

Section 4.4

Problem Solving Using Systems of Equations

Talk! Talk! Talk!



Many Americans say they cannot live without their cell phones. But choosing the best plan can be difficult.

In this section of the textbook, you will explore this question from a mathematical viewpoint.



First Steps:

- Take comprehensive notes** from your instructor's lecture and insert your notes into this section of the *Learning Guide*. Be sure to write down all examples, definitions, and other key concepts. Additional learning resources include the *Lecture Series on DVD*, the *PowerPoints*, and Section 4.4 of your textbook which begins on page 311.
- Complete the *Concept and Vocabulary Check* on page 321 of the textbook.

Guided Practice:

- Review each of the following *Solved Problems* and complete each *Pencil Problem*.

Objective #1: Solve problems using linear systems.

✓ *Solved Problem #1*

- 1a.** Socializing is a favorite leisure activity. Each weekend day, the sum of the average times spent socializing for men and women is 138 minutes. The difference between the average times spent socializing for women and men is 8 minutes. How many minutes per day on weekends do men and women devote to socializing?

Let x = average time per day women spend socializing.
Let y = average time per day men spend socializing.

$$x + y = 138$$

$$\underline{x - y = 8}$$

$$2x = 146$$

$$x = 73$$

Back-substitute 73 for x to find y .

$$x + y = 138$$

$$73 + y = 138$$

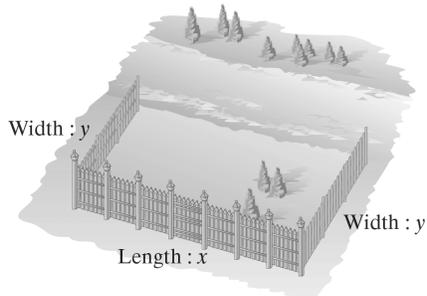
$$y = 65$$

Men average 65 minutes per day socializing and women average 73 minutes.

Pencil Problem #1

- 1a.** Each day, the sum of the average times spent on grooming for 20- to 24-year-old women and men is 86 minutes. The difference between grooming times for 20- to 24-year-old women and men is 12 minutes. How many minutes per day do 20- to 24-year-old women and men spend on grooming?

- 1b.** A rectangular lot whose perimeter is 360 feet is fenced along three sides.



An expensive fencing along the lot's length costs \$20 per foot. An inexpensive fencing along the two side widths costs only \$8 per foot. The total cost of the fencing along the three sides comes to \$3280. What are the lot's dimensions?

Let x = the length of the lot.
Let y = the width of the lot.

Use the formula for the perimeter of a rectangle to write the first equation.

$$P = 2l + 2w$$

$$360 = 2x + 2y$$

Use the other information in the problem to write the second equation.

$$20x + 8 \cdot 2y = 3280$$

The two equations form the system.

$$\begin{cases} 2x + 2y = 360 \\ 20x + 16y = 3280 \end{cases}$$

Multiply the first equation by -8 and add the result to the second equation.

$$-16x - 16y = -2880$$

$$\begin{array}{r} 20x + 16y = 3280 \\ \hline \end{array}$$

$$4x = 400$$

$$x = 100$$

Back-substitute to find y .

$$2x + 2y = 360$$

$$2(100) + 2y = 360$$

$$200 + 2y = 360$$

$$2y = 160$$

$$y = 80$$

The length is 100 feet and the width is 80 feet.

- 1b.** A rectangular lot whose perimeter is 320 feet is fenced along three sides. An expensive fencing along the lot's length costs \$16 per foot. An inexpensive fencing along the two side widths costs only \$5 per foot. The total cost of the fencing along the three sides comes to \$2140. What are the lot's dimensions?

Objective #2: Solve simple interest problems.
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 **Solved Problem #2**

2. Suppose that you invested \$25,000, part at 9% simple interest and the remainder at 12%. If the total yearly interest from these investments was \$2550, find the amount invested at each rate.

Let x = the amount invested at 9%.
 Let y = the amount invested at 12%.

$$\begin{cases} x + y = 25,000 \\ 0.09x + 0.12y = 2550 \end{cases}$$

This system can be solved by substitution.
 Solve for y in terms of x .

$$\begin{aligned} x + y &= 25,000 \\ y &= -x + 25,000 \end{aligned}$$

Substitute this value into the other equation.

$$\begin{aligned} 0.09x + 0.12y &= 2550 \\ 0.09x + 0.12(\overbrace{-x + 25,000}^y) &= 2550 \\ 0.09x - 0.12x + 3000 &= 2550 \\ -0.03x + 3000 &= 2550 \\ -0.03x &= -450 \\ x &= 15,000 \end{aligned}$$

Back-substitute to find y .

$$\begin{aligned} y &= -x + 25,000 \\ y &= -(15,000) + 25,000 \\ y &= 10,000 \end{aligned}$$

There was \$15,000 invested at 9% and \$10,000 invested at 12%.

 **Pencil Problem #2** 

2. A bank loaned out \$120,000, part of it at the rate of 8% annual mortgage interest and the rest at the rate of 18% annual credit card interest. The interest received on both loans totaled \$10,000. How much was loaned at each rate?

Objective #3: Solve mixture problems.**✓ Solved Problem #3**

3. A chemist needs to mix a 10% acid solution with a 60% acid solution to obtain 50 milliliters of a 30% acid solution. How many milliliters of each of the acid solutions must be used?

Let x = the number of milliliters of 10% acid solution.
Let y = the number of milliliters of 60% acid solution.

$$\begin{cases} x + y = 50 \\ 0.10x + 0.60y = 0.30(50) \end{cases}$$

This system can be solved by substitution.
Solve for y in terms of x .

$$\begin{aligned} x + y &= 50 \\ y &= -x + 50 \end{aligned}$$

Substitute this value into the other equation.

$$\begin{aligned} 0.10x + 0.60y &= 0.30(50) \\ 0.10x + 0.60y &= 15 \\ 0.10x + 0.60(\overbrace{-x + 50}^y) &= 15 \\ 0.10x - 0.60x + 30 &= 15 \\ -0.50x + 30 &= 15 \\ -0.50x &= -15 \\ x &= 30 \end{aligned}$$

Back-substitute to find y .

$$\begin{aligned} y &= -x + 50 \\ y &= -(30) + 50 \\ y &= 20 \end{aligned}$$

The chemist should mix 30 milliliters of the 10% acid solution and 20 milliliters of the 60% acid solution.

✎ Pencil Problem #3

3. A lab technician needs to mix a 5% fungicide solution with a 10% fungicide solution to obtain a 50-liter mixture consisting of 8% fungicide. How many liters of each of the fungicide solutions must be used?

Answers for Pencil Problems (Textbook Exercise references in parentheses):

- 1a. 20- to 24-year-old women averaged 49 minutes grooming and men averaged 37 minutes. (4.4 #5)
1b. 90 feet by 70 feet (4.4 #15) 2. \$116,000 was loaned at 8% and \$4000 was loaned at 18%. (4.4 #31)
3. 20 liters of 5% fungicide solution and 30 liters of 10% fungicide solution (4.4 #37)

Homework:

- Review the Section 4.4 summary** on page 334 of the textbook.
- Insert your homework** into this section of the *Learning Guide*. Show all work neatly and check your answers. Strive to work through difficulties when possible, making note of any exercises where you need additional help. Remember, even if your instructor assigns homework through *MyMathLab*, you should still write out your work.