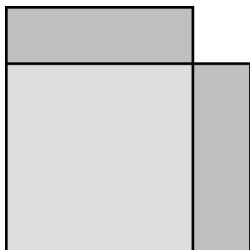


## Section 9.2

### Solving Quadratic Equations by Completing the Square

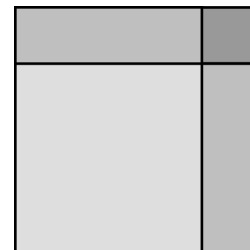
#### ***Completing the Square!!***



What is the difference between the figure on the left and the figure on the right?

The square on the right was *completed* by adding a small square to its upper right-hand corner.

In this section of the textbook, we will explore how this geometrical concept directly relates to an important algebraic procedure called “*completing the square.*”



#### **First Steps:**

- Take comprehensive notes** from your instructor’s lecture and insert your notes into this section of the *Learning Guide*. Be sure to write down all examples, definitions, and other key concepts. Additional learning resources include the *Lecture Series on DVD*, the *PowerPoints*, and Section 9.2 of your textbook which begins on page 630.
- Complete the *Concept and Vocabulary Check* on page 634 of the textbook.

#### **Guided Practice:**

- Review each of the following *Solved Problems* and complete each *Pencil Problem*.

**Objective #1:** Complete the square of a binomial.

#### ✓ ***Solved Problem #1***

- 1a.** Complete the square for the binomial. Then factor the resulting trinomial:  $x^2 + 8x$

$$x^2 + 8x$$

The coefficient of the  $x$ -term is 8.  
Half of 8 is 4, and  $4^2 = 16$ . Add 16.

$$x^2 + 8x + 16 = (x + 4)^2$$

#### ✎ ***Pencil Problem #1*** ✎

- 1a.** Complete the square for the binomial. Then factor the resulting trinomial:  $x^2 + 10x$

**1b.** Complete the square for the binomial. Then factor the resulting trinomial:  $x^2 - 6x$

$$x^2 - 6x$$

The coefficient of the  $x$ -term is  $-6$ .

Half of  $-6$  is  $-3$ , and  $(-3)^2 = 9$ . Add 9.

$$x^2 - 6x + 9 = (x - 3)^2$$

**1b.** Complete the square for the binomial. Then factor the resulting trinomial:  $x^2 - 2x$

**1c.** Complete the square for the binomial. Then factor the resulting trinomial:  $x^2 + 3x$

$$x^2 + 3x$$

The coefficient of the  $x$ -term is 3.

Half of 3 is  $\frac{3}{2}$ , and  $\left(\frac{3}{2}\right)^2 = \frac{9}{4}$ . Add  $\frac{9}{4}$ .

$$x^2 + 3x + \frac{9}{4} = \left(x + \frac{3}{2}\right)^2$$

**1c.** Complete the square for the binomial. Then factor the resulting trinomial:  $x^2 - 7x$

**Objective #2:** Solve quadratic equations by completing the square.

 **Solved Problem #2**

**2a.** Solve by completing the square:  $x^2 + 6x = 7$

The coefficient of the  $x$ -term is 6. Half of 6 is 3, and  $3^2 = 9$ . Add 9 to both sides of the equation.

$$x^2 + 6x = 7$$

$$x^2 + 6x + 9 = 7 + 9$$

Factor, and then solve using the square root property.

$$x^2 + 6x + 9 = 7 + 9$$

$$(x + 3)^2 = 16$$

$$x + 3 = \sqrt{16} \quad \text{or} \quad x + 3 = -\sqrt{16}$$

$$x + 3 = 4 \quad \quad \quad x + 3 = -4$$

$$x = 1 \quad \quad \quad x = -7$$

The solution set is  $\{-7, 1\}$ .

 **Pencil Problem #2** 

**2a.** Solve by completing the square:  $x^2 - 10x = -24$

**2b.** Solve by completing the square:  $x^2 - 10x + 18 = 0$

First subtract 18 from both sides to isolate the binomial.

$$x^2 - 10x + 18 = 0$$

$$x^2 - 10x = -18$$

Next, complete the square by adding  $\left(\frac{-10}{2}\right)^2$  to both sides.

$$x^2 - 10x + 25 = -18 + 25$$

$$x^2 - 10x + 25 = 7$$

Factor, and then solve using the square root property.

$$x^2 - 10x + 25 = 7$$

$$(x - 5)^2 = 7$$

$$x - 5 = \sqrt{7} \quad \text{or} \quad x - 5 = -\sqrt{7}$$

$$x = 5 + \sqrt{7} \quad \quad \quad x = 5 - \sqrt{7}$$

The solution set is  $\{5 \pm \sqrt{7}\}$ .

**2b.** Solve by completing the square:  $x^2 + 4x + 1 = 0$

**2c.** Solve by completing the square:  $2x^2 - 10x - 1 = 0$

$$2x^2 - 10x - 1 = 0$$

First, divide both sides of the equation by 2 so that the coefficient of the  $x^2$  term will be 1.

$$\frac{2x^2}{2} - \frac{10x}{2} - \frac{1}{2} = \frac{0}{2}$$

$$x^2 - 5x - \frac{1}{2} = 0$$

Next, add  $\frac{1}{2}$  to both sides to isolate the binomial.

$$x^2 - 5x - \frac{1}{2} = 0$$

$$x^2 - 5x = \frac{1}{2}$$

(continued on next page)

**2c.** Solve by completing the square:  $2x^2 - 2x - 6 = 0$

Complete the square:

The coefficient of the  $x$ -term is  $-5$ , and  $\frac{1}{2}(-5) = -\frac{5}{2}$ , so

add  $\left(-\frac{5}{2}\right)^2 = \frac{25}{4}$  to both sides.

$$x^2 - 5x = \frac{1}{2}$$

$$x^2 - 5x + \frac{25}{4} = \frac{1}{2} + \frac{25}{4}$$

$$x^2 - 5x + \frac{25}{4} = \frac{2}{4} + \frac{25}{4}$$

$$\left(x - \frac{5}{2}\right)^2 = \frac{27}{4}$$

$$x - \frac{5}{2} = \sqrt{\frac{27}{4}} \quad \text{or} \quad x - \frac{5}{2} = -\sqrt{\frac{27}{4}}$$

$$x = \frac{5}{2} + \frac{\sqrt{27}}{2} \quad x = \frac{5}{2} - \frac{\sqrt{27}}{2}$$

$$x = \frac{5}{2} + \frac{3\sqrt{3}}{2} \quad x = \frac{5}{2} - \frac{3\sqrt{3}}{2}$$

$$x = \frac{5 + 3\sqrt{3}}{2} \quad x = \frac{5 - 3\sqrt{3}}{2}$$

The solution set is  $\left\{\frac{5 \pm 3\sqrt{3}}{2}\right\}$ .

**Answers for Pencil Problems** (*Textbook Exercise references in parentheses*):

**1a.**  $x^2 + 10x + 25 = (x + 5)^2$  (9.2 #1)    **1b.**  $x^2 - 2x + 1 = (x - 1)^2$  (9.2 #3)

**1c.**  $x^2 - 7x + \frac{49}{4} = \left(x - \frac{7}{2}\right)^2$  (9.2 #7)

**2a.**  $\{4, 6\}$  (9.2 #15)    **2b.**  $\{-2 \pm \sqrt{3}\}$  (9.2 #19)    **2c.**  $\left\{\frac{1 \pm \sqrt{13}}{2}\right\}$  (9.2 #27)

**Homework:**

- Review the Section 9.2 summary** on page 676 of the textbook.
- Insert your homework** into this section of the *Learning Guide*. Show all work neatly and check your answers. Strive to work through difficulties when possible, making note of any exercises where you need additional help. Remember, even if your instructor assigns homework through *MyMathLab*, you should still write out your work.