Section 4.2
Solving Systems of Linear Equations by the Substitution Method

Pay at the Pump !

Other than outrage, what is going on at the gas pumps?
Is surging demand creating the increasing oil prices?
Like all things in a free-market economy, the price of a commodity is based on supply and demand.

In this section, we use a second method for solving linear systems, the substitution method, to understand this economic phenomenon.

First Steps:

- ☐ **Take comprehensive notes** from your instructor's lecture and insert your notes into this section of the *Learning Guide*. Be sure to write down all examples, definitions, and other key concepts. Additional learning resources include the *Lecture Series on DVD*, the *PowerPoints*, and Section 4.2 of your textbook which begins on page 292.
- ☐ Complete the ***Concept and Vocabulary Check*** on page 298 of the textbook.

Guided Practice:

- ☐ Review each of the following ***Solved Problems*** and complete each ***Pencil Problem***.

Objective #1: Solve linear systems by the substitution method.

✔ *Solved Problem #1*

1a. Solve by the substitution method:

$$\begin{cases} y = 5x - 13 \\ 2x + 3y = 12 \end{cases}$$

Since the first equation is solved for *y*, substitute $5x-13$ for *y* in the second equation.

$$2x + 3\overbrace{(5x-13)}^{y} = 12$$
$$2x + 15x - 39 = 12$$
$$17x - 39 = 12$$
$$17x = 51$$
$$x = 3$$

Back-substitute 3 for *x* into the first equation.

$$y = 5x - 13$$
$$y = 5(3) - 13$$
$$= 2$$

The solution set is $\{(3,2)\}$.

✎ *Pencil Problem #1* ✎

1a. Solve by the substitution method:

$$\begin{cases} x + 3y = 8 \\ y = 2x - 9 \end{cases}$$

1b. Solve by the substitution method:

$$\begin{cases} 3x+2y=-1 \\ x-y=3 \end{cases}$$

Solve the second equation for *x*.

$$x-y=3$$
$$x=y+3$$

Substitute $y+3$ for *x* in the first equation.

$$3x+2y=-1$$
$$3(\overbrace{y+3}^{x})+2y=-1$$
$$3y+9+2y=-1$$
$$5y+9=-1$$
$$5y=-10$$
$$y=-2$$

Back-substitute –2 for *y* into $x=y+3$.

$$x=y+3$$
$$x=-2+3=1$$

The solution set is $\{(1,-2)\}$.

1b. Solve by the substitution method:

$$\begin{cases} 2x-y=-5 \\ x+5y=14 \end{cases}$$

Objective #2:

Use the substitution method to identify systems with no solution or infinitely many solutions.

✔ *Solved Problem #2*

2a. Solve by the substitution method:

$$\begin{cases} 3x+y=-5 \\ y=-3x+3 \end{cases}$$

Since the second equation is solved for *y*, substitute $-3x+3$ for *y* in the first equation.

$$3x+y=-5$$
$$3x+(\overbrace{-3x+3}^{y})=-5$$
$$3x-3x+3=-5$$
$$3=-5, \text{ false}$$

The false statement indicates that the system is inconsistent and has no solution.

The solution set is $\{\ \}$.

✎ *Pencil Problem #2* ✐

2a. Solve by the substitution method:

$$\begin{cases} x=9-2y \\ x+2y=13 \end{cases}$$

2b. Solve by the substitution method:

$$\begin{cases} y = 3x - 4 \\ 9x - 3y = 12 \end{cases}$$

Substitute $3x - 4$ for y in the second equation.

$$9x - 3y = 12$$
$$9x - 3(\overbrace{3x - 4}^{y}) = 12$$
$$9x - 9x + 12 = 12$$
$$12 = 12, \text{ true}$$

The true statement indicates that the system contains dependent equations and has infinitely many solutions.

The solution set is $\{(x, y) \mid y = 3x - 4\}$ or $\{(x, y) \mid 9x - 3y = 12\}$.

2b. Solve by the substitution method:

$$\begin{cases} y = 3x - 5 \\ 21x - 35 = 7y \end{cases}$$

Objective #3: Solve problems using the substitution method.

✔ *Solved Problem #3*

3. The following models describe demand and supply for two-bedroom rental apartments, where p is the monthly rental price and x is the number of apartments.

$$\overbrace{p = -30x + 1800}^{\text{Demand Model}} \qquad \overbrace{p = 30x}^{\text{Supply Model}}$$

3a. Solve the system and find the equilibrium quantity and the equilibrium price.

Substitute $30x$ for p in the first equation.

$$p = -30x + 1800$$
$$\overbrace{30x}^{p} = -30x + 1800$$
$$60x = 1800$$
$$x = 30$$

Back-substitute to find p.

$$p = 30x$$
$$p = 30(30) = 900$$

The solution set is $\{(30, 900)\}$.

Equilibrium quantity: 30
Equilibrium price: $900

✎ *Pencil Problem #3* ✎

3. The following models describe wages for low-skilled labor, where p is the hourly price of labor and x is the number of workers, in millions.

$$\overbrace{p = -0.325x + 5.8}^{\text{Demand Model}} \qquad \overbrace{p = 0.375x + 3}^{\text{Supply Model}}$$

3a. Solve the system, and find the equilibrium number of workers, in millions, and the equilibrium hourly wage.

3b. Use your answer from part (a) to complete this statement:

When rents are ______ per month, consumers will demand ______ apartments and suppliers will offer ______ apartments for rent.

When rents are <u>$900</u> per month, consumers will demand <u>30</u> apartments and suppliers will offer <u>30</u> apartments for rent.

3b. Use your answer from part (a) to complete the statement:

If workers are paid ________ per hour, there will be ________ million available workers and ________ million workers will be hired. In this state of market equilibrium, there is no unemployment.

<u>**Answers** for Pencil Problems *(Textbook Exercise references in parentheses)*</u>:

1a. $\{(5,1)\}$ *(4.2 #3)*

1b. $\{(-1,3)\}$ *(4.2 #7)*

2a. no solution or { } *(4.2 #13)*

2b. infinitely many solutions or $\{(x, y) | y = 3x-5\}$ or $\{(x, y) | 21x-35 = 7y\}$ *(4.2 #15)*

3a. Ordered pair: (4,4.5). Equilibrium number of workers: 4 million Equilibrium hourly wage: $4.50 *(4.2 #41a)*

3b. If workers are paid <u>$4.50</u> per hour, there will be <u>4</u> million available workers and <u>4</u> million workers will be hired. In this state of market equilibrium, there is no unemployment. *(4.2 #41b)*

<u>Homework</u>:

- ☐ **Review the Section 4.2 summary** on page 334 of the textbook.
- ☐ **Insert your homework** into this section of the *Learning Guide*. Show all work neatly and check your answers. Strive to work through difficulties when possible, making note of any exercises where you need additional help. Remember, even if your instructor assigns homework through *MyMathLab*, you should still write out your work.