Section 6.3
Factoring Trinomials Whose Leading Coefficient Is Not 1

Is ONE the Loneliest Number?

The number 1 does not appear to be lonely when it comes to language.

The words "unit," "unity," "union," "unique," and "universal" are all derived from the Latin word for "one."

For the ancient Greeks, 1 was the indivisible unit from which all other numbers arose.

The Greek's philosophy of 1 applies to our work in this section. Factoring trinomials whose leading coefficient is 1 is the basic technique from which other methods of factoring will follow.

In this section of the text, we factor trinomials with leading coefficients that are not 1.

First Steps:

- ☐ **Take comprehensive notes** from your instructor's lecture and insert your notes into this section of the *Learning Guide*. Be sure to write down all examples, definitions, and other key concepts. Additional learning resources include the *Lecture Series on DVD*, the *PowerPoints*, and Section 6.3 of your textbook which begins on page 436.
- ☐ Complete the ***Concept and Vocabulary Check*** on page 441 of the textbook.

Here is an observation that sometimes helps narrow down the list of possible factorizations.

If a polynomial does not have a GCF other than 1 or if you have factored out the GCF, there will be no common factor within any of its binomial factors.

Here is an example:

$6x^2 - 17x + 12$ — There is no GCF other than 1.

$(2x - 4)(3x + 3)$ — This is not a possible factorization.

This binomial has a common factor of 2.

This binomial has a common factor of 3.

Guided Practice:

☐ Review each of the following ***Solved Problems*** and complete each ***Pencil Problem***.

Objective #1: Factor trinomials by trial and error.

✔ *Solved Problem #1*

1a. Factor $6x^2+19x-7$ by trial and error.

Step 1 Find two first terms whose product is $6x^2$.

$6x^2+19x-7=(6x\quad)(x\quad)$

$6x^2+19x-7=(3x\quad)(2x\quad)$

Step 2 The last term, –7, has possible factorizations of $1(-7)$ and $-1(7)$.

Step 3

Possible Factors of $6x^2+19x-7$	Sum of Outside and Inside Products
$(6x+1)(x-7)$	$-42x+x=-41x$
$(6x-7)(x+1)$	$6x-7x=-x$
$(6x-1)(x+7)$	$42x-x=41x$
$(6x+7)(x-1)$	$-6x+7x=x$
$(3x+1)(2x-7)$	$-21x+2x=-19x$
$(3x-7)(2x+1)$	$3x-14x=-11x$
$(3x-1)(2x+7)$	$\overbrace{21x-2x}^{\text{required middle term}}=19x$
$(3x+7)(2x-1)$	$-3x+14x=11x$

The required middle term is obtained by using the factors $(3x-1)(2x+7)$.

Check: $(3x-1)(2x+7)=6x^2+21x-2x-7$

$=6x^2+19x-7$

Thus, $6x^2+19x-7=(3x-1)(2x+7)$

✎ *Pencil Problem #1* ✎

1a. Factor $2x^2+5x+3$ by trial and error.

1b. Factor $3x^2 - 13xy + 4y^2$ by trial and error.

Step 1 Find two First terms whose product is $3x^2$.

$$3x^2 - 13xy + 4y^2 = (3x \quad)(x \quad)$$

Step 2 The last term, $4y^2$, has pairs of factors that are either both positive or both negative. Because the middle term, $-13xy$, is negative, both factors must be negative. Thus the last term has possible factorizations of $-2y(-2y)$ or $-y(-4y)$.

Step 3

Possible Factors of $3x^2 - 13xy + 4y^2$	Sum of Outside and Inside Products
$(3x-4y)(x-y)$	$-3xy - 4xy = -7xy$
$(3x-y)(x-4y)$	$\overbrace{-12xy - xy}^{\text{required middle term}} = -13xy$
$(3x-2y)(x-2y)$	$-6xy - 2xy = -8xy$

The required middle term is obtained by using the factors $(3x-y)(x-4y)$.

Check: $(3x-y)(x-4y) = 3x^2 - 12xy - xy + 4y^2$

$$= 3x^2 - 13xy + 4y^2$$

Thus, $3x^2 - 13xy + 4y^2 = (3x-y)(x-4y)$

1b. Factor $3x^2 + 5xy + 2y^2$ by trial and error.

Objective #2: Factor trinomials by grouping.

✔ *Solved Problem #2*

2a. Factor $3x^2 - x - 10$ by grouping.

$a = 3$ and $c = -10$, so $ac = 3(-10) = -30$.

The factors of –30 whose sum is –1 are 5 and –6.

$$3x^2 - x - 10 = 3x^2 + 5x - 6x - 10$$
$$= x(3x+5) - 2(3x+5)$$
$$= (3x+5)(x-2)$$

Check: $(3x+5)(x-2) = 3x^2 - 6x + 5x - 10$

$$= 3x^2 - x - 10$$

Thus, $3x^2 - x - 10 = (3x+5)(x-2)$

✎ *Pencil Problem #2*

2a. Factor $9y^2 - 9y + 2$ by grouping.

2b. Factor $8x^2 - 10x + 3$ by grouping.

$a = 8$ and $c = 3$, so $ac = 8(3) = 24$.

The factors of 24 whose sum is –10 are –6 and –4.

$$\begin{aligned} 8x^2 - 10x + 3 &= 8x^2 - 4x - 6x + 3 \\ &= 4x(2x-1) - 3(2x-1) \\ &= (2x-1)(4x-3) \end{aligned}$$

2b. Factor $20x^2 + 27x - 8$ by grouping.

2c. Factor completely: $5y^4 + 13y^3 + 6y^2$

First factor out the greatest common factor (GCF) of y^2.

$$5y^4 + 13y^3 + 6y^2 = y^2(5y^2 + 13y + 6)$$

Then factor the resulting trinomial.

$$\begin{aligned} 5y^4 + 13y^3 + 6y^2 &= y^2(5y^2 + 13y + 6) \\ &= y^2(5y+3)(y+2) \end{aligned}$$

2c. Factor completely: $4x^2 + 26x + 30$

Answers for Pencil Problems *(Textbook Exercise references in parentheses)*:

1a. $(2x+3)(x+1)$ *(6.3 #1)* **1b.** $(3x+2y)(x+y)$ *(6.3 #45)*

2a. $(3y-1)(3y-2)$ *(6.3 #39)* **2b.** $(4x-1)(5x+8)$ *(6.3 #41)* **2c.** $2(2x+3)(x+5)$ *(6.3 #59)*

Homework:

- ☐ **Review the Section 6.3 summary** on page 476 of the textbook.
- ☐ **Insert your homework** into this section of the *Learning Guide*. Show all work neatly and check your answers. Strive to work through difficulties when possible, making note of any exercises where you need additional help. Remember, even if your instructor assigns homework through *MyMathLab*, you should still write out your work.