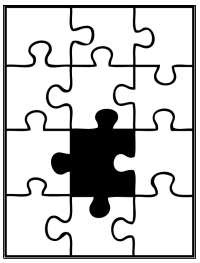
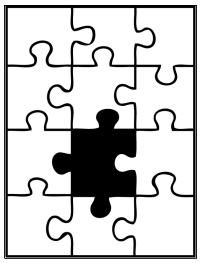
Section 6.4
Factoring Special Forms

Do you enjoy Solving P_z_l_s?



The process of solving puzzles is a natural way to develop problem-solving skills that are important to every area of our lives. Engaging in problem solving for sheer pleasure releases chemicals in the brain that enhance our feeling of well-being. Perhaps this is why puzzles date back 12,000 years.

In this section, we develop factoring techniques by reversing the formulas for special products discussed in the previous chapter. These factorizations can be visualized by fitting pieces of a puzzle together to form rectangles.



First Steps:

- ☐ **Take comprehensive notes** from your instructor's lecture and insert your notes into this section of the *Learning Guide*. Be sure to write down all examples, definitions, and other key concepts. Additional learning resources include the *Lecture Series on DVD*, the *PowerPoints*, and Section 6.4 of your textbook which begins on page 444.
- ☐ Complete the ***Concept and Vocabulary Check*** on page 451 of the textbook.

Guided Practice:

- ☐ Review each of the following ***Solved Problems*** and complete each ***Pencil Problem***.

Objective #1: Factor the difference of two squares.

✔ *Solved Problem #1*

1a. Factor: $x^2 - 81$

Notice that the trinomial fits the form $A^2 - B^2$.

Thus, factor using $A^2 - B^2 = (A + B)(A - B)$.

$$x^2 - 81 = x^2 - 9^2$$
$$= (x + 9)(x - 9)$$

✎ *Pencil Problem #1* ✎

1a. Factor: $x^2 - 25$

1b. Factor: $36x^2 - 25$

Notice that the trinomial fits the form $A^2 - B^2$.

Thus, factor using $A^2 - B^2 = (A + B)(A - B)$.

$$36x^2 - 25 = (6x)^2 - 5^2$$
$$= (6x + 5)(6x - 5)$$

1b. Factor: $x^2 + 36$

1c. Factor: $25 - 4x^{10}$

Notice that the trinomial fits the form $A^2 - B^2$.

Thus, factor using $A^2 - B^2 = (A + B)(A - B)$.

$$25 - 4x^{10} = 5^2 - (2x^5)^2$$
$$= (5 + 2x^5)(5 - 2x^5)$$

1c. Factor: $49y^4 - 16$

1d. Factor: $18x^3 - 2x$

First factor out the GCF.

$$18x^3 - 2x = 2x(9x^2 - 1)$$

Next, factor the difference of two squares.

$$18x^3 - 2x = 2x(9x^2 - 1)$$
$$= 2x(3x + 1)(3x - 1)$$

1d. Factor: $2x^3 - 72x$

1e. Factor: $81x^4 - 16$

First, factor the difference of two squares.

$$81x^4 - 16 = (9x^2 + 4)(9x^2 - 4)$$

The factor of $9x^2 - 4$ is the difference of two squares and can be factored.

$$81x^4 - 16 = (9x^2 + 4)(9x^2 - 4)$$
$$= (9x^2 + 4)(3x + 2)(3x - 2)$$

1e. Factor: $x^4 - 16$

Objective #2: Factor perfect square trinomials.

✔ *Solved Problem #2*

2a. Factor: $x^2 + 14x + 49$

Notice that the trinomial fits the form $A^2 + 2AB + B^2$.

Thus, factor using $A^2 + 2AB + B^2 = (A + B)^2$.

$x^2 + 14x + 49 = (x + 7)^2$

✎ *Pencil Problem #2*

2a. Factor: $x^2 + 2x + 1$

2b. Factor: $16x^2 - 56x + 49$

Notice that the trinomial fits the form $A^2 - 2AB + B^2$.

Thus, factor using $A^2 - 2AB + B^2 = (A - B)^2$.

$16x^2 - 56x + 49 = (4x - 7)^2$

2b. Factor: $25y^2 - 10y + 1$

2c. Factor: $4x^2 + 12xy + 9y^2$

Notice that the trinomial fits the form $A^2 + 2AB + B^2$.

Thus, factor using $A^2 + 2AB + B^2 = (A + B)^2$.

$4x^2 + 12xy + 9y^2 = (2x + 3y)^2$

2c. Factor: $16x^2 - 40xy + 25y^2$

Objective #3: Factor the sum or difference of two cubes.

✔ *Solved Problem #3*

3a. Factor: $x^3 + 27$

Notice that the polynomial fits the form $A^3 + B^3$.

Thus it factors as $(A + B)(A^2 - AB + B^2)$.

$$x^3 + 27 = x^3 + 3^3$$
$$= (x + 3)(x^2 - 3x + 3^2)$$
$$= (x + 3)(x^2 - 3x + 9)$$

✎ *Pencil Problem #3*

3a. Factor: $x^3 + 1$

3b. Factor: $1-y^3$

Notice that the polynomial fits the form A^3-B^3.

Thus it factors as $(A-B)(A^2+AB+B^2)$.

$$\begin{aligned}1-y^3 &= 1^3-y^3\\ &= (1-y)(1^2+1\cdot y+y^2)\\ &= (1-y)(1+y+y^2)\end{aligned}$$

3b. Factor: $8y^3-1$

3c. Factor: $125x^3+8$

Notice that the polynomial fits the form A^3+B^3.

Thus it factors as $(A+B)(A^2-AB+B^2)$.

$$\begin{aligned}125x^3+8 &= (5x)^3+2^3\\ &= (5x+2)\left[(5x)^2-(5x)(2)+2^2\right]\\ &= (5x+2)(25x^2-10x+4)\end{aligned}$$

3c. Factor: $64x^3+27y^3$

Answers for Pencil Problems *(Textbook Exercise references in parentheses)*:

1a. $(x+5)(x-5)$ *(6.4 #1)* **1b.** prime *(6.4 #31)* **1c.** $(7y^2+4)(7y^2-4)$ *(6.4 #15)*

1d. $2x(x+6)(x-6)$ *(6.4 #29)* **1e.** $(x^2+4)(x+2)(x-2)$ *(6.4 #23)* **2a.** $(x+1)^2$ *(6.4 #45)*

2b. $(5y-1)^2$ *(6.4 #55)* **2c.** $(4x-5y)^2$ *(6.4 #65)* **3a.** $(x+1)(x^2-x+1)$ *(6.4 #79)*

3b. $(2y-1)(4y^2+2y+1)$ *(6.4 #83)* **3c.** $(4x+3y)(16x^2-12xy+9y^2)$ *(6.4 #93)*

Homework:

- ☐ **Review the Section 6.4 summary** on page 476 of the textbook.
- ☐ **Insert your homework** into this section of the *Learning Guide*. Show all work neatly and check your answers. Strive to work through difficulties when possible, making note of any exercises where you need additional help. Remember, even if your instructor assigns homework through *MyMathLab*, you should still write out your work.

