

## Section 8.4

### Rationalizing the Denominator

#### Are You Getting a Tax Refund?



In most tax years, approximately 59% of all taxpayers receive a tax refund, whereas 41% must pay more taxes than were withheld.



In this section's Exercise Set, you will use a formula that models the percentage of taxpayers who must pay more taxes based on the taxpayer's age. In the section itself, you will learn how to rewrite the formula without a radical in its denominator using a technique called rationalizing the denominator.

#### First Steps:

- Take **comprehensive notes** from your instructor's lecture and insert your notes into this section of the *Learning Guide*. Be sure to write down all examples, definitions, and other key concepts. Additional learning resources include the *Lecture Series on DVD*, the *PowerPoints*, and Section 8.4 of your textbook which begins on page 592.
- Complete the *Concept and Vocabulary Check* on page 595 of the textbook.

#### Guided Practice:

- Review each of the following *Solved Problems* and complete each *Pencil Problem*.

**Objective #1:** Rationalize denominators containing one term.

#### ✓ *Solved Problem #1*

1a. Rationalize the denominator:  $\frac{25}{\sqrt{10}}$

Multiply the numerator and denominator by  $\sqrt{10}$ .  
Then simplify.

$$\begin{aligned}\frac{25}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} &= \frac{25\sqrt{10}}{10} \\ &= \frac{5\sqrt{10}}{2}\end{aligned}$$

#### *Pencil Problem #1*

1a. Rationalize the denominator:  $\frac{2}{\sqrt{6}}$

**1b.** Rationalize the denominator:  $\sqrt{\frac{2}{7}}$

The square root of a quotient is the quotient of the square roots.

$$\sqrt{\frac{2}{7}} = \frac{\sqrt{2}}{\sqrt{7}}$$

Multiply the numerator and denominator by  $\sqrt{7}$  and then simplify.

$$\begin{aligned}\sqrt{\frac{2}{7}} &= \frac{\sqrt{2}}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} \\ &= \frac{\sqrt{14}}{7}\end{aligned}$$

**1b.** Rationalize the denominator:  $\sqrt{\frac{3}{5}}$

**1c.** Rationalize the denominator:  $\frac{15}{\sqrt{18}}$

Begin by simplifying  $\sqrt{18}$ .

$$\frac{15}{\sqrt{18}} = \frac{15}{3\sqrt{2}}$$

Multiply the numerator and denominator by  $\sqrt{2}$  and then simplify.

$$\begin{aligned}\frac{15}{\sqrt{18}} &= \frac{15}{3\sqrt{2}} \\ &= \frac{15}{3\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \\ &= \frac{15\sqrt{2}}{3 \cdot 2} \\ &= \frac{5\sqrt{2}}{2}\end{aligned}$$

**1c.** Rationalize the denominator:  $\frac{1}{\sqrt{20}}$

1d. Rationalize the denominator:  $\sqrt{\frac{7x}{20}}$

The square root of a quotient is the quotient of the square roots.

$$\sqrt{\frac{7x}{20}} = \frac{\sqrt{7x}}{\sqrt{20}}$$

Next, simplify  $\sqrt{20}$ .

$$\begin{aligned}\sqrt{\frac{7x}{20}} &= \frac{\sqrt{7x}}{\sqrt{20}} \\ &= \frac{\sqrt{7x}}{2\sqrt{5}}\end{aligned}$$

Multiply the numerator and denominator by  $\sqrt{5}$  and then simplify.

$$\begin{aligned}\sqrt{\frac{7x}{20}} &= \frac{\sqrt{7x}}{\sqrt{20}} \\ &= \frac{\sqrt{7x}}{2\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} \\ &= \frac{\sqrt{35x}}{2 \cdot 5} \\ &= \frac{\sqrt{35x}}{10}\end{aligned}$$

1d. Rationalize the denominator:  $\sqrt{\frac{x^2}{3}}$

**Objective #2:** Rationalize denominators containing two terms.

 **Solved Problem #2**

2a. Rationalize the denominator:  $\frac{8}{4+\sqrt{5}}$

Multiply the numerator and the denominator by the conjugate of the denominator.

$$\begin{aligned}\frac{8}{4+\sqrt{5}} &= \frac{8}{4+\sqrt{5}} \cdot \frac{4-\sqrt{5}}{4-\sqrt{5}} \\ &= \frac{32-8\sqrt{5}}{16-5} \\ &= \frac{32-8\sqrt{5}}{11}\end{aligned}$$

 **Pencil Problem #2** 

2a. Rationalize the denominator:  $\frac{1}{4+\sqrt{3}}$

2b. Rationalize the denominator:  $\frac{8}{\sqrt{7}-\sqrt{3}}$

Multiply the numerator and the denominator by the conjugate of the denominator.

$$\begin{aligned}\frac{8}{\sqrt{7}-\sqrt{3}} &= \frac{8}{\sqrt{7}-\sqrt{3}} \cdot \frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}+\sqrt{3}} \\ &= \frac{8\sqrt{7}+8\sqrt{3}}{(\sqrt{7})^2-(\sqrt{3})^2} \\ &= \frac{8\sqrt{7}+8\sqrt{3}}{7-3} \\ &= \frac{4(2\sqrt{7}+2\sqrt{3})}{4} \\ &= 2\sqrt{7}+2\sqrt{3}\end{aligned}$$

2b. Rationalize the denominator:  $\frac{6}{\sqrt{6}+\sqrt{3}}$

**Answers for Pencil Problems** (*Textbook Exercise references in parentheses*):

1a.  $\frac{\sqrt{6}}{3}$  (8.4 #5)

1b.  $\frac{\sqrt{15}}{5}$  (8.4 #9)

1c.  $\frac{\sqrt{5}}{10}$  (8.4 #25)

1d.  $\frac{x\sqrt{3}}{3}$  (8.4 #13)

2a.  $\frac{4-\sqrt{3}}{13}$  (8.4 #53)

2b.  $2\sqrt{6}-2\sqrt{3}$  (8.4 #65)

**Homework:**

- Review the Section 8.4 summary** which begins on page 614 of the textbook.
- Insert your homework** into this section of the *Learning Guide*. Show all work neatly and check your answers. Strive to work through difficulties when possible, making note of any exercises where you need additional help. Remember, even if your instructor assigns homework through *MyMathLab*, you should still write out your work.