

Chapter 7 - Factoring

Why do we need to factor?

We know how to solve linear equations like $3x - 8 = 1$ but we can't use the same method to solve $x^2 + 7x = -10$. We need some new tools.

Section 7.1 Greatest Common Factor (GCF) and Factoring by grouping.

Step 1

Find the GCF - the most the terms have in common

2. 5 and $15x$: $GCF = 5$

4. $20x^2$ and $15x$: $GCF = 5x$

8. $10y^5$, $20y^2$, $-5y$: $GCF = 5y$

12. $18x^5y^4$, $6x^6y^3$, $12x^4y^5$: $GCF = 6x^4y^3$

Step 2
Factor out the GCF

What is factoring? The opposite of distributing

$$9(x+1) = 9x+9 \quad \text{distribute the 9}$$

$$\frac{9x+9}{\text{GCF}=9} = 9(x+1) \quad \text{factor out the 9. Divide each term by 9.}$$

14. $9x+9 = 9(x+1)$

16. $5y-5 = 5(y-1)$

Distribute to check

18. $10x+30 = 10(x+3)$

22. $x^2+6x = x(x+6)$

30. $10y^4+15y^6 = 5y^4(2+3y^2)$

when multiplying we add exponents
when factoring/dividing we subtract exponents.

34. $15y^2-3y+9 = 3(5y^2-y+3)$

36. $32x^4+2x^3+8x^2 = 2x^2(16x^2+x+4)$

42. $12x^2-25$ cannot be factored

48. $18x^3y^2-12x^3y-24x^2y = 6x^2y(3xy-2x-4)$

more complicated: The common factor is
a binomial (2 terms)

50. $\boxed{x(x+7)} + \boxed{10(x+7)}$ 2 terms in the polynomial

common factor
is $x+7$

factor out the $x+7$

$$= (x+7)(x+10)$$

52. $\textcircled{x(x+3)} - \textcircled{8(x+3)}$

$$= (x+3)(x-8)$$

54. $x(y+9) - 11(y+9)$

$$= (y+9)(x-11)$$

56. $7x(x+y) - (x+y)$

no number here
what is it?

$$= 7x(x+y) - 1(x+y)$$

Put in the invisible 1

$$= (x+y)(7x-1)$$

60. $9x^2(7x+2) + 7x+2$

no parentheses here
add them and the 1

$$= 9x^2(7x+2) + 1(7x+2)$$
$$= (7x+2)(9x^2+1)$$

Factor by grouping \rightarrow need 4 terms

$$62. \quad \underline{x^2 + 3x} + \underline{5x + 15}$$

$$\begin{array}{c} \uparrow \\ \text{GCF} = x \end{array}$$

$$\begin{array}{c} \uparrow \\ \text{GCF} = 5 \end{array}$$

$$= x(x+3) + 5(x+3)$$

$$= (x+3)(x+5)$$

① group the 1st 2 terms and the last 2 terms

② Factor out the GCF from each pair

③ Factor out the common binomial [They must be the same!]

~~65.~~

$$66. \quad \underline{x^3 - 3x^2} + \underline{4x - 12}$$

$$\text{GCF} = x^2$$

$$\text{GCF} = 4$$

$$= x^2(x-3) + 4(x-3)$$

$$= (x-3)(x^2+4)$$

$$74. \quad \underline{10x^2 - 12xy} + \underline{35xy - 42y^2}$$

$$= 2x(5x - 6y) + 7y(5x - 6y)$$

$$= (5x - 6y)(2x + 7y)$$

Factoring out a negative

Remember when distributing a negative it changes the signs:

$$-3(x+4) = -3x-12$$

$$-2(2x-3) = -4x+6$$

Factoring does the same thing:

$$-3x-12 = -3(x+4)$$

$$-4x+6 = -2(2x-3)$$

When the middle sign is negative, factor out the negative sign

$$64. \quad \underline{x^2+7x} - \underline{4x-28}$$

$$= x(x+7) - 4(x+7)$$

$$= (x+7)(x-4)$$

72: changed the problem = $\underline{xy-x} - \underline{7y+7}$

$$= x(y-1) - 7(y-1)$$

$$= (y-1)(x-7)$$

$$76. \quad \underline{4x^3-x^2} - \underline{12x+3}$$

$$= x^2(4x-1) - 3(4x-1)$$

$$= (4x-1)(x^2-3)$$

Application

90. $72x - 16x^2$ is the height after x seconds

a) $x=3$

$$72(3) - 16(3)^2$$

$$= 216 - 16(9)$$

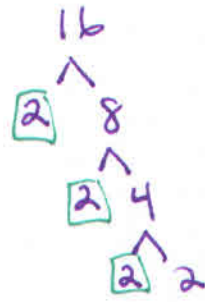
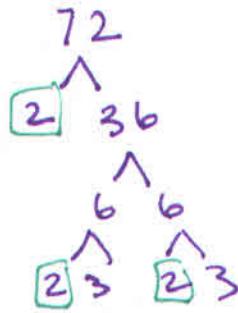
$$= 216 - 144$$

$$= 72$$

The debris is 72 feet high after 3 seconds.

b) factor the polynomial

$$72x - 16x^2$$
$$= 8x(9 - 2x)$$



3 2's in common
 $2 \cdot 2 \cdot 2 = 8$

c) Calculate using the factored polynomial

$$x = 3$$

$$8(3)(9 - 2(3))$$

$$= 24(9 - 6)$$

$$= 24(3)$$

$$= 72 \text{ feet}$$

Yes, our factorization is correct

because they gave us the same answer when we evaluated them at $x = 3$.