Section 6.6
Solving Quadratic Equations by Factoring

See you later, Alligator?

Fortunately, the alligator is no longer an endangered species. At one time the alligator was the subject of a protection program at Florida's Everglades National Park.

Park rangers used a formula to estimate the alligator population during the years of the program.

In this section of the textbook, you will solve a quadratic equation in order to estimate the number of years it took for the alligator population to reach various levels.

First Steps:

- ☐ **Take comprehensive notes** from your instructor's lecture and insert your notes into this section of the *Learning Guide*. Be sure to write down all examples, definitions, and other key concepts. Additional learning resources include the *Lecture Series on DVD*, the *PowerPoints*, and Section 6.6 of your textbook which begins on page 462.
- ☐ Complete the ***Concept and Vocabulary Check*** on page 471 of the textbook.

Guided Practice:

- ☐ Review each of the following ***Solved Problems*** and complete each ***Pencil Problem***.

Objective #1: Use the zero-product principle.

✔ *Solved Problem #1*

1. Solve the equation: $(2x+1)(x-4)=0$

The equation is in factored form on the left side with zero on the right side. Thus, set each factor equal to zero and solve the resulting equations.

$2x+1=0$ or $x-4=0$

$2x=-1$ $\qquad$ $x=4$

$x=-\frac{1}{2}$

The solution set is $\left\{-\frac{1}{2},4\right\}$.

✎ *Pencil Problem #1* ✎

1. Solve the equation: $(x-6)(x+4)=0$

Objective #2: Solve quadratic equations by factoring.

✔ *Solved Problem #2*

2a. Solve: $x^2 - 6x + 5 = 0$

All the terms are on one side and zero is on the other side.

$x^2 - 6x + 5 = 0$

Thus, factor the left side of the equation.

$x^2 - 6x + 5 = 0$

$(x-1)(x-5) = 0$

Next, set each factor equal to zero and solve the resulting equations.

$x - 1 = 0$ or $x - 5 = 0$

$x = 1$ $\quad$ $x = 5$

The solution set is $\{1, 5\}$.

✎ *Pencil Problem #2* ✎

2a. Solve: $x^2 - 5x = 0$

2b. Solve: $4x^2 = 2x$

Move all terms to one side and obtain zero on the other side.

$4x^2 = 2x$

$4x^2 - 2x = 0$

Then factor the left side of the equation.

$4x^2 - 2x = 0$

$2x(2x-1) = 0$

Next, set each factor equal to zero and solve the resulting equations.

$2x = 0$ or $2x - 1 = 0$

$x = 0$ $\quad$ $2x = 1$

$x = \frac{1}{2}$

The solution set is $\left\{0, \frac{1}{2}\right\}$.

2b. Solve: $2x^2 = 7x + 4$

2c. Solve: $x^2 = 10x - 25$

Move all terms to one side and obtain zero on the other side.

$$x^2 = 10x - 25$$
$$x^2 - 10x + 25 = 0$$

Then factor the left side of the equation.

$$x^2 - 10x + 25 = 0$$
$$(x-5)^2 = 0$$

Because both factors are the same, it is only necessary to set one of them equal to zero.

$$x - 5 = 0$$
$$x = 5$$

The solution set is $\{5\}$.

2c. Solve: $x^2 + 4x + 4 = 0$

2d. Solve: $(x-5)(x-2) = 28$

Write the equation in standard form by finding the product on the left side and then subtracting 28 from both sides.

$$(x-5)(x-2) = 28$$
$$x^2 - 7x + 10 = 28$$
$$x^2 - 7x - 18 = 0$$

Then factor the left side of the equation.

$$x^2 - 7x - 18 = 0$$
$$(x-9)(x+2) = 0$$

Set each factor equal to zero and solve the resulting equations.

$$x^2 - 7x - 18 = 0$$
$$(x-9)(x+2) = 0$$
$$x - 9 = 0 \quad \text{or} \quad x + 2 = 0$$
$$x = 9 \qquad\qquad x = -2$$

The solution set is $\{-2, 9\}$.

2d. Solve: $x(x-4) = 21$

Objective #3: Solve problems using quadratic equations.

✔ *Solved Problem #3*

3. The length of a rectangular sign is 3 feet longer than the width. If the sign's area is 54 square feet, find its length and width.

Let $x =$ the width of the sign.
Let $x + 3 =$ the length of the sign.

The area of 54 square feet can be represented as follows.

$A = l \cdot w$

$54 = (x+3) \cdot x$

Write the equation in standard form by finding the product on the right side and then subtracting 54 from both sides.

$54 = (x+3) \cdot x$

$54 = x^2 + 3x$

$0 = x^2 + 3x - 54$

$0 = (x-6)(x+9)$

Set each factor equal to zero and solve the resulting equations.

$x - 6 = 0$ or $x + 9 = 0$

$x = 6$ $\qquad$ $x = -9$

Reject –9 because the width cannot be negative.

The width of the sign is 6 feet and the length is $6 + 3,$ or 9 feet.

✎ *Pencil Problem #3* ✎

3. The number of football games, *N*, that must be played in a league with *t* teams if each team is to play every other team once is described by

$$N = \frac{t^2 - t}{2}.$$

If a league has 45 games scheduled, how many teams belong to the league, assuming that each team plays every other team once?

Answers for Pencil Problems *(Textbook Exercise references in parentheses)*:

1. $\{-4, 6\}$ *(6.6 #3)* **2a.** $\{0, 5\}$ *(6.6 #19)* **2b.** $\left\{-\frac{1}{2}, 4\right\}$ *(6.6 #33)* **2c.** $\{-2\}$ *(6.6 #27)*

2d. $\{-3, 7\}$ *(6.6 #43)* **3.** 10 teams *(6.6 #81)*

Homework:

- ☐ **Review the Section 6.6 summary** on page 477 of the textbook.
- ☐ **Insert your homework** into this section of the *Learning Guide*. Show all work neatly and check your answers. Strive to work through difficulties when possible, making note of any exercises where you need additional help. Remember, even if your instructor assigns homework through *MyMathLab*, you should still write out your work.