

Section 5.6

Dividing Polynomials by Binomials

Are You Having a Déjà Vu?

In this section of the textbook, we will study the process for dividing a polynomial by a binomial. This process may remind you of long division of whole numbers.

Let's review long division of whole numbers by dividing 3983 by 26.

Divisor	$26 \overline{)3983}$	Dividend
$\begin{array}{r} 1 \\ 26 \overline{)3983} \\ \underline{26} \\ 138 \end{array}$	\downarrow	\downarrow
	<p>DIVIDE: $\frac{39}{26} = 1$ plus a remainder. MULTIPLY: $1 \cdot 26 = 26$. SUBTRACT: $39 - 26 = 13$. BRING DOWN the next digit in the dividend.</p>	
$\begin{array}{r} 15 \\ 26 \overline{)3983} \\ \underline{26} \\ 138 \\ \underline{130} \\ 83 \end{array}$	\downarrow	\downarrow
	<p>DIVIDE: $\frac{138}{26} = 5$ plus a remainder. MULTIPLY: $5 \cdot 26 = 130$. SUBTRACT: $138 - 130 = 8$. BRING DOWN the next digit in the dividend.</p>	
$\begin{array}{r} 153 \\ 26 \overline{)3983} \\ \underline{26} \\ 138 \\ \underline{130} \\ 83 \\ \underline{78} \\ 5 \end{array}$	\downarrow	\downarrow
	<p>DIVIDE: $\frac{83}{26} = 3$ plus a remainder. MULTIPLY: $3 \cdot 26 = 78$. SUBTRACT: $83 - 78 = 5$. There are no more digits to bring down, so the remainder is 5.</p>	

The quotient is 153 and the remainder is 5. This can be written as $153\frac{5}{26}$.

First Steps:

- Take comprehensive notes** from your instructor's lecture and insert your notes into this section of the *Learning Guide*. Be sure to write down all examples, definitions, and other key concepts. Additional learning resources include the *Lecture Series on DVD*, the *PowerPoints*, and Section 5.6 of your textbook which begins on page 386.
- Complete the *Concept and Vocabulary Check* on page 393 of the textbook.

Guided Practice:

□ Review each of the following *Solved Problems* and complete each *Pencil Problem*.

Objective #1: Divide polynomials by binomials.

 **Solved Problem #1**

1a. Divide $x^2 + 14x + 45$ by $x + 9$.

Arrange the terms in the dividend, $x^2 + 14x + 45$, and the divisor, $x + 9$, in descending order.

$$x + 9 \overline{) x^2 + 14x + 45}$$

Divide x^2 by x : $\frac{x^2}{x} = x$. Align like terms.

$$x + 9 \overline{) x^2 + 14x + 45} \quad \begin{array}{r} x \\ \end{array}$$

Multiply each term in the divisor, $x + 9$, by x .

$$x + 9 \overline{) x^2 + 14x + 45} \quad \begin{array}{r} x \\ x^2 + 9x \\ \hline \end{array}$$

Subtract $x^2 + 9x$ from $x^2 + 14x$ and bring down the 45.

$$x + 9 \overline{) x^2 + 14x + 45} \quad \begin{array}{r} x \\ x^2 + 9x \\ \hline 5x + 45 \end{array}$$

Divide $5x$ by x : $\frac{5x}{x} = 5$. Next, multiply each term in the divisor by 5, and then subtract.

$$x + 9 \overline{) x^2 + 14x + 45} \quad \begin{array}{r} x + 5 \\ x^2 + 9x \\ \hline 5x + 45 \\ 5x + 45 \\ \hline 0 \end{array}$$

The quotient is $x + 5$ and the remainder is 0.

Thus, $\frac{x^2 + 14x + 45}{x + 9} = x + 5$

 **Pencil Problem #1** 

1a. Divide $x^2 - 5x + 6$ by $x - 3$.

1b. Divide: $\frac{6x+8x^2-12}{2x+3}$

Arrange the terms in the numerator (dividend),
in descending order:

$$\frac{6x+8x^2-12}{2x+3} = \frac{8x^2+6x-12}{2x+3}$$

$$\begin{array}{r} 4x-3 \\ 2x+3 \overline{)8x^2+6x-12} \\ \underline{8x^2+12x} \\ -6x-12 \\ \underline{-6x-9} \\ -3 \end{array}$$

Write your answer in the following format:

$$\frac{\text{Dividend}}{\text{Divisor}} = \text{Quotient} + \frac{\text{Remainder}}{\text{Divisor}}$$

Thus,

$$\begin{aligned} \frac{6x+8x^2-12}{2x+3} &= \frac{\text{Quotient}}{4x-3} - \frac{\text{Remainder}}{\underbrace{2x+3}_{\text{Divisor}}} \\ &= 4x-3 - \frac{3}{2x+3} \end{aligned}$$

1b. Divide: $\frac{5y+10+y^2}{y+2}$

1c. Divide: $\frac{x^3-1}{x-1}$

Rewrite x^3-1 using coefficients of 0 on the missing terms gives x^3+0x^2+0x-1 .

$$\begin{array}{r} x^2+x+1 \\ x-1 \overline{)x^3+0x^2+0x-1} \\ \underline{x^3-x^2} \\ x^2+0x \\ \underline{x^2-x} \\ x-1 \\ \underline{x-1} \\ 0 \end{array}$$

The quotient is x^2+x+1 and the remainder is 0.

Thus,

$$\frac{x^3-1}{x-1} = x^2+x+1$$

1c. Divide: $\frac{y^4-2y^2+5}{y-1}$

Answers for Pencil Problems (Textbook Exercise references in parentheses):

1a. $x-2$ (5.6 #5)

1b. $y+3+\frac{4}{y+2}$ (5.6 #11)

1c. $y^3+y^2-y-1+\frac{4}{y-1}$ (5.6 #35)

Homework:

- Review the Section 5.6 summary** on page 412 of the textbook.
- Insert your homework** into this section of the *Learning Guide*. Show all work neatly and check your answers. Strive to work through difficulties when possible, making note of any exercises where you need additional help. Remember, even if your instructor assigns homework through *MyMathLab*, you should still write out your work.