Section 7.6 - Solving Quadratic Equations by Factoring

A quadratic equation has an x^2 in it.

Examples: $x^2+3x-2=7$

$2x^2-5x+7=-11$

$ax^2+bx+c=0$ ← standard form
x^2 term, then x then constant.

We need new methods to solve quadratics. We are going to learn 3 ways:

Factoring zero product property section 7.6	Square Root method section 10.1	Quadratic Formula section 10.3

Solving by Factoring:

If we have $x^2-7x+10=0$, we can't isolate the x. But if we factor it we can:

$$(x-5)(x-2)=0$$

Now we need the Zero Product Property:

$4\cdot3=12$ $\quad$ $4\cdot0=0$ $\quad$ $8\cdot0=0$
$0\cdot3=0$ $\quad$ $8\cdot6=48$ $\quad$ $6\cdot0=0$

The only way to get a zero is if one number is zero

$(x-5)(x-2)=0$

Two factors multiplied to equal zero. One or the other must equal zero.

$x-5=0$ or $x-2=0$

Now we can solve

$+5 \quad +5 \qquad +2 \quad +2$

$x=5$ or $x=2$

$\{2, 5\}$ solution set

Quadratics often have 2 solutions

Examples:

4. $(x-3)(x+8)=0$

$x-3=0$ or $x+8=0$ mini-equations

$+3 \quad +3 \qquad -8 \quad -8$

$x=3 \qquad x=-8$

8. $8(x-5)(3x+11)=0$

$8=0$ or $x-5=0$ or $3x+11=0$

doesn't give a solution

$+5 \quad +5 \qquad -11 \quad -11$

$x=5$

$\frac{3x}{3}=\frac{-11}{3}$

$x=-\frac{11}{3}$

$\{-\frac{11}{3}, 5\}$

2. $x(x-3)=0$

$x=0$ or $x-3=0$

$+3 \quad +3$

$x=0$ or $x=3$

$\{0,3\}$

Factor First:

10. $x^2+5x+6=0$

$(x+2)(x+3)=0$

$x+2=0$ or $x+3=0$

$-2 \quad -2 \qquad -3 \quad -3$

$x=-2$ or $x=-3$

$\{-3,-2\}$

check:

$(-3)^2+5(-3)+6 \stackrel{?}{=} 0$

$9-15+6 \stackrel{?}{=} 0$

$0=0$ ✓

$(-2)^2+5(-2)+6 \stackrel{?}{=} 0$

$4-10+6 \stackrel{?}{=} 0$

$0=0$ ✓

12. $x^2+x-42=0$

$(x+7)(x-6)=0$

$x+7=0$ or $x-6=0$

$x=-7$ or $x=6$

$\{-7,6\}$

14. $x^2 + 7x = 18$ ← we need a zero on the right to use the zero product property

$$\begin{array}{r} x^2+7x=18 \\ -18\ \ -18 \\ \hline x^2+7x-18=0 \end{array}$$

$(x+9)(x-2)=0$

$x+9=0$ or $x-2=0$

$x=-9$ or $x=2$

$\{-9, 2\}$

18. $x^2-6x=0$ ← 2 terms so look for the GCF = x

$x(x-6)=0$

$x=0$ or $x-6=0$

$x=0$ or $x=6$

$\{0, 6\}$

26. $2x^2=-3x$ ← Get a zero first

$2x^2+3x=0$ ← Factor out the GCF

$x(2x+3)=0$

$x=0$ or $2x+3=0$

$\phantom{x=0 \text{ or } 2x}-3\ \ -3$

$\frac{2x}{2}=\frac{-3}{2}$

$x=0$ or $x=-\frac{3}{2}$

$\left\{-\frac{3}{2}, 0\right\}$

28. $x^2+6x+9=0$

$(x+3)(x+3)=0$

$x+3=0$ or $x+3=0$

$x=-3$ or $x=-3$ ← The 2 solutions come out the same

$\{-3\}$

Quadratic equations can have 0, 1 or 2 solutions.

34. $3x^2=x+4$

$3x^2-x-4=0$ use the AC method

$\underline{3x^2+3x}\underline{-4x-4}=0$

$3\cdot -4=-12$

$1\cdot 12$
$2\cdot 6$
$3\cdot -4$

$3x(x+1)-4(x+1)=0$

$(x+1)(3x-4)=0$ Now solve

$x+1=0$ or $3x-4=0$

$+4\ \ +4$

$x=-1$ or $\frac{3x}{3}=\frac{4}{3}$

$x=\frac{4}{3}$

$\{-1,\frac{4}{3}\}$

38. $x^2-25=0$

$(x+5)(x-5)=0$

$x+5=0$ or $x-5=0$

$x=-5$ or $x=5$

$\{-5,5\}$

44. $x(x-3)=18$

It's factored but we don't have a zero.

We need to rearrange this equation.

$x^2-3x=18$

$x^2-3x-18=0$

$(x-6)(x+3)=0$

$x-6=0$ or $x+3=0$

$x=6$ or $x=-3$

$\{-3,6\}$

48. $(x-3)(x+8)=-30$ ← need to get a zero

FOIL

$x^2+8x-3x-24=-30$

$x^2+5x-24=-30$

$\quad +30 \quad +30$

$x^2+5x+6=0$ Factor

$(x+3)(x+2)=0$

$x=-3$ or $x=-2$

$\{-3,-2\}$

52. $y(y+9)=4(2y+5)$

$y^2+9y=8y+20$

$-8y\ -20 \quad -8y \quad -20$

$y^2+y-20=0$

$(y+5)(y-4)=0$

$y=-5$ or $y=4$

$\{-5,4\}$

56. $25w^2 = 80w - 64$

$25w^2 - 80w + 64 = 0$ Perfect squares

$(5w-8)(5w-8) = 0$

$5w - 8 = 0$ same factors

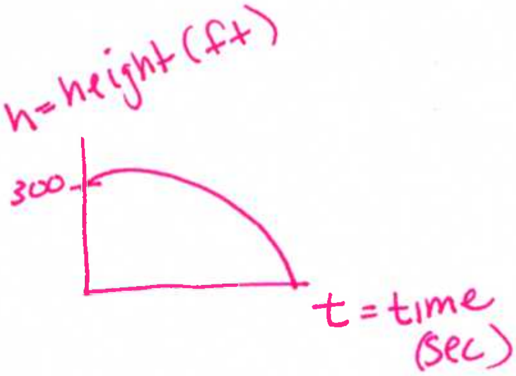
$\quad +8 \quad +8$

$\frac{5w}{5} = \frac{8}{5}$

$w = \frac{8}{5}$

$\left\{ \frac{8}{5} \right\}$

Applications



67. $h = -16t^2 + 20t + 300$

The ground is $h=0$

$0 = -16t^2 + 20t + 300$

$0 = -4(4t^2 - 5t - 75)$ ac method

$0 = -4(4t^2 - 20t + 15t - 75)$

$0 = -4(4t(t-5) + 15(t-5))$

$0 = -4(t-5)(4t+15)$

$t - 5 = 0$ or $4t + 15 = 0$

$\boxed{t = 5}$ or $\frac{4t}{4} = \frac{-15}{4}$

$t = \cancel{-\frac{15}{4}}$ can't have a negative time

$4 \cdot 75 = -300$

$1 \cdot 300$
$2 \cdot 150$
$3 \cdot 100$
$4 \cdot 75$
$5 \cdot 60$
$6 \cdot 50$
$10 \cdot 30$
$12 \cdot 25$
$15 \cdot 20$

The ball hit the ground after 5 seconds.