

Section 9.4

Imaginary Numbers as Solutions of Quadratic Equations

Why Study Something if it is *IMAGINARY*???

i

Great Question!

The numbers that we study in this section were given the name “*imaginary*” at a time when mathematicians believed such numbers to be useless.

Since that time, many *real-life* applications for so-called imaginary numbers have been discovered, but the name they were originally given has endured.

i

First Steps:

- Take comprehensive notes** from your instructor’s lecture and insert your notes into this section of the *Learning Guide*. Be sure to write down all examples, definitions, and other key concepts. Additional learning resources include the *Lecture Series on DVD*, the *PowerPoints*, and Section 9.4 of your textbook which begins on page 647.
- Complete the *Concept and Vocabulary Check* on page 650 of the textbook.

Guided Practice:

- Review each of the following *Solved Problems* and complete each *Pencil Problem*.

Objective #1: Express square roots of negative numbers in terms of *i*.

✓ *Solved Problem #1*

1a. Write as a multiple of *i*: $\sqrt{-16}$

$$\begin{aligned}\sqrt{-16} &= \sqrt{16(-1)} \\ &= \sqrt{16}\sqrt{-1} \\ &= 4i\end{aligned}$$

1b. Write as a multiple of *i*: $\sqrt{-50}$

$$\begin{aligned}\sqrt{-50} &= \sqrt{50(-1)} \\ &= \sqrt{25 \cdot 2}\sqrt{-1} \\ &= 5i\sqrt{2}\end{aligned}$$

✎ *Pencil Problem #1* ✎

1a. Write as a multiple of *i*: $\sqrt{-36}$

1b. Write as a multiple of *i*: $\sqrt{-20}$

Objective #2: Solve quadratic equations with imaginary solutions. **Solved Problem #2**

2a. Solve: $(x+2)^2 = -25$

Use the square root property.

$$x+2 = \sqrt{-25} \quad \text{or} \quad x+2 = -\sqrt{-25}$$

$$x+2 = 5i \qquad x+2 = -5i$$

$$x = -2 + 5i \qquad x = -2 - 5i$$

The solution set is $\{-2 \pm 5i\}$. **Pencil Problem #2** 

2a. Solve: $(x+7)^2 = -64$

2b. Solve: $x^2 + 6x + 13 = 0$

Use the quadratic formula with $a = 1$, $b = 6$, and $c = 13$.

$$\begin{aligned}
 x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 &= \frac{-6 \pm \sqrt{6^2 - 4(1)(13)}}{2 \cdot 1} \\
 &= \frac{-6 \pm \sqrt{-16}}{2} \\
 &= \frac{-6 \pm 4i}{2} \\
 &= -3 \pm 2i
 \end{aligned}$$

The solution set is $\{-3 \pm 2i\}$.

2b. Solve: $x^2 - 12x + 40 = 0$

Answers for Pencil Problems (Textbook Exercise references in parentheses):

1a. $6i$ (9.4 #1) **1b.** $2i\sqrt{5}$ (9.4 #7) **2a.** $\{-7 \pm 8i\}$ (9.4 #19) **2b.** $\{6 \pm 2i\}$ (9.4 #29)

Homework:

- Review the Section 9.4 summary** on page 677 of the textbook.
- Insert your homework** into this section of the *Learning Guide*. Show all work neatly and check your answers. Strive to work through difficulties when possible, making note of any exercises where you need additional help. Remember, even if your instructor assigns homework through *MyMathLab*, you should still write out your work.