Section 9.5
Graphs of Quadratic Equations

Heads UP!!!

Many sports involve objects that are thrown, kicked, or hit, and then proceed with no additional force of their own. Such objects are called projectiles.

In this section of your textbook, you will learn to use graphs of quadratic equations to gain a visual understanding of various projectile sports.

First Steps:

- ☐ **Take comprehensive notes** from your instructor's lecture and insert your notes into this section of the *Learning Guide*. Be sure to write down all examples, definitions, and other key concepts. Additional learning resources include the *Lecture Series on DVD*, the *PowerPoints*, and Section 9.5 of your textbook which begins on page 652.
- ☐ Complete the ***Concept and Vocabulary Check*** on page 661 of the textbook.

Guided Practice:

- ☐ Review each of the following ***Solved Problems*** and complete each ***Pencil Problem***.

Objective #1: Understand the characteristics of graphs of quadratic equations.

✔ *Solved Problem #1*

1. Determine whether the graph of $y = x^2 - 6x + 8$ is a parabola that opens upward or downward.

Because *a*, the coefficient of x^2, is 1, which is greater than 0, the parabola opens upward.

✎ *Pencil Problem #1* ✎

1. Determine whether the graph of $y = -2x^2 + x + 6$ is a parabola that opens upward or downward.

Objective #2: Find a parabola's intercepts.

✔ *Solved Problem #2*

2a. Find the *x*-intercepts for the parabola whose equation is $y = x^2 - 6x + 8$.

Replace *y* with 0 and solve the resulting equation.

$0 = x^2 - 6x + 8$

$0 = (x - 4)(x - 2)$

$x - 4 = 0$ or $x - 2 = 0$

$x = 4$ $\qquad$ $x = 2$

The *x*-intercepts are 2 and 4.

✎ *Pencil Problem #2* ✎

2a. Find the *x*-intercepts for the parabola whose equation is $y = x^2 - 4x + 3$.

2b. Find the y-intercept for the parabola whose equation is $y = x^2 - 6x + 8$.

To find the y-intercept, replace x with 0.

$$y = x^2 - 6x + 8$$
$$y = 0^2 - 6(0) + 8$$
$$= 8$$

The y-intercept is 8.

2b. Find the y-intercept for the parabola whose equation is $y = -x^2 + 8x - 12$.

Objective #3: Find a parabola's vertex.

✔ *Solved Problem #3*

3. Find the vertex for the parabola whose equation is $y = x^2 - 6x + 8$.

Note that $a = 1,\ b = -6,\ c = 8$.

x-coordinate of vertex: $x = \dfrac{-b}{2a} = \dfrac{-(-6)}{2(1)} = \dfrac{6}{2} = 3$

y-coordinate of vertex: $y = x^2 - 6x + 8$

$$y = 3^2 - 6(3) + 8$$
$$= 9 - 18 + 8$$
$$= -1$$

The vertex is $(3, -1)$.

✎ *Pencil Problem #3*

3. Find the vertex for the parabola whose equation is $y = 2x^2 + 4x - 6$.

Objective #4: Graph quadratic equations.

✔ *Solved Problem #4*

4. Graph the quadratic equation: $y = x^2 + 6x + 5$

Step 1 Determine how the parabola opens.

Here a, the coefficient of x^2, is 1.

Because $a > 0$, the parabola opens upward.

Step 2 Find the vertex.

For this equation, $a = 1, b = 6$, and $c = 5$.

x-coordinate of vertex: $x = \dfrac{-b}{2a} = \dfrac{-6}{2(1)} = \dfrac{-6}{2} = -3$

y-coordinate of vertex: $y = x^2 + 6x + 5$

$$y = (-3)^2 + 6(-3) + 5 = -4$$

The vertex is $(-3, -4)$.

✎ *Pencil Problem #4*

4. Graph the quadratic equation: $y = -x^2 + 4x - 3$

Step 3 Find the x-intercepts.

Replace y with 0 in $y = x^2 + 6x + 5$ and solve for x.

$$x^2 + 6x + 5 = 0$$
$$(x+5)(x+1) = 0$$
$$x + 5 = 0 \quad \text{or} \quad x + 1 = 0$$
$$x = -5 \qquad x = -1$$

The x-intercepts are -5 and -1.

Step 4 Find the y-intercept.

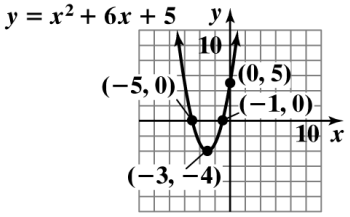
Replace x with 0 and solve for y.

$$y = 0^2 + 6(0) + 5 = 5$$

The y-intercept is 5.

Steps 5 and 6 Plot the intercepts and the vertex.

Plot $(-3,-4), (-5,0), (-1,0)$, and $(0,5)$, and connect them with a smooth curve.



Objective #5: Solve problems using a parabola's vertex.

✔ *Solved Problem #5*

5. An archer's arrow follows a parabolic path. The height of the arrow, y, in feet, can be modeled by $y = -0.005x^2 + 2x + 5$, where x is the arrow's horizontal distance, in feet.

5a. What is the maximum height of the arrow and how far from its release does this occur?

The information needed is found at the vertex.

x-coordinate of vertex:

$$x = \frac{-b}{2a} = \frac{-2}{2(-0.005)} = 200$$

y-coordinate of vertex:

$$y = -0.005x^2 + 2x + 5$$
$$y = -0.005(200)^2 + 2(200) + 5 = 205$$

The vertex is (200,205).

The maximum height of the arrow is 205 feet.
This occurs 200 feet from its release.

✎ *Pencil Problem #5* ✎

5. A ball is thrown upward and outward from a height of 6 feet. The height of the ball, y, in feet, can be modeled by $y = -0.8x^2 + 3.2x + 6$ where x is the ball's horizontal distance, in feet, from where it was thrown.

5a. What is the maximum height of the ball and how far from where it was thrown does this occur?

5b. Find the horizontal distance the arrow travels before it hits the ground. Round to the nearest foot.

The arrow will hit the ground when the height reaches 0.

$$y = -0.005x^2 + 2x + 5$$
$$0 = -0.005x^2 + 2x + 5$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-2 \pm \sqrt{2^2 - 4(-0.005)(5)}}{2(-0.005)}$$
$$x \approx -2 \text{ or } x \approx 402$$

The arrow travels 402 feet before hitting the ground.

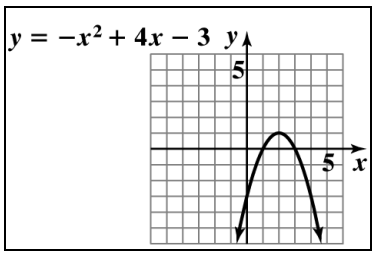
5b. How far does the ball travel horizontally before hitting the ground?
Round to the nearest tenth of a foot.

Answers for Pencil Problems *(Textbook Exercise references in parentheses)*:

1. downward *(9.5 #3)*

2a. 1 and 3 *(9.5 #5)* **2b.** −12 *(9.5 #13)*

3. $(-1, -8)$ *(9.5 #21)*

4. 

(9.5 #29)

5a. The maximum height is 9.2 feet; this occurs 2 feet from its release. *(9.5 #49a)* **5b.** 5.4 feet *(9.5 #49b)*

Homework:

- ☐ **Review the Section 9.5 summary** on page 677 of the textbook.
- ☐ **Insert your homework** into this section of the *Learning Guide*. Show all work neatly and check your answers. Strive to work through difficulties when possible, making note of any exercises where you need additional help. Remember, even if your instructor assigns homework through *MyMathLab*, you should still write out your work.