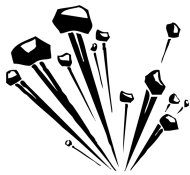


## Section 6.1

# The Greatest Common Factor and Factoring by Grouping

### **KABOOM!!!!**



An explosion caused debris to rise vertically with an initial velocity of 64 feet per second. It is possible to calculate the height of the debris at any given time after the explosion.



In this section of the textbook, we will look at how a polynomial can be used to model this situation, and we will apply the concept of factoring to such polynomials.

#### **First Steps:**

- Take comprehensive notes** from your instructor's lecture and insert your notes into this section of the *Learning Guide*. Be sure to write down all examples, definitions, and other key concepts. Additional learning resources include the *Lecture Series on DVD*, the *PowerPoints*, and Section 6.1 of your textbook which begins on page 418.
- Complete the *Concept and Vocabulary Check* on page 424 of the textbook.

#### **Guided Practice:**

- Review each of the following *Solved Problems* and complete each *Pencil Problem*.

**Objective #1:** Find the greatest common factor.

#### ✓ **Solved Problem #1**

- 1a.** Find the greatest common factor of the following list of monomials:  $18x^3$  and  $15x^2$

$$18x^3 = 3x^2 \cdot 6x$$

$$15x^2 = 3x^2 \cdot 5$$

The GCF is  $3x^2$ .

- 1b.** Find the greatest common factor of the following list of monomials:  $x^4y$ ,  $x^3y^2$ , and  $x^2y$

$$x^4y = x^2y \cdot x^2$$

$$x^3y^2 = x^2y \cdot xy$$

$$x^2y = x^2y$$

The GCF is  $x^2y$ .

#### ✎ **Pencil Problem #1** ✎

- 1a.** Find the greatest common factor of the following list of monomials:  $12x^2$  and  $8x$

- 1b.** Find the greatest common factor of the following list of monomials:  $16x^5y^4$ ,  $8x^6y^3$ , and  $20x^4y^5$

<b>Objective #2:</b> Factor out the greatest common factor of a polynomial.
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<p style="text-align: center;"> <b>Solved Problem #2</b></p> <p><b>2a.</b> Factor: <math>6x^2 + 18</math></p> <p>The GCF is 6.</p> $6x^2 + 18 = 6 \cdot x^2 + 6 \cdot 3$ $= 6(x^2 + 3)$ <hr/> <p><b>2b.</b> Factor: <math>25x^2 + 35x^3</math></p> <p>The GCF is <math>5x^2</math>.</p> $25x^2 + 35x^3 = 5x^2 \cdot 5 + 5x^2 \cdot 7x$ $= 5x^2(5 + 7x)$ <hr/> <p><b>2c.</b> Factor: <math>15x^5 + 12x^4 - 27x^3</math></p> <p>The GCF is <math>3x^3</math>.</p> $15x^5 + 12x^4 - 27x^3 = 3x^3 \cdot 5x^2 + 3x^3 \cdot 4x - 3x^3 \cdot 9$ $= 3x^3(5x^2 + 4x - 9)$ <hr/> <p><b>2d.</b> Factor: <math>8x^3y^2 - 14x^2y + 2xy</math></p> <p>The GCF is <math>2xy</math>.</p> $8x^3y^2 - 14x^2y + 2xy = 2xy \cdot 4x^2y - 2xy \cdot 7x + 2xy \cdot 1$ $= 2xy(4x^2y - 7x + 1)$	<p style="text-align: center;"> <b>Pencil Problem #2</b></p> <p><b>2a.</b> Factor: <math>18y^2 + 12</math></p> <hr/> <p><b>2b.</b> Factor: <math>8x^2 - 4x^4</math></p> <hr/> <p><b>2c.</b> Factor: <math>100y^5 - 50y^3 + 100y^2</math></p> <hr/> <p><b>2d.</b> Factor: <math>11x^2 - 23</math></p>
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**Objective #3:** Factor out the negative of the greatest common factor of a polynomial.

 **Solved Problem #3**

3. Factor:  $-16a^4b^5 + 24a^3b^4 - 20ab^2$

It is preferable to have a first term with a positive coefficient inside parentheses.

Thus, we will factor out  $-4ab^2$ .

$$\begin{aligned} & -16a^4b^5 + 24a^3b^4 - 20ab^2 \\ &= -4ab^2 \cdot 4a^3b^3 - 4ab^2 \cdot (-6a^2b^2) - 4ab^2 \cdot 5 \\ &= -4ab^2(4a^3b^3 - 6a^2b^2 + 5) \end{aligned}$$

 **Pencil Problem #3** 

3. Factor:  $-4a^3b^2 + 6ab$

**Objective #4:** Factor by grouping.

 **Solved Problem #4**

4a. Factor:  $x^2(x+1) + 7(x+1)$

Factor out the greatest common factor of  $x+1$ .

$$x^2 \overbrace{(x+1)}^{\text{GCF}} + 7 \overbrace{(x+1)}^{\text{GCF}} = (x+1)(x^2 + 7)$$

 **Pencil Problem #4** 

4a. Factor:  $x(x+5) + 3(x+5)$

4b. Factor:  $x(y+4) - 7(y+4)$

Factor out the greatest common factor of  $y+4$ .

$$x \overbrace{(y+4)}^{\text{GCF}} - 7 \overbrace{(y+4)}^{\text{GCF}} = (y+4)(x-7)$$

4b. Factor:  $3x(x+y) - (x+y)$

**4c.** Factor:  $x^3 + 5x^2 + 2x + 10$ 

There is no factor other than 1 common to all four terms. However, we can group terms that have a common factor.

$$x^3 + 5x^2 + 2x + 10 = (x^3 + 5x^2) + (2x + 10)$$

Factor out the greatest common factor from the grouped terms. The remaining two terms have  $x + 5$  as a common binomial factor, which should then be factored out.

$$\begin{aligned} x^3 + 5x^2 + 2x + 10 &= (x^3 + 5x^2) + (2x + 10) \\ &= x^2(x + 5) + 2(x + 5) \\ &= (x + 5)(x^2 + 2) \end{aligned}$$

**4c.** Factor:  $x^2 + 2x + 4x + 8$ **4d.** Factor:  $xy + 3x - 5y - 15$ 

There is no factor other than 1 common to all four terms. However, we can group terms that have a common factor.

$$xy + 3x - 5y - 15 = (xy + 3x) + (-5y - 15)$$

Factor out  $x$  from the first two grouped terms and  $-5$  from the last two grouped terms.

$$xy + 3x - 5y - 15 = x(y + 3) - 5(y + 3)$$

Finally, factor out the common factor of  $y + 3$ .

$$\begin{aligned} xy + 3x - 5y - 15 &= x(y + 3) - 5(y + 3) \\ &= (y + 3)(x - 5) \end{aligned}$$

**4d.** Factor:  $3x^3 - 2x^2 - 6x + 4$ 

**Answers for Pencil Problems (Textbook Exercise references in parentheses):**

**1a.**  $4x$  (6.1 #3)   **1b.**  $4x^4y^3$  (6.1 #11)   **2a.**  $6(3y^2 + 2)$  (6.1 #23)   **2b.**  $4x^2(2 - x^2)$  (6.1 #31)

**2c.**  $50y^2(2y^3 - y + 2)$  (6.1 #37)   **2d.** cannot be factored (6.1 #41)   **3.**  $-2ab(2a^2b - 3)$  (6.1 #53)

**4a.**  $(x + 5)(x + 3)$  (6.1 #57)   **4b.**  $(x + y)(3x - 1)$  (6.1 #63)

**4c.**  $(x + 2)(x + 4)$  (6.1 #69)   **4d.**  $(3x - 2)(x^2 - 2)$  (6.1 #83)

**Homework:**

- Review the Section 6.1 summary** which begins on page 475 of the textbook.
- Insert your homework** into this section of the *Learning Guide*. Show all work neatly and check your answers. Strive to work through difficulties when possible, making note of any exercises where you need additional help. Remember, even if your instructor assigns homework through *MyMathLab*, you should still write out your work.