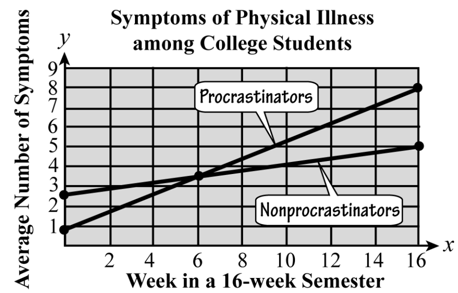
Section 4.3
Solving Systems of Linear Equations by the Addition Method

Procrastination makes you sick!

Researchers compared college students who were procrastinators and nonprocrastinators. Early in the semester, procrastinators reported fewer symptoms of illness, but late in the semester, they reported more symptoms than their nonprocrastinating peers.

In this section of the textbook, a third solution method, called addition, will verify (6, 3.5) as the point of intersection.



Source: Gerrig and Zimbardo, Psychology and Life, 18th Edition, Allyn and Bacon, 2008.

First Steps:

- ☐ **Take comprehensive notes** from your instructor's lecture and insert your notes into this section of the *Learning Guide*. Be sure to write down all examples, definitions, and other key concepts. Additional learning resources include the *Lecture Series on DVD*, the *PowerPoints*, and Section 4.3 of your textbook which begins on page 301.

- ☐ Complete the ***Concept and Vocabulary Check*** on page 307 of the textbook.

Guided Practice:

- ☐ Review each of the following ***Solved Problems*** and complete each ***Pencil Problem***.

Objective #1: Solve linear systems by the addition method.

✔ *Solved Problem #1*

1a. Solve the system: $\begin{cases} x + y = 5 \\ x - y = 9 \end{cases}$

Add the equations to elimininate the y-terms.

$$\begin{array}{l} x + y = 5 \\ \underline{x - y = 9} \\ 2x \quad = 14 \end{array}$$

Now solve for x.

$$2x = 14$$
$$x = 7$$

Back-substitute into either of the original equations to solve for y.

$$x + y = 5$$
$$7 + y = 5$$
$$y = -2$$

The solution set is $\{(7, -2)\}$.

✎ *Pencil Problem #1* ✎

1a. Solve the system: $\begin{cases} x + y = -3 \\ x - y = 11 \end{cases}$

1b. Solve the system: $\begin{cases} 4x - y = 22 \\ 3x + 4y = 26 \end{cases}$

Multiply each term of the first equation by 4 and add the equations to eliminate *y*.

$$\begin{array}{r} 16x - 4y = 88 \\ \underline{3x + 4y = 26} \\ 19x \quad = 114 \\ x = 6 \end{array}$$

Back-substitute into either of the original equations to solve for *y*.

$$\begin{aligned} 4x - y &= 22 \\ 4(6) - y &= 22 \\ 24 - y &= 22 \\ -y &= -2 \\ y &= 2 \end{aligned}$$

The solution set is $\{(6, 2)\}$.

1b. Solve the system: $\begin{cases} 3x + y = 7 \\ 2x - 5y = -1 \end{cases}$

1c. Solve the system: $\begin{cases} 4x + 5y = 3 \\ 2x - 3y = 7 \end{cases}$

Multiply each term of the second equation by –2 and add the equations to eliminate *x*.

$$\begin{array}{r} 4x + 5y = 3 \\ \underline{-4x + 6y = -14} \\ 11y = -11 \\ y = -1 \end{array}$$

Back-substitute into either of the original equations to solve for *x*.

$$\begin{aligned} 2x - 3y &= 7 \\ 2x - 3(-1) &= 7 \\ 2x + 3 &= 7 \\ 2x &= 4 \\ x &= 2 \end{aligned}$$

The solution set is $\{(2, -1)\}$.

1c. Solve the system: $\begin{cases} 3x - 4y = 11 \\ 2x + 3y = -4 \end{cases}$

1d. Solve the system: $\begin{cases} 2x = 9 + 3y \\ 4y = 8 - 3x \end{cases}$

Rewrite each equation in the form $Ax + By = C$.

$2x - 3y = 9$

$3x + 4y = 8$

Multiply the top equation by 4 and multiply the bottom equation by 3.

$$\begin{array}{r} 8x - 12y = 36 \\ \underline{9x + 12y = 24} \\ 17x \qquad = 60 \\ x = \dfrac{60}{17} \end{array}$$

Back-substitution of $\dfrac{60}{17}$ to find y would cause cumbersome arithmetic. Instead, use the system that is in the form $Ax + By = C$ to eliminate x and find y.

$2x - 3y = 9$

$3x + 4y = 8$

Multiply the top equation by –3 and multiply the bottom equation by 2.

$$\begin{array}{r} -6x + 9y = -27 \\ \underline{6x + 8y = \ \ 16} \\ 17y = -11 \\ y = \dfrac{-11}{17} \end{array}$$

The solution set is $\left\{\left(\dfrac{60}{17}, -\dfrac{11}{17}\right)\right\}$.

1d. Solve the system: $\begin{cases} 2x - y = 3 \\ 4x + 4y = -1 \end{cases}$

Objective #2:

Use the addition method to identify systems with no solution or infinitely many solutions.

✔ *Solved Problem #2*

2a. Solve the system: $\begin{cases} x + 2y = 4 \\ 3x + 6y = 13 \end{cases}$

Multiply the first equation by −3 and then add.

$$\begin{array}{r} -3x - 6y = -12 \\ \underline{3x + 6y = \ \ 13} \\ 0 = \ \ 1, \text{ false} \end{array}$$

The false statement indicates that the system is inconsistent and has no solution.

The solution set is $\{ \ \}$.

✎ *Pencil Problem #2* ✐

2a. Solve the system: $\begin{cases} 3x - y = 1 \\ 3x - y = 2 \end{cases}$

2b. Solve the system: $\begin{cases} x-5y=7 \\ 3x-15y=21 \end{cases}$

Multiply the first equation by −3 and then add.

$$\begin{array}{r} -3x+15y=-21 \\ \underline{3x-15y=\ \ 21} \\ 0=\ \ 0,\ \text{true} \end{array}$$

The true statement indicates that the system has infinitely many solutions.

The solution set is $\{(x,y)|x-5y=7\}$ or $\{(x,y)|3x-15y=21\}$.

2b. Solve the system: $\begin{cases} x+3y=2 \\ 3x+9y=6 \end{cases}$

Objective #3: Determine the most efficient method for solving a linear system.

✔ *Solved Problem #3*

3. True or False: If the solutions of a linear system do not involve integers, then it can be difficult to determine the exact solution when using the Graphing Method.

True

✎ *Pencil Problem #3* ✐

3. True or False: The Substitution Method is often the best choice when solving a linear system that has an equation with a variable that is on one side by itself.

Answers for Pencil Problems *(Textbook Exercise references in parentheses)*:

1a. $\{(4,-7)\}$ *(4.3 #1)* **1b.** $\{(2,1)\}$ *(4.3 #9)* **1c.** $\{(1,-2)\}$ *(4.3 #17)* **1d.** $\left\{\left(\frac{11}{12},-\frac{7}{6}\right)\right\}$ *(4.3 #25)*

2a. no solution or { } *(4.3 #29)*

2b. The solution set is $\{(x,y)|x+3y=2\}$ or $\{(x,y)|3x+9y=6\}$. *(4.3 #31)*

3. true *(4.3 #79)*

Homework:

- ☐ **Review the Section 4.3 summary** on page 334 of the textbook.
- ☐ **Insert your homework** into this section of the *Learning Guide*. Show all work neatly and check your answers. Strive to work through difficulties when possible, making note of any exercises where you need additional help. Remember, even if your instructor assigns homework through *MyMathLab*, you should still write out your work.