Section 5.7
Negative Exponents and Scientific Notation

What is YOUR Share?

In a recent year there was a budget deficit of about $1,350,000,000,000 and there were approximately 307,000,000 Americans.

In this section of the textbook, we will use exponents and scientific notation to explore these values.

First Steps:

- ☐ **Take comprehensive notes** from your instructor's lecture and insert your notes into this section of the *Learning Guide*. Be sure to write down all examples, definitions, and other key concepts. Additional learning resources include the *Lecture Series on DVD*, the *PowerPoints*, and Section 5.7 of your textbook which begins on page 396.
- ☐ Complete the ***Concept and Vocabulary Check*** on page 406 of the textbook.

Guided Practice:

- ☐ Review each of the following ***Solved Problems*** and complete each ***Pencil Problem***.

Objective #1: Use the negative exponent rule.

✔ *Solved Problem #1*

1a. Use the negative exponent rule to write 5^{-3} with a positive exponent and then simplify.

$$5^{-3} = \frac{1}{5^3}$$
$$= \frac{1}{125}$$

1b. Use the negative exponent rule to write $(-3)^{-4}$ with a positive exponent and then simplify.

$$(-3)^{-4} = \frac{1}{(-3)^4}$$
$$= \frac{1}{81}$$

Pencil Problem #1

1a. Use the negative exponent rule to write 8^{-2} with a positive exponent and then simplify.

1b. Use the negative exponent rule to write $(-6)^{-2}$ with a positive exponent and then simplify.

1c. Use the negative exponent rule to write 8^{-1} with a positive exponent and then simplify.

$$8^{-1} = \frac{1}{8^1}$$
$$= \frac{1}{8}$$

1c. Use the negative exponent rule to write -6^{-2} with a positive exponent and then simplify.

1d. Write $\left(\frac{4}{5}\right)^{-2}$ with positive exponents only. Then simplify, if possible.

$$\left(\frac{4}{5}\right)^{-2} = \frac{5^2}{4^2}$$
$$= \frac{25}{16}$$

1d. Write $\left(\frac{1}{4}\right)^{-2}$ with positive exponents only. Then simplify, if possible.

1e. Write $\frac{1}{7y^{-2}}$ with positive exponents only. Then simplify, if possible.

$$\frac{1}{7y^{-2}} = \frac{y^2}{7}$$

1e. Write $\frac{1}{6x^{-5}}$ with positive exponents only. Then simplify, if possible.

1f. Write $\frac{x^{-1}}{y^{-8}}$ with positive exponents only. Then simplify, if possible.

$$\frac{x^{-1}}{y^{-8}} = \frac{y^8}{x^1}$$
$$= \frac{y^8}{x}$$

1f. Write $\frac{x^{-8}}{y^{-1}}$ with positive exponents only. Then simplify, if possible.

Objective #2: Simplify exponential expressions.

✔ *Solved Problem #2*

2a. Simplify: $x^{-12} \cdot x^2$

$$\begin{aligned} x^{-12} \cdot x^2 &= x^{-12+2} \\ &= x^{-10} \\ &= \frac{1}{x^{10}} \end{aligned}$$

2b. Simplify: $\dfrac{75x^3}{5x^9}$

$$\begin{aligned} \frac{75x^3}{5x^9} &= \frac{75}{5} \cdot \frac{x^3}{x^9} \\ &= 15x^{3-9} \\ &= 15x^{-6} \\ &= \frac{15}{x^6} \end{aligned}$$

2c. Simplify: $\dfrac{(6x^4)^2}{x^{11}}$

$$\begin{aligned} \frac{(6x^4)^2}{x^{11}} &= \frac{6^2(x^4)^2}{x^{11}} \\ &= \frac{36x^{4 \cdot 2}}{x^{11}} \\ &= \frac{36x^8}{x^{11}} \\ &= 36x^{8-11} \\ &= 36x^{-3} \\ &= \frac{36}{x^3} \end{aligned}$$

✎ *Pencil Problem #2* ✎

2a. Simplify: $x^{-8} \cdot x^3$

2b. Simplify: $\dfrac{30z^5}{10z^{10}}$

2c. Simplify: $\dfrac{y^{-3}}{(y^4)^2}$

2d. Simplify: $\left(\frac{x^8}{x^4}\right)^{-5}$

$$\left(\frac{x^8}{x^4}\right)^{-5} = \left(x^4\right)^{-5}$$
$$= x^{-20}$$
$$= \frac{1}{x^{20}}$$

2d. Simplify: $\left(\frac{x^4}{x^2}\right)^{-3}$

Objective #3: Convert from scientific notation to decimal notation.

✔ *Solved Problem #3*

3a. Write 7.4×10^9 in decimal notation.

The exponent is positive so we move the decimal point nine places to the right.

$7.4\times10^9 = 7,400,000,000$

✎ *Pencil Problem #3*

3a. Write 9.23×10^5 in decimal notation.

3b. Write 3.017×10^{-6} in decimal notation.

The exponent is negative so we move the decimal point six places to the left.

$3.017\times10^{-6} = 0.000003017$

3b. Write 7.86×10^{-4} in decimal notation.

Objective #4: Convert from decimal notation to scientific notation.

✔ *Solved Problem #4*

4a. Write $7,410,000,000$ in scientific notation.

$7,410,000,000 = 7.41\times10^9$

✎ *Pencil Problem #4*

4a. Write $220,000,000$ in scientific notation.

4b. Write 0.000000092 in scientific notation.

$0.000000092 = 9.2\times10^{-8}$

4b. Write 0.0000202 in scientific notation.

Objective #5: Compute with scientific notation.

✔ *Solved Problem #5*

5a. Perform the indicated computation, writing the answer in scientific notation: $(3\times10^{8})(2\times10^{2})$

$$\begin{aligned}(3\times10^{8})(2\times10^{2}) &= (3\times2)\times(10^{8}\times10^{2})\\ &= 6\times10^{8+2}\\ &= 6\times10^{10}\end{aligned}$$

✎ *Pencil Problem #5* ✎

5a. Perform the indicated computation, writing the answer in scientific notation: $(2\times10^{3})(3\times10^{2})$

5b. Perform the indicated computation, writing the answer in scientific notation: $\dfrac{8.4\times10^{7}}{4\times10^{-4}}$

$$\begin{aligned}\frac{8.4\times10^{7}}{4\times10^{-4}} &= \frac{8.4}{4}\cdot\frac{10^{7}}{10^{-4}}\\ &= 2.1\times10^{7-(-4)}\\ &= 2.1\times10^{11}\end{aligned}$$

5b. Perform the indicated computation, writing the answer in scientific notation: $\dfrac{15\times10^{-4}}{5\times10^{2}}$

5c. Perform the indicated computation, writing the answer in scientific notation: $(4\times10^{-2})^{3}$

$$\begin{aligned}(4\times10^{-2})^{3} &= 4^{3}\times(10^{-2})^{3}\\ &= 64\times10^{-6}\\ &= 6.4\times10^{-5}\end{aligned}$$

5c. Perform the indicated computation, writing the answer in scientific notation: $(3\times10^{-2})^{4}$

Objective #6: Solve applied problems using scientific notation.

✔ *Solved Problem #6*

6. The cost of President Obama's 2009 economic stimulus package was \$787 billion, or 7.87×10^{11} dollars. If this cost were evenly divided among every individual in the United States (approximately 3.07×10^{8} people), how much would each citizen have to pay?

$$\frac{7.87\times10^{11}}{3.07\times10^{8}}=\frac{7.87}{3.07}\times\frac{10^{11}}{10^{8}}\approx 2.56\times10^{3}=2560$$

Each citizen would have to pay about \$2560.

✎ *Pencil Problem #6* ✎

6. If there are approximately 3.2×10^{7} seconds in a year, approximately how many years is 1.35 trillion seconds?

Answers for Pencil Problems *(Textbook Exercise references in parentheses)*:

1a. $\frac{1}{8^2}=\frac{1}{64}$ *(5.7 #1)* **1b.** $\frac{1}{(-6)^2}=\frac{1}{36}$ *(5.7 #5)* **1c.** $-\frac{1}{6^2}=-\frac{1}{36}$ *(5.7 #7)*

1d. 16 *(5.7 #19)* **1e.** $\frac{x^5}{6}$ *(5.7 #23)* **1f.** $\frac{y}{x^8}$ *(5.7 # 25)*

2a. $\frac{1}{x^5}$ *(5.7 #29)* **2b.** $\frac{3}{z^5}$ *(5.7 # 37)* **2c.** $\frac{1}{y^{11}}$ *(5.7 # 47)* **2d.** $\frac{1}{x^6}$ *(5.7 #53)*

3a. 923,000 *(5.7 #81)* **3b.** 0.000786 *(5.7 #89)*

4a. 2.2×10^{8} *(5.7 #93)* **4b.** 2.02×10^{-5} *(5.7 #101)*

5a. 6×10^{5} *(5.7 #107)* **5b.** 3×10^{-6} *(5.7 #115)* **5c.** 8.1×10^{-7} *(5.7 #123)*

6. 42,000 years *(5.7 #141)*

Homework:

- ☐ **Review the Section 5.7 summary** on page 412 of the textbook.
- ☐ **Insert your homework** into this section of the *Learning Guide*. Show all work neatly and check your answers. Strive to work through difficulties when possible, making note of any exercises where you need additional help. Remember, even if your instructor assigns homework through *MyMathLab*, you should still write out your work.