Section 7.2 - Factoring Trinomials where $a = 1$

Standard form of a trinomial (3 terms)

$$ax^2 + bx + c$$

section 7.2: when $a = 1$ it is easier to factor because we have $x^2 + bx + c$.

section 7.3: when $a \neq 1$ we will use a different method called the ac or grouping method

This type of factoring is the opposite of FOIL

Review FOIL:

$(x+2)(x+5)$

$= x^2 + 5x + 2x + 10$ (like terms)

$= x^2 + 7x + 10$

7 comes from $2+5$; 10 comes from $2 \cdot 5$

$(x-3)(x-8)$

$= x^2 - 8x - 3x + 24$

$= x^2 - 11x + 24$

-11 comes from $-3+(-8)$; 24 comes from $-3 \cdot -8$

$(x+4)(x-6)$

$= x^2 - 6x + 4x - 24$

$= x^2 - 2x - 24$

-2 comes from $4+(-6)$; -24 comes from $4 \cdot -6$

The 2 numbers in the factors multiply to get the last term (c) and add to make the middle term (b).

In reverse:

$x^2+7x+10$

$(x+2)(x+5)$

multiply to get x^2

→ what 2 numbers multiply to get 10 and add to 7?

2 and 5

$(x+2)(x+5)$

$x^2-11x+24$

$(x\quad)(x\quad)$

multiply to get x^2

→ what 2 numbers multiply to get 24 and add to −11?

−3 and −8

$(x-3)(x-8)$

$x^2-2x-24$

$(x\quad)(x\quad)$

multiply to get x

→ what 2 numbers multiply to −24 and add to −2?

−6 and 4

$(x-6)(x+4)$

Does the order of factors matter?

Examples: (c is positive)

4. $x^2+9x+14$

$=(x+2)(x+7)$

or

$(x+7)(x+2)$

14

1·14

2·7

← make a systematic list of all factors

$2\cdot 7=14$

$2+7=9$

6. $x^2+13x+12$

$=(x+1)(x+12)$

12

1·12

2·6

3·4

$1+12=13$

8. $x^2 - 13x + 40$

b is negative so the 2 numbers must be negative

$= (x-8)(x-5)$

40
$-1 \cdot -40$
$-2 \cdot -20$
$-4 \cdot -10$
$-5 \cdot -8$ $-5+(-8) = -13$

10. $x^2 - 8x + 16$

$= (x-4)(x-4)$

$= (x-4)^2$

Factors are the same

16
$-1 \cdot -16$
$-2 \cdot -8$
$-4 \cdot -4$ $-4+(-4) = -8$

When c is negative:

14. $x^2 + 3x - 28$

$= (x-4)(x+7)$

-28 ← The 2 numbers must have opposite signs

$-1 \cdot 28$ or $1 \cdot -28$
$-2 \cdot 14$ or $2 \cdot -14$
$-4 \cdot 7$ or $4 \cdot -7$
$-4+7=3$

16. $y^2 + 5y - 24$

$= (y-3)(y+8)$

-24
$1 \cdot 24$
$2 \cdot 12$
$3 \cdot 8$ Look for a difference of 5 $-3+8=5$
$4 \cdot 6$

22. $x^2 + 4x + 5$

prime

This trinomial doesn't factor

5
$1 \cdot 5$ no way to get 4

34. $y^2 - 15y + 5$

prime

5
1·5 no way to get -15

Two variables:

36. $x^2 + 6xy + 8y^2$

$= (x + 2y)(x + 4y)$

8
1·8
2·4 $2+4=6$

Put the y's in to make $8y^2$

Check with FOIL:

$(x+2y)(x+4y)$

$= x^2 + 4xy + 2xy + 8y^2$

$= x^2 + 6xy + 8y^2$

Two-Step Factoring (Factor completely)

Look for the GCF first!

44. $3x^2 + 21x + 36$ GCF = 3

$= 3(x^2 + 7x + 12)$ Now factor the inside

Bring down the 3 ↓

$= 3(x+3)(x+4)$

12
1·12
2·6
3·4

52. $2r^3+8r^2+6r$ GCF $=2r$

$=2r(r^2+4r+3)$ $\quad 3$, $1\cdot 3$

$=2r(r+1)(r+3)$

66. $x^3y-2x^2y^2-3xy^3$ GCF $=xy$

$=xy(x^2-2xy-3y^2)$ $\quad -3$, $1\cdot 3$ $\quad 1+(-3)=-2$

$=xy(x+1)(x-3)$

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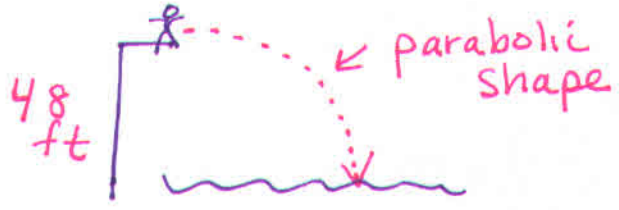
78. $-16t^2+32t+48$ is your height, t = seconds after you dive from the diving board 48 ft high

a) Factor (GCF $=-16$)

$-16(t^2-2t-3)$ $\quad -3$, $1\cdot 3$

$=-16(t+1)(t-3)$

Factoring out a negative changes the signs inside



b) Evaluate both forms for $t=3$

original

$-16(3)^2+32(3)+48$

$=-16(9)+96+48$

$=-144+96+48$

$=-144+144$

$=0$

Factored

$-16(3+1)(3-3)$

$=-16(4)(0)$

$=0$

we got the same answer. what does a height $=0$ mean?

You enter the water at 3 seconds.

Challenge Problems:

70. $(a+b)x^2 - 13(a+b)x + 36(a+b)$ The GCF is a binomial $(a+b)$

$= (a+b)(x^2 - 13x + 36)$

$= (a+b)(x-4)(x-9)$

36
$1 \cdot 36$
$2 \cdot 18$
$3 \cdot 12$
$(-4 \cdot -9)$ $-4 + (-9) = -13$
$6 \cdot 6$

72. $x^2 - .5x - .06$

$= (x - .2)(x - .3)$

$.06$
$.1 \cdot .6$
$(-.2 \cdot -.3)$ $-.2 + -.3 = -.5$

Think 6
$1 \cdot 6$
$2 \cdot 3$

74. $x^2 + \frac{2}{3}x + \frac{1}{9}$

$= (x + \frac{1}{3})(x + \frac{1}{3})$

$\frac{1}{9}$
$(\frac{1}{3} \cdot \frac{1}{3})$

Think $\frac{9}{3 \cdot 3}$

76. $-x^2 - 4x + 45$ when a is negative, factor out -1

$= -1(x^2 + 4x - 45)$

$= -1(x-5)(x+9)$

or

$-(x-5)(x+9)$

-45
$1 \cdot 45$
$3 \cdot 15$
$(-5 \cdot 9)$ $-5 + 9 = 4$