SUPPLEMENT TO §1.8

RULES OF EXPONENTS

i.	$x^a \cdot x^b = x^{a+b}$	When we multiply like bases raised to powers, we add the powers.
ii.	$(x^a)^b = x^{a \cdot b}$	When we raise a base-with-a-power to a power, we multiply the powers together.
iii.	$(xy)^a = x^a y^a$	We apply the exponent to each factor inside the parentheses.

Note that one of the most common mistakes is to confuse rules *i* and *ii*. To avoid this mistake, notice that if there are **two** factors with like bases, you should use rule *i*.

EXAMPLE: Simplify the following expressions using the rules of exponents.

a. $-2t^3 \cdot 4t^5$ b. $5\left(v^4\right)^2$ c. $4(3u)^2$ d. x^3y^2

Solutions:

a. $-2t^3 \cdot 4t^5 = -8t^3 \cdot t^5$ Multiply the -2 and the 4 together.

$= -8t^{3+5}$ Add the two exponents together using rule *i*.

$= -8t^8$

b. $5\left(v^4\right)^2 = 5 \cdot v^{4 \cdot 2}$ Apply rule *ii*; note that the power "2" doesn't affect the coefficient "5" since we apply exponents before multiplication.

$= 5v^8$

c. $4(3u)^2 = 4\left(3^2u^2\right)$ Apply rule *iii*.

$= 4(9u^2)$ (simplify)

$= 36u^2$ Multiply the 4 and 9 together; note that we don't distribute 4 to 9 and u^2 since there is no addition or subtraction.

d. x^3y^2 cannot be simplified further since the bases are different variables.

■

EXERCISES:

1. Simplify the following expression using the exponent rules.

a. $x^5 \cdot x^6$

b. $7y^7 \cdot 3y^3$

c. $2x^2(3x - 2)$

d. $x^5 + x^5$

e. $5t^3 - 2t^3$

f. $9x^5 - x^5$

g. $(x^5)^6$

h. $2(x^5)^3$

i. $x(x^2)^3$

j. $3(2y)^4$

k. $(2x^2)^3$

l. $(3x)^2$

2. True of False. If a statement is false, change one side of the equation to make it true.

a. $x^2 + x^3 = x^5$

b. $3^2 \cdot 3^4 = 3^6$

c. $3^2 \cdot 2^3 = 6^5$

d. $x^2 x^3 = 2x^5$

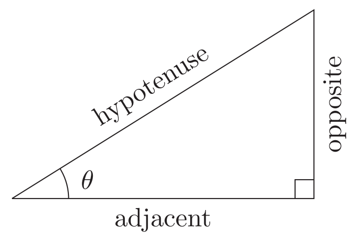
e. $(5^2)^3 = 5^5$

f. $x^3 + x^3 = 2x^3$

SUPPLEMENT TO §2.8

Right Triangle Trigonometry:

Consider a right triangle, one of whose acute angles (an angle whose measure is less than 90°) is θ (the Greek letter theta). The three sides of the triangle are the called the hypotenuse (across from the right angle), the side opposite θ, and the side adjacent to θ.



Ratios of a right triangle's three sides are used to define the six trigonometric functions: sine, cosine, tangent, cosecant, secant, and cotangent. These six functions are abbreviated sin, cos, tan, csc, sec, and cot, respectively.

Right Triangle Definition of Trigonometric Functions

Let θ be an acute angle of a right triangle. The six trigonometric functions of θ are defined as follows.

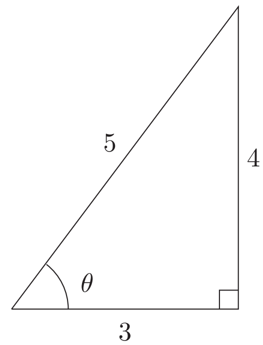
$$\sin(\theta) = \frac{\text{opp}}{\text{hyp}} \qquad \cos(\theta) = \frac{\text{adj}}{\text{hyp}} \qquad \tan(\theta) = \frac{\text{opp}}{\text{adj}}$$

$$\csc(\theta) = \frac{\text{hyp}}{\text{opp}} \qquad \sec(\theta) = \frac{\text{hyp}}{\text{adj}} \qquad \cot(\theta) = \frac{\text{adj}}{\text{opp}}$$

The abbreviations "opp," "adj," and "hyp" stand for length of the "opposite," "adjacent," and "hypotenuse" respectively, as seen in the diagram of the right triangle above. Note that the ratios in the second row are the reciprocals of the ratios in the first row. That is:

$$\csc(\theta) = \frac{1}{\sin(\theta)} \qquad \sec(\theta) = \frac{1}{\cos(\theta)} \qquad \cot(\theta) = \frac{1}{\tan(\theta)}$$

EXAMPLE: Evaluate the six trigonometric functions of the angle θ shown in the given right triangle.



Solutions: Using adj = 3, opp = 4, and hyp = 5, you can write the following:

$$\sin(\theta) = \frac{\text{opp}}{\text{hyp}} = \frac{4}{5} \qquad \cos(\theta) = \frac{\text{adj}}{\text{hyp}} = \frac{3}{5} \qquad \tan(\theta) = \frac{\text{opp}}{\text{adj}} = \frac{4}{3}$$

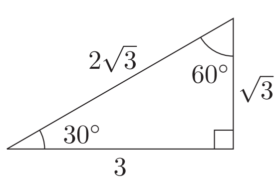
$$\csc(\theta) = \frac{\text{hyp}}{\text{opp}} = \frac{5}{4} \qquad \sec(\theta) = \frac{\text{hyp}}{\text{adj}} = \frac{5}{3} \qquad \cot(\theta) = \frac{\text{adj}}{\text{opp}} = \frac{3}{4}$$

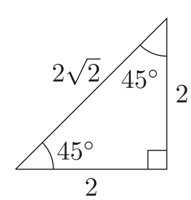
■

Some people like to use the mnemonic device SOH-CAH-TOA to remember the relationships. The SOH reminds us that sine outputs the opposite over the hypotenuse, the CAH reminds us that cosine outputs the adjacent over the hypotenuse, and the TOA reminds us that tangent outputs the opposite over the adjacent.

EXERCISES:

1. The angles 30°, 45°, and 60° occur frequently in trigonometry. Use the given triangles to fill out Table 1.

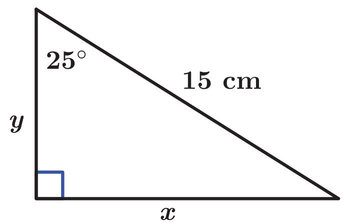




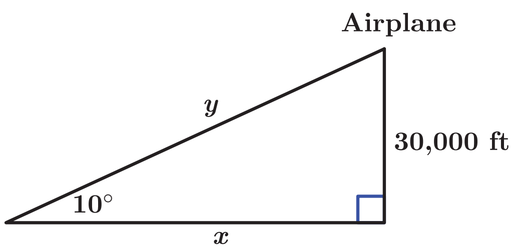
θ	$\sin(\theta)$	$\cos(\theta)$	$\tan(\theta)$	$\csc(\theta)$	$\sec(\theta)$	$\cot(\theta)$
30°						
60°						
45°						

Table 1

2. Find the lengths of the sides of the triangle (x and y) accurate to 3 decimal places.



3. An airplane is flying at an altitude of 30,000 feet. When the airplane is at a 10° angle of elevation from the airport, it will start to receive radar signals from the airport's landing system. How far away, measured along the ground as x, is the plane from the airport when it receives the radar signals? How far away, measured by line of sight as y, is the plane from the airport when it receives the radar signals? Answer both questions accurate to the nearest hundred feet.



SUPPLEMENT TO §3.3

1. State whether the rate is positive, negative, or zero for the given scenario.

   a. The rate at which a child's height changes with respect to time.

   b. The rate at which an elderly person's height changes with respect to time.

   c. The rate at which a middle aged person's height changes with respect to time.

   d. The rate at which a pond's water level changes during a drought with respect to time.

   e. The rate at which a pond's water level changes during the rainy season with respect to time.

   f. The rate at which a healthy person's heart beats while they are at rest with respect to time.

2. Interpret the slope of each of the following formulas as a rate of change in practical terms. Make sure you include the unit of the slope in your interpretation.

   a. The temperature in Mathville on November 5, 2013 is given by the formula

   $$T = -2.1t + 54$$

   where T represents the temperature in degrees Fahrenheit and t represents the time (in minutes) since 9 : 00am.

   b. The number of calories a runner burns is given by the formula

   $$C = 15t$$

   where C represents the number of calories burned and t represents the time (in minutes) spent running.

   c. The average price of a new laptop computer is given by the formula

   $$A = -\frac{50}{3}t + 1200$$

   where A represents the average price, in dollars, of a new laptop computer and t represent the number of months since December 2006.

   d. The number of dollars a professional blogger is paid is given by the formula

   $$A = 25p$$

   where p represents the number of posts they create.

3. Elijah, Logan, and Savannah each go out for separate walks. The following graph, table, and formula describe the number of kilometers, D, that Elijah, Logan, and Savannah have walked in t hours respectively.

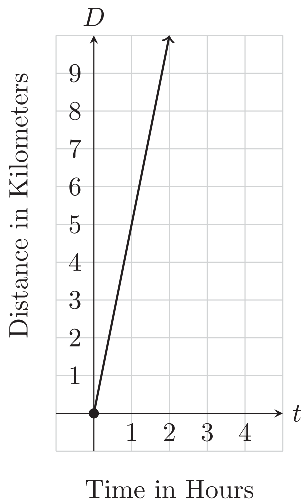


Figure 1: Elijah's distance walked.

t (hours)	D (km)
0	0
2	9
4	18
6	27

Table 1: Logan's distance walked.

The distance Savannah walked is given by the formula

$$D = \frac{13}{4}t.$$

Given this information, who walks fastest and who walks slowest? How do you know?

SUPPLEMENT TO §3.4

1. For the following equations, state what the slope and y-intercept are and then use that slope and y-intercept to graph the solutions to the equation in the space provided. Label the y-intercept and label the graph with its corresponding equation. You will need to choose an appropriate scale for the grid on the coordinate plane.

   a. $y = 6x + 30$

   b. $y = -50x + 250$

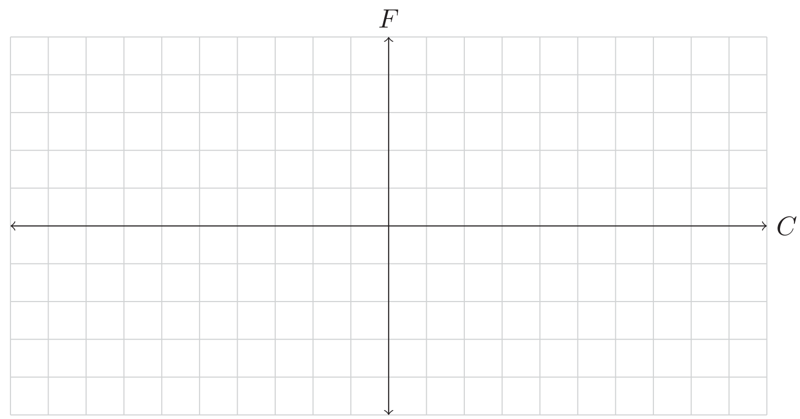
   c. $y = \frac{1}{12}x - 4$

   d. $y = -\frac{25}{4}x + 25$

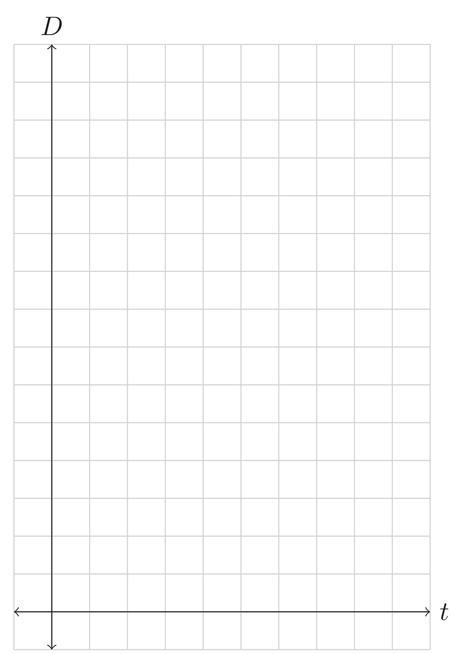
2. The linear equation

$$F = \frac{9}{5}C + 32$$

models the temperature in degrees Fahrenheit, F, given the temperature in degrees Celsius, C. Graph the equation in the provided coordinate plane. You will need to set up an appropriate scale on the coordinate plane for your graph to fit properly. Label the C- and F-intercepts and the equation of the line.



3. You're leaving Eugene from a Ducks game and driving back home to Portland. Eugene is about 104 miles from Portland and the traffic keeps you driving an average of 52 miles per hour. Model an equation which gives the distance, D, you are from home t hours after leaving Eugene and then graph that equation in the provided coordinate plane. You will need to set up an appropriate scale on the coordinate plane for your graph to fit properly. Label the t- and D-intercepts and the equation of the line.



SUPPLEMENT TO §3.5

1. Determine the equations for the lines graphed in the following figures.

a.

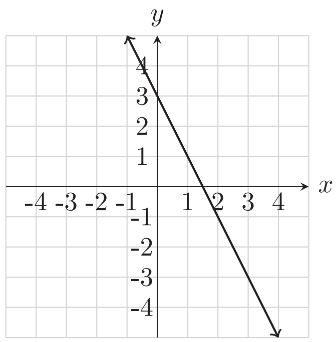


Figure 1

b.

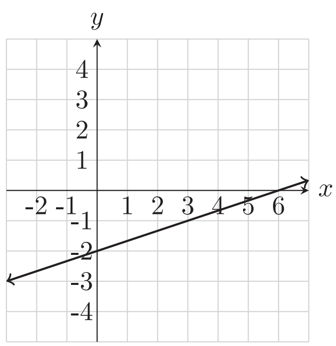


Figure 2

c.

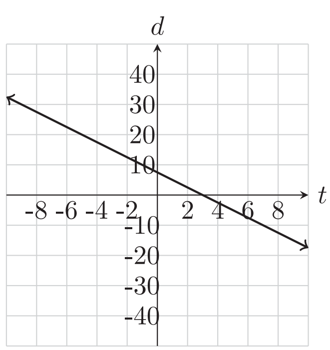


Figure 3

d.

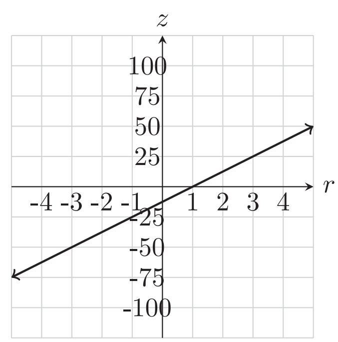


Figure 4

e.

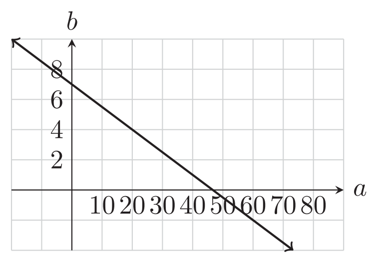


Figure 5

f.

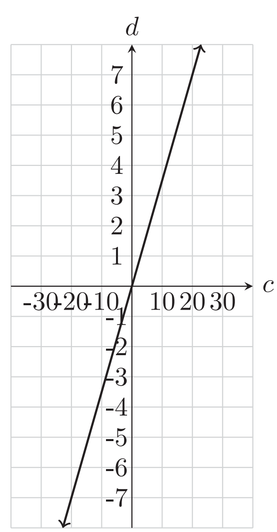


Figure 6

2. The population of a suburb of Portland was 12,500 in 1990. The population of the suburb has been increasing at an average rate of 750 people per year.

    a. Write a linear equation that gives the town population in terms of the year using P to represent the population of the Portland suburb t years since 1990.

    b. Use the linear equation found in part (a) to predict the suburb's population in 2005. Show all of your work and then state a conclusion using a complete sentence.

3. A storage tank at a production factory holds 250 gallons of a liquid chemical at the beginning of a day. Over the course of the day the chemical is used for production purposes at a constant rate. Two hours after the production begins there are 210 gallons remaining.

    a. Determine the linear equation which models the amount of chemical remaining in the tank in terms of the number of hours of production time during this day. Be sure to define any variable you use.

    b. Use the linear equation you found in part (a) to determine the number of hours of production which will result in the tank being half full.

4. The life span of an insect can be modified by the temperature of the environment. Assume that the relationship between temperature of the environment, in degrees Celsius, and life span of the fruit fly, in days, is linear. Suppose a population of fruit flies has a life span of 80 days at a temperature of 10 degrees and a life span of 50 days at a temperature of 20 degrees.

   a. Write a linear relationship between the temperature and the life span.

   b. What is the life span at a temperature of 25 degrees?

   c. At what temperature is the life span 92 days?

5. Savannah worked 12 hours one week and earned \$137.40. The next week she worked 17 hours and earned \$194.65.

   a. Write a linear equation that gives Savannah's weekly wages based on the number of hours she worked that week.

   b. If Savannah works 15 hours in a week, how much does she make?

   c. If Savannah earns \$240.45 in a week, how many hours does she work?

ANSWERS TO SUPPLEMENT §1.8:

1.
a. $x^5 \cdot x^6 = x^{11}$

b. $7y^7 \cdot 3y^3 = 21y^{10}$

c. $2x^2(3x - 2) = 6x^3 - 4x^2$

d. $x^5 + x^5 = 2x^5$

e. $5t^3 - 2t^3 = 3t^3$

f. $9x^5 - x^5 = 8x^5$

g. $\left(x^5\right)^6 = x^{30}$

h. $2\left(x^5\right)^3 = 2x^{15}$

i. $x\left(x^2\right)^3 = x^7$

j. $3(2y)^4 = 48y^4$

k. $\left(2x^2\right)^3 = 8x^6$

l. $(3x)^2 = 9x^2$

2.
a. $x^2 + x^3 = x^5$ is *false.*
There is no way to combine $x^2 + x^3$ because $x^2 + x^3 = x \cdot x + x \cdot x \cdot x$ while $x^5 = x \cdot x \cdot x \cdot x \cdot x$.

b. $3^2 \cdot 3^4 = 3^6$ is *true.*

c. $3^2 \cdot 2^3 = 6^5$ is *false.*
We can add the exponents only when the bases are the same. By order of operations $3^2 \cdot 2^3 = 9 \cdot 8 = 72$ while $6^5 = 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 = 7776$.

d. $x^2x^3 = 2x^5$ is *false.*
$x^2 \cdot x^3 = x \cdot x \cdot x \cdot x \cdot x = x^5$ by the definition of exponents.

e. $\left(5^2\right)^3 = 5^5$ is *false.*
$\left(5^2\right)^3 = 5^2 \cdot 5^2 \cdot 5^2 = 5^6$ by the definition of exponents.

f. $x^3 + x^3 = 2x^3$ is *true.*

ANSWERS TO SUPPLEMENT §2.8**:**

1.

θ	$\sin(\theta)$	$\cos(\theta)$	$\tan(\theta)$	$\csc(\theta)$	$\sec(\theta)$	$\cot(\theta)$
$30°$	$\frac{1}{2}$	$\frac{3}{2\sqrt{3}}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\frac{3}{\sqrt{3}}$
$60°$	$\frac{3}{2\sqrt{3}}$	$\frac{\sqrt{3}}{2\sqrt{3}}$	$\frac{3}{\sqrt{3}}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$
$45°$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1	$\sqrt{2}$	$\sqrt{2}$	1

Table 1

2. The base length, x, is $15\sin(25°) \approx 6.339$ centimeters. The height length, y, is $15\cos(25°) \approx 13.595$ centimeters.

3. The **ground** distance of the plane from airport, x, is $\frac{30,000}{\tan(10°)} \approx 170,100$ feet and the **line of sight** distance of the plane from the airport, y, is $\frac{30,000}{\sin(10°)} \approx 172,800$ feet when the plane starts receiving radar signals.

ANSWERS TO SUPPLEMENT §3.3**:**

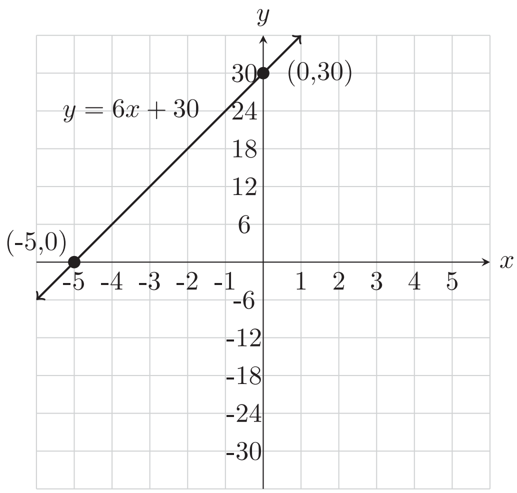
1. a. Positive
   b. Negative
   c. Zero
   d. Negative
   e. Positive
   f. Zero

2. a. The temperature in Mathville was decreasing at a rate of 2.1 $\frac{{}^\circ\text{F}}{\text{minute}}$.
   b. The number of calories the runner burns increases at a rate of 15 $\frac{\text{calories}}{\text{minute}}$.
   c. The average price of a new laptop computer is decreasing at a rate of $\frac{50}{3}\frac{\text{dollars}}{\text{month}}$.
   d. The amount a professional blogger is paid increases at a rate of 25 $\frac{\text{dollars}}{\text{post}}$.

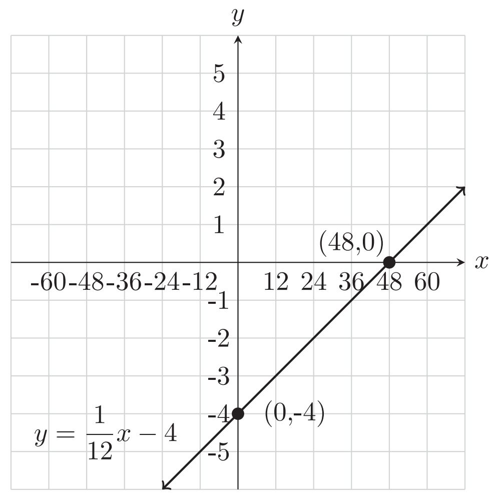
3. Elijah is walking at a rate of 5 kilometers per hour, Logan at a rate of 4.5 kilometers per hour, and Savannah at a rate of 3.25 kilometers per hours, thus Elijah is walking the fastest and Savannah the slowest.

ANSWERS TO SUPPLEMENT §3.4**:**

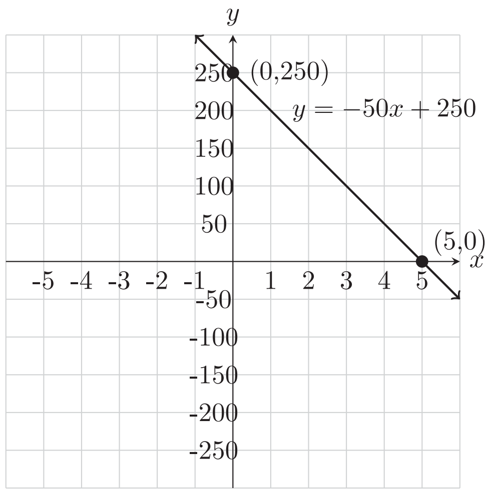
1. a.



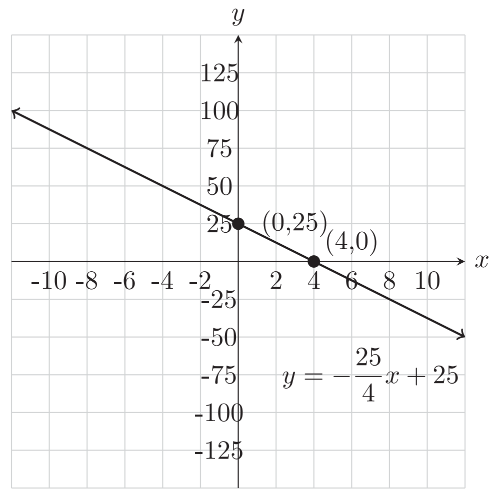
c.



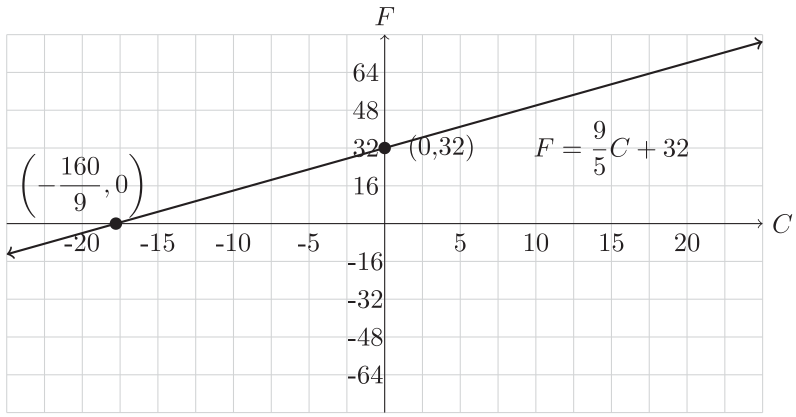
b.



d.

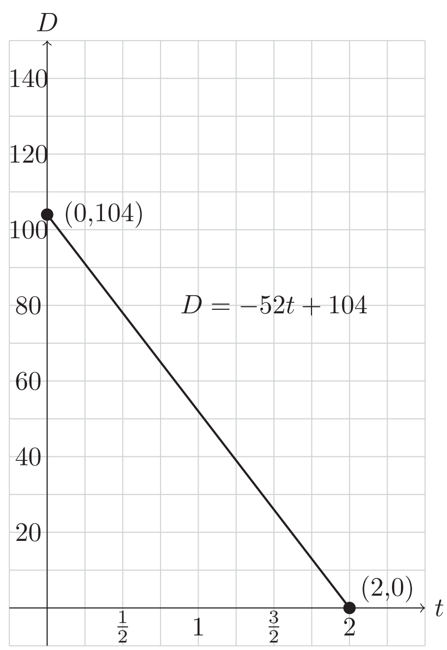


2. The F-intercept is $(0, 32)$ and the slope is $m = \frac{9}{5}$.



3. The D-intercept is $(0, 104)$ since you are 104 miles from Portland when you start driving. You're getting closer to Portland so your distance is decreasing at a rate of 52 miles per hour thus the slope is -52. Hence the equation which models your distance from Portland given the number of hours you've been driving is

$$D = -52t + 104.$$



ANSWERS TO SUPPLEMENT §3.5**:**

1. a. $y = -2x + 3$

   b. $y = \frac{1}{3}x - 2$

   c. $d = -\frac{5}{2}t + \frac{15}{2}$

   d. $z = \frac{25}{2}r - \frac{25}{2}$

   e. $b = -\frac{3}{20}a + 7$

   f. $d = \frac{7}{20}c$

2. a. $P = 750t + 12,500$

   b. According to the model, the population of the suburb will be 23,750 in 2005.

3. a. Let t represent the number of hours of production and let A represent the amount of chemical (in gallons) remaining in the tank.

   $$A = -20t + 250$$

   b. The tank will be half empty after 6 hours and 15 minutes of production.

4. a. Let L represent the life span of the fruit fly (in days) at a temperature of T in degrees Celsius. Then the linear model which describes the fruit fly's lifespan in terms of the environment's temperature in degrees Celsius is given by

   $$L = -3T + 110.$$

   b. The life span is 35 days when the temperature is 25 degrees Celsius.

   c. The temperature is 6 degrees Celsius when the life span is 92 days.

5. a. Let W represent Savannah's weekly pay, in dollars, when she works h hours. Then the linear model of Savannah's wages in terms of hours is given by

   $$W = 11.45h.$$

   b. Savannah earns \$171.75 if she works 15 hours in a week.

   c. Savannah worked 21 hours if she earned \$240.45 for the week.