Section 4.1
Solving Systems of Linear Equations by Graphing

The CO$T to Cross

Visitors to the world's great bridges are frequently inspired by their beauty. This feeling is not always shared by daily commuters dealing with escalating toll costs and peak-hour congestion. For these frequent users, most bridge authorities provide the option of a fixed cost that reduces the toll. In this section, you will see how toll options in these situations can be modeled by two linear equations in two variables and their graphs.

First Steps:

☐ **Take comprehensive notes** from your instructor's lecture and insert your notes into this section of the *Learning Guide*. Be sure to write down all examples, definitions, and other key concepts. Additional learning resources include the *Lecture Series on DVD*, the *PowerPoints*, and Section 4.1 of your textbook which begins on page 280.

☐ Complete the ***Concept and Vocabulary Check*** on page 289 of the textbook.

Guided Practice:

☐ Review each of the following ***Solved Problems*** and complete each ***Pencil Problem***.

Objective #1: Decide whether an ordered pair is a solution of a linear system.

✔ *Solved Problem #1*

1. Determine if the ordered pair $(7,6)$ is a solution of the system: $\begin{cases} 2x-3y=-4 \\ 2x+y=4 \end{cases}$

To determine if $(7,6)$ is a solution to the system, replace x with 7 and y with 6 in both equations.

$$\begin{aligned} 2x-3y&=-4 \\ 2(7)-3(6)&=-4 \\ 14-18&=-4 \\ -4&=-4, \text{ true} \end{aligned} \qquad \begin{aligned} 2x+y&=4 \\ 2(7)+6&=4 \\ 14+6&=4 \\ 20&=4, \text{ false} \end{aligned}$$

The ordered pair does not satisfy both equations, so it is not a solution to the system.

✎ *Pencil Problem #1* ✎

1. Determine if the ordered pair $(2,-3)$ is a solution of the system: $\begin{cases} 2x+3y=-5 \\ 7x-3y=23 \end{cases}$

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Objective #2: Solve systems of linear equations by graphing.

✔ *Solved Problem #2*

2. Solve by graphing: $\begin{cases} 2x + y = 6 \\ 2x - y = -2 \end{cases}$

Graph $2x + y = 6$ by using intercepts.

x-intercept (Set $y = 0.$)

$$\begin{aligned} 2x + y &= 6 \\ 2x + 0 &= 6 \\ 2x &= 6 \\ x &= 3 \end{aligned}$$

y-intercept (Set $x = 0.$)

$$\begin{aligned} 2x + y &= 6 \\ 2(0) + y &= 6 \\ 0 + y &= 6 \\ y &= 6 \end{aligned}$$

Graph $2x - y = -2$ by using intercepts.

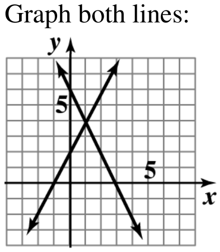
x-intercept (Set $y = 0.$)

$$\begin{aligned} 2x - y &= -2 \\ 2x - 0 &= -2 \\ 2x &= -2 \\ x &= -1 \end{aligned}$$

y-intercept (Set $x = 0.$)

$$\begin{aligned} 2x - y &= -2 \\ 2(0) - y &= -2 \\ -y &= -2 \\ y &= 2 \end{aligned}$$

Graph both lines:



The lines intersect at (1,4).

The solution set is $\{(1,4)\}$.

✎ *Pencil Problem #2* ✎

2. Solve by graphing: $\begin{cases} x + y = 1 \\ y - x = 3 \end{cases}$

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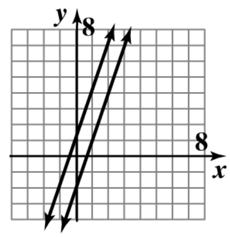
Objective #3: Use graphing to identify systems with no solution or infinitely many solutions.

✔ *Solved Problem #3*

3a. Solve by graphing: $\begin{cases} y = 3x - 2 \\ y = 3x + 1 \end{cases}$

Graph $y = 3x - 2$ by using the y-intercept of –2 and the slope of 3.

Graph $y = 3x + 1$ by using the y-intercept of 1 and the slope of 3.



Because both equations have the same slope, 3, but different y-intercepts, the lines are parallel.

Thus, the system is inconsistent and has no solution.

The solution set is the empty set, $\{ \}$.

3b. Solve by graphing: $\begin{cases} x + y = 3 \\ 2x + 2y = 6 \end{cases}$

Graph $x + y = 3$ by using intercepts.

x-intercept (Set $y = 0.$)
$$x + y = 3$$
$$x + 0 = 3$$
$$x = 3$$

y-intercept (Set $x = 0.$)
$$x + y = 3$$
$$0 + y = 3$$
$$y = 3$$

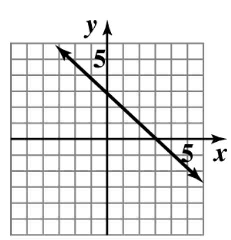
Graph $2x + 2y = 6$ by using intercepts.

x-intercept (Set $y = 0.$)
$$2x + 2y = 6$$
$$2x + 2(0) = 6$$
$$2x = 6$$
$$x = 3$$

y-intercept (Set $x = 0.$)
$$2x + 2y = 6$$
$$2(0) + 2y = 6$$
$$2y = 6$$
$$y = 3$$

Both lines have the same x-intercept and the same y-intercept.

Thus, the graphs of the two equations in the system are the same line.



Any ordered pair that is a solution to one equation is a solution to the other, and, consequently, a solution of the system. The system has an infinite number of solutions, namely all points that are solutions of either line.
The solution set is $\{(x, y) | x + y = 3\}$.

Pencil Problem #3

3a. Solve by graphing: $\begin{cases} y = 2x - 1 \\ y = 2x + 1 \end{cases}$

3b. Solve by graphing: $\begin{cases} x - 2y = 4 \\ 2x - 4y = 8 \end{cases}$

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Objective #4: Use graphs of linear systems to solve problems.

✔ Solved Problem #4

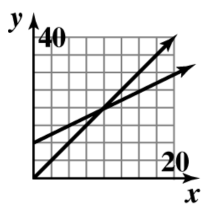
4. The toll to a bridge costs \$2.00. If you use the bridge x times in a month, the monthly cost, y, is $y = 2x$. With a \$10 discount pass, the toll is reduced to \$1.00. The monthly cost, y, of using the bridge x times in a month with the discount pass is $y = x + 10$.

4a. Solve by graphing the system: $\begin{cases} y = 2x \\ y = x + 10 \end{cases}$

Suggestion: Let the x-axis extend from 0 to 20 and let the y-axis extend from 0 to 40.

Graph $y = 2x$ by using the y-intercept of 0 and the slope of 2.

Graph $y = x + 10$ by using the y-intercept of 10 and the slope of 1.



The solution is the ordered pair (10,20).

4b. Interpret the coordinates of the solution in practical terms.

If the bridge is used 10 times in a month, the total monthly cost without the discount pass is the same as the monthly cost with the discount pass, namely \$20.

✎ Pencil Problem #4 ✎

4. You plan to start taking an aerobics class. Nonmembers pay \$4 per class. Members pay a \$10 monthly fee plus an additional \$2 per class. The monthly cost, y, of taking x classes can be modeled by the linear system $\begin{cases} y = 4x \\ y = 2x + 10 \end{cases}$

4a. Solve the system by graphing.

4b. Interpret the coordinates of the solution in practical terms.

Answers for Pencil Problems *(Textbook Exercise references in parentheses)*:

1. $(2, -3)$ is a solution of the system *(4.1 #1)* **2.** $\{(-1, 2)\}$ *(4.1 #13)* **3a.** no solution or $\{\ \}$ *(4.1 #25)*

3b. infinitely many solutions; $\{(x, y) | x - 2y = 4\}$ or $\{(x, y) | 2x - 4y = 8\}$. *(4.1 #29)* **4a.** $\{(5, 20)\}$ *(4.1 #53a)*

4b. Nonmembers and members pay the same amount per month for taking 5 classes, namely \$20. *(4.1 #53b)*

Homework:

- ☐ **Review the Section 4.1 summary** on page 333 of the textbook.
- ☐ **Insert your homework** into this section of the *Learning Guide*. Show all work neatly and check your answers. Strive to work through difficulties when possible, making note of any exercises where you need additional help. Remember, even if your instructor assigns homework through *MyMathLab*, you should still write out your work.

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