

## Section 5.4

### Polynomials in Several Variables

#### ***Timber !!!***



The next time you visit a lumberyard and go rummaging through piles of wood, think polynomials! But these polynomials are a bit different from those we have encountered so far.

In this section of the textbook, we will explore a polynomial in two variables that the construction industry uses to determine the number of board feet that can be manufactured from a tree with a diameter of  $x$  inches and a length of  $y$  feet.



#### **First Steps:**

- Take comprehensive notes** from your instructor's lecture and insert your notes into this section of the *Learning Guide*. Be sure to write down all examples, definitions, and other key concepts. Additional learning resources include the *Lecture Series on DVD*, the *PowerPoints*, and Section 5.4 of your textbook which begins on page 368.
- Complete the *Concept and Vocabulary Check* on page 373 of the textbook.

#### **Guided Practice:**

- Review each of the following *Solved Problems* and complete each *Pencil Problem*.

**Objective #1:** Evaluate polynomials in several variables.

#### **✓ Solved Problem #1**

1. Evaluate  $3x^3y + xy^2 + 5y + 6$  for  $x = -1$  and  $y = 5$ .

Begin by substituting  $-1$  in for  $x$  and  $5$  in for  $y$ .

$$\begin{aligned} 3x^3y + xy^2 + 5y + 6 &= 3(-1)^3(5) + (-1)(5)^2 + 5(5) + 6 \\ &= 3(-1)(5) + (-1)(25) + 5(5) + 6 \\ &= -15 - 25 + 25 + 6 \\ &= -9 \end{aligned}$$

#### **Pencil Problem #1**

1. Evaluate  $2x^2y - 5y + 3$  for  $x = 2$  and  $y = -3$ .

**Objective #2:** Understand the vocabulary of polynomials in two variables.

 **Solved Problem #2**

2. Determine the coefficient of each term, the degree of each term, and the degree of the polynomial:

$$8x^4y^5 - 7x^3y^2 - x^2y - 5x + 11$$

$$\overbrace{8x^4y^5}^{\text{Term}} - \overbrace{7x^3y^2}^{\text{Term}} - \overbrace{x^2y}^{\text{Term}} - \overbrace{5x}^{\text{Term}} + \overbrace{11}^{\text{Term}}$$

Term	Coefficient	Degree
$8x^4y^5$	8	$4 + 5 = 9$
$-7x^3y^2$	-7	$3 + 2 = 5$
$-x^2y$	-1	$2 + 1 = 3$
$-5x$	-5	1
11	11	0

The degree of the polynomial is the highest degree of all its terms, which is 9.

 **Pencil Problem #2** 

2. Determine the coefficient of each term, the degree of each term, and the degree of the polynomial:

$$x^3y^2 - 5x^2y^7 + 6y^2 - 3$$

**Objective #3:** Add and subtract polynomials in several variables.

 **Solved Problem #3**

- 3a. Add:  $(-8x^2y - 3xy + 6) + (10x^2y + 5xy - 10)$

$$\begin{aligned} &(-8x^2y - 3xy + 6) + (10x^2y + 5xy - 10) \\ &= -8x^2y - 3xy + 6 + 10x^2y + 5xy - 10 \\ &= -8x^2y + 10x^2y - 3xy + 5xy + 6 - 10 \\ &= 2x^2y + 2xy - 4 \end{aligned}$$

- 3b. Subtract:

$$(7x^3 - 10x^2y + 2xy^2 - 5) - (4x^3 - 12x^2y - 3xy^2 + 5)$$

$$\begin{aligned} &(7x^3 - 10x^2y + 2xy^2 - 5) - (4x^3 - 12x^2y - 3xy^2 + 5) \\ &= 7x^3 - 10x^2y + 2xy^2 - 5 - 4x^3 + 12x^2y + 3xy^2 - 5 \\ &= 7x^3 - 4x^3 - 10x^2y + 12x^2y + 2xy^2 + 3xy^2 - 5 - 5 \\ &= 3x^3 + 2x^2y + 5xy^2 - 10 \end{aligned}$$

 **Pencil Problem #3** 

- 3a. Add:  $(4x^2y + 8xy + 11) + (-2x^2y + 5xy + 2)$

- 3b. Subtract:  $(x^3 + 7xy - 5y^2) - (6x^3 - xy + 4y^2)$

<b>Objective #4:</b> Multiply polynomials in several variables.
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**Solved Problem #4**

**4a.** Multiply:  $(6xy^3)(10x^4y^2)$

$$\begin{aligned}(6xy^3)(10x^4y^2) &= (6 \cdot 10)(x \cdot x^4)(y^3 \cdot y^2) \\ &= 60x^{1+4}y^{3+2} \\ &= 60x^5y^5\end{aligned}$$

**Pencil Problem #4**

**4a.** Multiply:  $(-8x^3y^4)(3x^2y^5)$

**4b.** Multiply:  $6xy^2(10x^4y^5 - 2x^2y + 3)$

$$\begin{aligned}6xy^2(10x^4y^5 - 2x^2y + 3) \\ &= 6xy^2 \cdot 10x^4y^5 - 6xy^2 \cdot 2x^2y + 6xy^2 \cdot 3 \\ &= 60x^{1+4}y^{2+5} - 12x^{1+2}y^{2+1} + 18xy^2 \\ &= 60x^5y^7 - 12x^3y^3 + 18xy^2\end{aligned}$$

**4b.** Multiply:  $4ab^2(7a^2b^3 + 2ab)$

**4c.** Multiply:  $(7x - 6y)(3x - y)$

$$\begin{aligned}(7x - 6y)(3x - y) \\ &= \overbrace{(7x)(3x)}^{\text{F}} + \overbrace{(7x)(-y)}^{\text{O}} + \overbrace{(-6y)(3x)}^{\text{I}} + \overbrace{(-6y)(-y)}^{\text{L}} \\ &= 21x^2 - 7xy - 18xy + 6y^2 \\ &= 21x^2 - 25xy + 6y^2\end{aligned}$$

**4c.** Multiply:  $(x - 3y)(2x + 7y)$

4d. Multiply:  $(6xy^2 + 5x)(6xy^2 - 5x)$

$$\begin{aligned} \overbrace{(6xy^2 + 5x)(6xy^2 - 5x)}^{(A+B)(A-B)} &= \overbrace{(6xy^2)^2}^{A^2} - \overbrace{(5x)^2}^{B^2} \\ &= 36x^2y^4 - 25x^2 \end{aligned}$$

4d. Multiply:  $(x^2 - 2y^2)^2$

**Answers for Pencil Problems (Textbook Exercise references in parentheses):**

1. -6 (5.4 #5)

Term	Coefficient	Degree
$x^3y^2$	1	$3 + 2 = 5$
$-5x^2y^7$	-5	$2 + 7 = 9$
$6y^2$	6	2
-3	-3	0

2. The degree of the polynomial is 9. (5.4 #7)

3a.  $2x^2y + 13xy + 13$  (5.4 #11)    3b.  $-5x^3 + 8xy - 9y^2$  (5.4 #15)

4a.  $-24x^5y^9$  (5.4 #29)    4b.  $28a^3b^5 + 8a^2b^3$  (5.4 #35)

4c.  $2x^2 + xy - 21y^2$  (5.4 #41)    4d.  $x^4 - 4x^2y^2 + 4y^4$  (5.4 #51)

**Homework:**

- Review the Section 5.4 summary on page 411 of the textbook.
- Insert your homework into this section of the *Learning Guide*. Show all work neatly and check your answers. Strive to work through difficulties when possible, making note of any exercises where you need additional help. Remember, even if your instructor assigns homework through *MyMathLab*, you should still write out your work.