the mortality of children under age five decrease? Data from the United Nations indicates that this is, indeed, the case.

In this section of the textbook, you will be given a graph for which each point represents one country. You will use the concept of slope to see how much the mortality rate decreases for each 1% increase in the literacy rate of adult females in a country.

First Steps:

☐ **Take comprehensive notes** from your instructor's lecture and insert your notes into this section of the *Learning Guide*. Be sure to write down all examples, definitions, and other key concepts. Additional learning resources include the *Lecture Series on DVD*, the *PowerPoints*, and Section 2.4 of your textbook which begins on page 135.

☐ Complete the *Concept and Vocabulary Check* on page 150 of the textbook.

Guided Practice:

☐ Review each of the following *Solved Problems* and complete each *Pencil Problem*.

Objective #1: Use intercepts to graph a linear function in standard form.

✔ *Solved Problem #1*	✎ *Pencil Problem #1*✎
1. Graph: $3x - 2y = 6$	**1a.** Graph: $6x - 2y = 12$

Find the *x*–intercept by setting $y = 0$.

$3x - 2y = 6$

$3x - 2(0) = 6$

$3x = 6$

$x = 2$

Find the *y*–intercept by setting $x = 0$.

$3x - 2y = 6$

$3(0) - 2y = 6$

$-2y = 6$
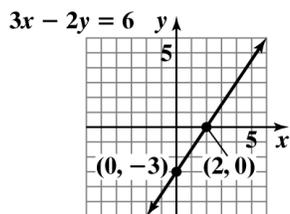
$y = -3$

Plot the points and draw the line that passes through them.

$3x - 2y = 6$

$(0, -3)$ $(2, 0)$

Objective #2: Compute a line's slope.

✔ *Solved Problem #2*	✎ *Pencil Problem #2*✎
2. Find the slope of the line passing through $(4, -2)$ and $(-1, 5)$.	**2.** Find the slope of the line passing through $(-7, 1)$ and $(-4, -3)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{5 - (-2)}{-1 - 4}$$

$$= \frac{7}{-5}$$

$$= -\frac{7}{5}$$

Objective #3: Find a line's slope and y-intercept from its equation.

✔ *Solved Problem #3*	✎ *Pencil Problem #3*✎
3. Give the slope and the y-intercept for the line whose equation is $8x - 4y = 20$.	**3.** Give the slope and the y-intercept for the line whose equation is $5x + 3y = 15$.

First, convert the equation to slope-intercept form by solving the equation for y.

$$8x - 4y = 20$$

$$-4y = -8x + 20$$

$$\frac{-4y}{-4} = \frac{-8x + 20}{-4}$$

$$y = 2x - 5$$

In this form, the coefficient of x is the line's slope and the constant term is the y-intercept.

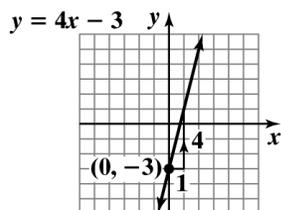The slope is 2 and the y-intercept is –5.

Objective #4: Graph linear functions in slope-intercept form.

✔ *Solved Problem #4*	✎ *Pencil Problem #4*✎
4. Graph the line whose equation is $y = 4x - 3$.	**4.** Graph the line whose equation is $y = -2x + 1$.

Begin by plotting the *y*-intercept of –3.
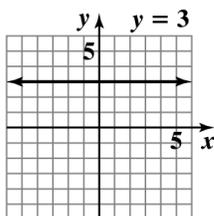Then use the slope of 4 to plot more points.

$y = 4x - 3$

Objective #5: Graph horizontal or vertical lines.

✔ *Solved Problem #5*	✎ *Pencil Problem #5*✎
5a. Graph $y = 3$ in the rectangular coordinate system.	**5a.** Graph $f(x) = -2$ in the rectangular coordinate system.

$y = 3$ is a horizontal line.

$y = 3$

5b. Graph $x = -3$ in the rectangular coordinate system.

5b. Graph $x = 5$ in the rectangular coordinate system.
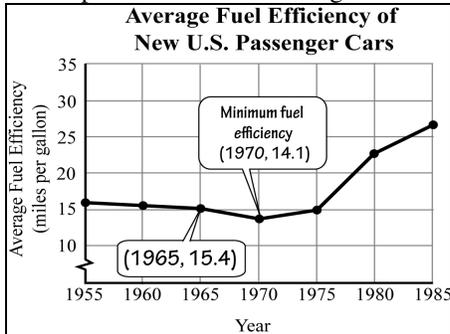
$x = -3$ is a vertical line.

$x = -3$

Objective #6: Interpret slope as rate of change.

✔ *Solved Problem #6*

6. Use the graph to find the slope of the line segment for the period from 1965 through 1970.

Average Fuel Efficiency of New U.S. Passenger Cars

Minimum fuel efficiency (1970, 14.1)

(1965, 15.4)

Average Fuel Efficiency (miles per gallon)

Year

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{14.1 - 15.4}{1970 - 1965} = \frac{-1.3}{5} = -0.26$$

✏ *Pencil Problem #6*

6. When the literacy rate for adult females in a country is 0%, the infant mortality rate is 254 (per thousand). When the literacy rate for adult females is 60%, the infant mortality rate is 110.
Use the points $(0, 254)$ and $(60, 110)$ to compute the slope of the line that represents this relationship.

Objective #7: Find a function's average rate of change.

✔ *Solved Problem #7*

7. When a person receives a drug injected into a muscle, the concentration of the drug in the body, measured in milligrams per 100 milliliters, is a function of the time elapsed after the injection, measured in hours. The graph below represents such a function, where x represents hours after the injection and $f(x)$ is the drug's concentration at time x. Find the average rate of change in the drug's concentration between 1 hour and 3 hours.

(3, 0.05)

(y = fx)

(1, 0.03)

Drug Concentration in the Blood (milligrams per 100 milliliters)
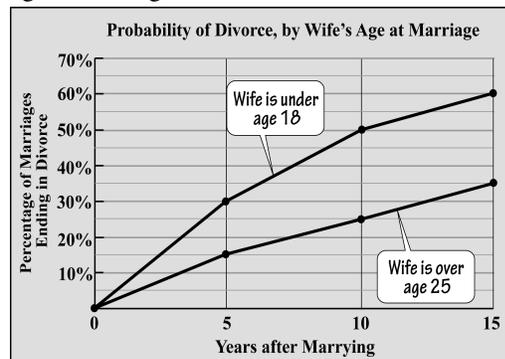
Time (hours)
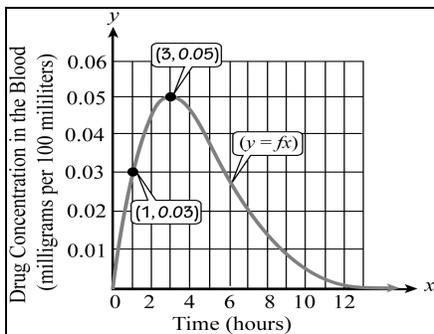
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0.05 - 0.03}{3 - 1} = \frac{0.02}{2} = 0.01$$

The average rate of change between 1 hour and 3 hours is 0.01. This means that the drug's concentration is increasing at an average rate of 0.01 milligram per 100 milliliters per hour.

✏ *Pencil Problem #7*

7. Divorce rates are typically higher for couples who marry in their teens. The graph shows the percentage of marriages ending in divorce by wife's age at marriage.

Probability of Divorce, by Wife's Age at Marriage

Wife is under age 18

Wife is over age 25

Percentage of Marriages Ending in Divorce

Years after Marrying

Find the average rate of change in the percentage of marriages ending in divorce between five and ten years of marriage in which the wife is under 18 when she marries.

Objective #8: Use slope and *y*-intercept to model data.

✔ Solved Problem #8

8. The amount of carbon dioxide in the atmosphere, measured in parts per million, has been increasing as a result of the burning of oil and coal. The buildup of gases and particles is believed to trap heat and raise the planet's temperature. The pre-industrial concentration of atmospheric carbon dioxide was 280 parts per million. The scatter plot shows the average atmospheric concentration of carbon dioxide, in parts per million, for selected years from 1950 through 2009. Use the line in the scatter plot to find a function in the form $C(x) = mx + b$ that models average atmospheric concentration of carbon dioxide, $C(x)$, in parts per million, x years after 1950. Round m to two decimal places.
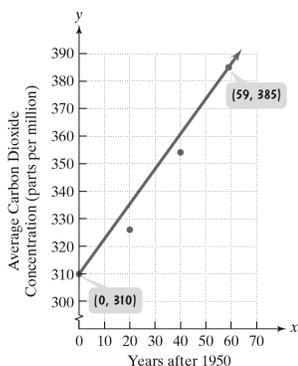
Use the line segment passing through $(59, 385)$ and $(0, 310)$ to obtain a model.

We need values for *m*, the slope, and *b*, the *y*-intercept.
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{385 - 310}{59 - 0} = \frac{75}{59} \approx 1.27$$

The point $(0, 310)$ gives us the *y*-intercept of 310.

Thus, $C(x) = mx + b$
$$C(x) = 1.27x + 310$$

✏ Pencil Problem #8 ✏

8. Recall from *Pencil Problem 6* that when the literacy rate for adult females in a country is 0%, the infant mortality rate is 254 (per thousand). When the literacy rate for adult females is 60%, the infant mortality rate is 110. Use the *y*-intercept and the slope that you found for the line passing through the points $(0, 254)$ and $(60, 110)$ to write a linear function that models child mortality, $f(x)$, per thousand, for children under five in a country where *x*% of adult women are literate.

Answers for Pencil Problems *(Textbook Exercise references in parentheses)*:

1. *(2.4 #5)*
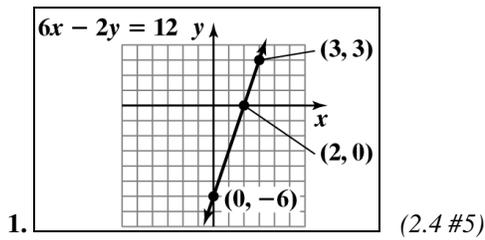
2. $m = -\dfrac{4}{3}$ *(2.4 #21)*

3. $m = -\dfrac{5}{3}$ and $b = 5$ *(2.4 #47b)*

4. *(2.4 #31)*

5a. *(2.4 #51)* **5b.** *(2.4 #57)*
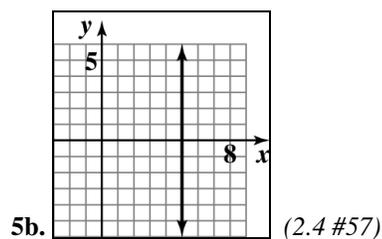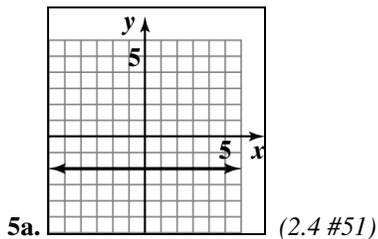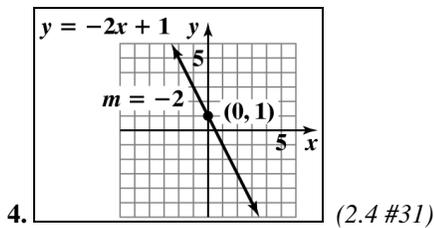
6. $m = -2.4$ *(2.4 #83b)*

7. $m = 4$; There is an average increase of 4% of marriages ending in divorce per year. *(2.4 #81)*

8. $f(x) = -2.4x + 254$ *(2.4 #83c)*

Homework:

☐ **Review the Section 2.4 summary** which begins on page 168 of the textbook.

☐ **Insert your homework** into this section of the *Learning Guide*. Show all work neatly and check your answers. Strive to work through difficulties when possible, making note of any exercises where you need additional help. Remember, even if your instructor assigns homework through *MyMathLab*, you should still write out your work.