Section 7.3 - Factoring Trinomials when $a \neq 1$

Recap - Section 7.1
Factoring out the GCF
Factoring by grouping ← need today

7.2 Factoring trinomials when $a = 1$

$x^2 + 7x + 12$
$= (x+3)(x+4)$ ↙ when $a = 1$ ~~go~~ factor directly

when $a \neq 1$ there are 2 methods

- Trial → Error
  - list and check all combinations
- AC method (grouping)
  - many people prefer this method

AC or grouping method

1. Multiply $a \cdot c$
2. find the factors of ac that add up to b
3. replace the middle term with the 2 numbers from step 2
4. factor the 4 terms by grouping

Examples:

2. $3x^2+5x+2$

$3 \cdot 2 = 6$

$1 \cdot 6$

$2 \cdot 3$ $\quad 2+3=5$

$2x+3x$

$= 3x^2+2x+3x+2$ Factor by grouping

$= x(3x+2)+1(3x+2)$

$= (3x+2)(x+1)$

6. $2x^2+19x+35$

$ac = 2 \cdot 35 = 70$

$1 \cdot 70$

$2 \cdot 35$

$5 \cdot 14$ $\quad 5+14=19$

$5x+14x$

$= 2x^2+5x+14x+35$

$= x(2x+5)+7(2x+5)$

$= (2x+5)(x+7)$

check using foil

$(2x+5)(x+7)$

$= 2x^2+14x+5x+35$

$= 2x^2+19x+35$ ✓

18. $3x^2-25x-28$

$ac = 3(-28) = -84$

$1 \cdot 84$

$2 \cdot 42$

$3 \cdot 28$ $\quad 3+(-28)=-25$

$4 \cdot 21$

$6 \cdot 14$

$7 \cdot 12$

↑ complete list

$= 3x^2+3x-28x-28$

$= 3x(x+1)-28(x+1)$

$= (x+1)(3x-28)$

52. $15x^2 - 31xy + 10y^2$

$= \underline{15x^2 - 6xy} \underline{- 25xy + 10y^2}$

$= 3x(5x - 2y) - 5y(5x - 2y)$

$= (5x - 2y)(3x - 5y)$

$a \cdot c = 15 \cdot 10 = 150$

$1 \cdot 150$

$2 \cdot 75$

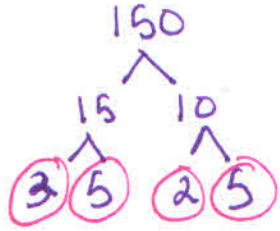
$3 \cdot 50$

$5 \cdot 30$

$(6 \cdot 25)$ $-6 + (-25) = -31$

$10 \cdot 15$

use a factor tree to make it easier for large numbers



2 step factoring - Factor out the GCF first

60. $4x^2 - 18x - 10$ $GCF = 2$

$= 2(2x^2 - 9x - 5)$ $ac = 2(-5) = -10$

$= 2[\underline{2x^2 + x} \underline{- 10x - 5}]$ $(1 \cdot 10)$ $1 + (-10) = -9$

$2 \cdot 5$

$= 2[x(2x + 1) - 5(2x + 1)]$

$= 2(2x + 1)(x - 5)$

76. $24x^4 + 10x^3 - 4x^2$ $GCF = 2x^2$

$= 2x^2(12x^2 + 5x - 2)$ $ac = 12 \cdot -2 = -24$

$= 2x^2[\underline{12x^2 - 3x} + \underline{8x - 2}]$

$= 2x^2[3x(4x - 1) + 2(4x - 1)]$

$= 2x^2(4x - 1)(3x + 2)$

$1 \cdot 24$

$2 \cdot 12$

$(3 \cdot 8)$ $-3 + 8 = 5$

$4 \cdot 6$

check with FOIL

Challenge:

88. $6(y+1)x^2 + 33(y+1)x + 15(y+1)$ GCF $= y+1$

$= (y+1)(6x^2 + 33x + 15)$ ~~$a+c=6$~~ There is still a 3 in common

$= 3(y+1)(2x^2 + 11x + 5)$ $a \cdot c = 2 \cdot 5 = 10$

$= 3(y+1)[2x^2 + x + 10x + 5]$ $1 \cdot 10$, $1+10=11$; $2 \cdot 5$

$= 3(y+1)[x(2x+1) + 5(2x+1)]$

$= 3(y+1)(2x+1)(x+5)$ factored completely

90. factor out leading negatives

$-10x^2y^4 + 14xy^4 + 12y^4$ GCF $= -2y^4$

$= -2y^4(5x^2 - 7x - 6)$ $a \cdot c = 5(-6) = -30$

$1 \cdot 30$

$2 \cdot 15$

$3 \cdot 10$, $3+(-10)=-7$

$5 \cdot 6$

$= -2y^4[5x^2 + 3x - 10x - 6]$

$= -2y^4[x(5x+3) - 2(5x+3)]$

$= -2y^4(5x+3)(x-2)$