SUPPLEMENT TO §10.6

We can use ***function notation*** to communicate the information contained in the graph of a function. For example, if the point $(5, 3)$ is on the graph of a function called f, we can write "$f(5) = 3$". This means that if we are asked to find $f(5)$ using the graph of the function f, we would need to search for the y-value that is associated with the x-value 5. Similarly, if we are asked to solve an equation like $f(x) = 4$ using a graph, we would need to search for the x-value(s) associated with the y-value 4.

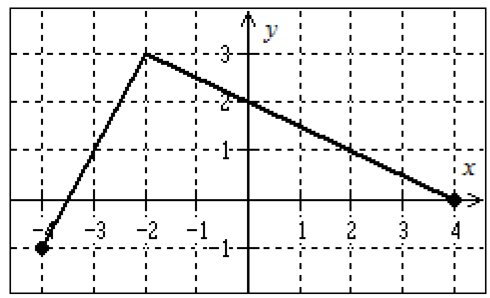
EXAMPLE: The graph of $y = g(x)$ is given at the right.

a. Find $g(-2)$.

b. Find $g(-3)$.

c. Solve $g(x) = -1$.
(Express your answer in a solution set.)

d. Solve $g(x) = 1$.
(Express your answer in a solution set.)



The graph of $y = g(x)$.

Solutions

a. $g(-2) = 3$. (At the x-value -2, the y-value is 3 on the graph.)

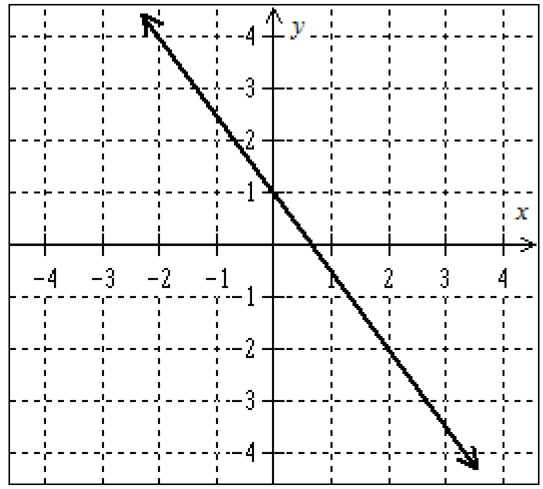
b. $g(-3) = 1$. (At the x-value -3, the y-value is 1 on the graph.)

c. The solution set is $\{-4\}$. (At the x-value -4, the y-value is -1 on the graph.)

d. The solution set is $\{-3, 2\}$. (At the x-values -3 and 2, the y-value is 1 on the graph.)

EXERCISES:

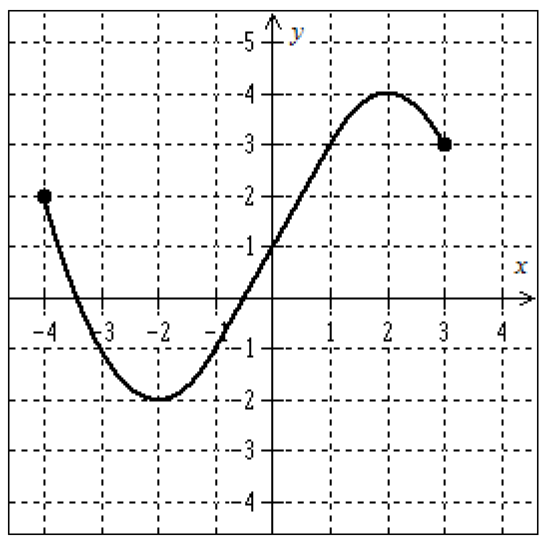
1. The graph of $y = G(x)$ is given at the right.



The graph of $y = G(x)$.

- **a.** On the graph, if $x = 0$, then $y =$____.
- **b.** Find $G(-2)$.
- **c.** Find $G(4)$.
- **d.** If $G(x) = -2$, then $x =$____.
- **e.** Solve $G(x) = 4$.
- **f.** Find a formula for $y = G(x)$. [§4.4 review]
- **g.** Use the formula you found in part **f** to find the exact coordinates of the horizontal intercept of the line. [§4.4 review]

2. The graph of $y = f(x)$ is given at the right.



The graph of $y = f(x)$.

- **a.** On the graph, if $x = 1$, then $y =$____.
- **b.** Find $f(0)$.
- **c.** Find $f(-4)$.
- **d.** From the graph we can see that $f(-3) = -1$. What point on the graph is this statement describing?
- **e.** From the graph we can see that $f(2) = 4$. What point on the graph is this statement describing?
- **f.** If $f(x) = 4$, then $x =$____.
- **g.** Solve $f(x) = -1$.

3. Find the following function values if the function H is defined by the set of ordered pairs $\{(2, 5), (7, 10), (-4, 3), (-1, 3), (5, 2)\}$.

- **a.** $H(-4)$
- **b.** $H(2)$

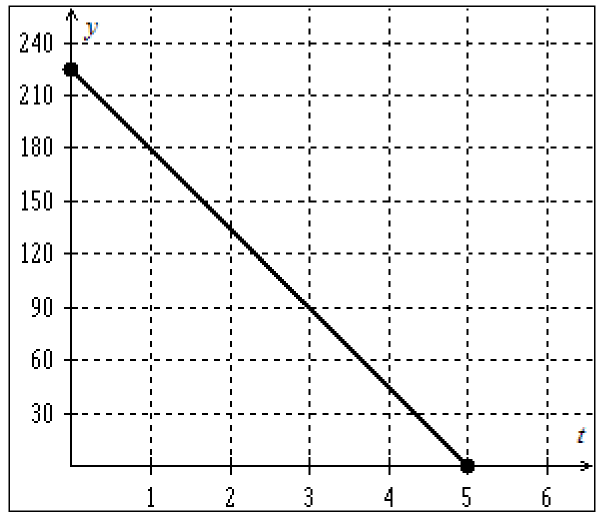
4. Billy Jean is driving home from a business meeting. Shown is the graph $y = d(t)$ where d represents her remaining distance (in miles) to home as a function of time, t, (in hours) since she left her meeting.

a. Find and interpret $d(1)$ in the context of the real-world situation. Write in a complete sentence.

b. Find and interpret $d(5)$ in the context of the real-world situation. Write in a complete sentence.

c. Find a formula for $y = d(t)$. [§4.5 review]

d. Using your formula from part **c**, find and interpret the exact value of $d(0)$ in the context of the real-world situation. Write in a complete sentence. [§4.5 review]



The graph of $y = d(t)$.

5. Instead of using graphs, we can use a *table* to define a function. For example the table below defines the function j. (The top row represents the input values and the bottom row represents the output values, and each column represents an ordered pair that satisfies the function, e.g., the last column implies that the ordered pair $(6, -100)$ satisfies the function.)

x	-3	1	3	6
$j(x)$	170	50	-10	-100

a. Find $j(3)$.

b. When $j(x) = 170$, then $x =$____.

c. Is j a linear function? Defend your answer with a strong argument. [§4.3 review]

d. Find a formula for $y = j(x)$ in slope intercept form. [§4.4 review]

e. Use your answer to part **d** to find $j(-8)$.

f. Make a graph of $y = j(x)$ on graph paper to verify that the formula found corresponds to the data in the table. [§4.4 review]

6. An independent tester examined several modern wind turbines (that generate power) to check if they met factory specifications. The Air 403 Wind Turbine came close to its expectations. The amount of power $P(x)$ (in Watts) generated by a wind-speed of x miles per hour blowing on the turbine can be approximated by the formula $P(x) = 20.8x - 262$.

a. Evaluate $P(25)$. Interpret your answer in a complete sentence in the context of the problem.

b. Solve $P(x) = 450$. (Round to the nearest integer). Interpret your answer in a complete sentence in the context of the problem.

c. If you need about 300 Watts to run the power to your heated shed, at what speed does the wind have to be blowing? (Round to the nearest integer.)

To get the turbine to turn at all (and thus generate power), the wind has to be more than just a light breeze. In parts ***d*** *and* ***e****, we will investigate this fact.*

d. Evaluate $P(10)$. Interpret your answer in a complete sentence in the context of the problem.

e. At what speed will the power production be 0 Watts? (Round to the nearest integer.) This speed is called the ***cut-in speed***: if the wind blows even just a tiny amount faster, the turbine will start generating power.

ANSWERS TO THE SUPPLEMENT TO §10.6:

1. **a.** $y = \underline{1}$

b. $G(-2) = 4$

c. $G(4) = -5$

d. $x = \underline{2}$

e. The solution set is $\{-2\}$.

f. Since this is a graph of a line with slope $m = -\frac{3}{2}$ and vertical intercept $(0, 1)$, the formula for the graph in slope intercept form is $y = -\frac{3}{2}x + 1$, and the formula for the function is $y = G(x) = -\frac{3}{2}x + 1$.

g. The horizontal intercept is the point where $y = 0$. Thus,

$$\begin{aligned} 0 &= -\tfrac{3}{2}x + 1 \\ -1 &= -\tfrac{3}{2}x \\ \tfrac{2}{3} &= x \end{aligned}$$

So the horizontal intercept is $\left(\frac{2}{3}, 0\right)$.

2. **a.** $y = \underline{3}$

b. $f(0) = 1$

c. $f(-4) = 2$

d. the point $(-3, -1)$

e. the point $(2, 4)$

f. $x = \underline{2}$

g. The solution set is $\{-1, 3\}$.

3. **a.** $H(-4) = 3$ **b.** $H(2) = 5$

4. **a.** $d(1) = 180$. This means that one hour after Billy Jean left her meeting, she still had 180 miles left to travel.

b. $d(5) = 0$. This means that 5 hours after Billy Jean left her meeting, she had 0 miles left to travel which implies that she must have arrived at her home.

c. This is a linear graph, so we need to find the slope first. Two points on the graph are $(1, 180)$ and $(5, 0)$. We can use these points to find the slope of the line"

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{180 - 0}{1 - 5} \\ &= \frac{180}{-4} \\ &= -45 \end{aligned}$$

Now we can plug this slope and the point $(5, 0)$ into the point-slope formula to find the formula for the line:

$$\begin{aligned} (5,0) \Rightarrow \; y &= -45x + b \\ 0 &= -45(5) + b \\ 0 &= -225 + b \\ b &= 225 \end{aligned}$$

Thus, the formula for the line is $y = -45x + 225$ **but** the question asks for "$d(t)$" so t is where the x usually is! This means that we should use t instead of x in our final formula: $y = d(t) = -45t + 225$.

d. $$\begin{aligned} d(0) &= -45(0) + 225 \\ &= 225 \end{aligned}$$

This means that just as Billy Jean was leaving her meeting (i.e., after driving for 0 hours), she had 225 miles left to drive to get home.

5. **a.** $j(3) = -10$.

b. $x = \underline{-3}$

c. If a function is linear, it must have constant slope; thus, we can determine if j is linear by finding the slope between each pair of points and see if the slope is constant. Recall that the slope of the line passing through two points is the ratio of the *rise* to the *run* between the points.

run		4		2		3	
x	-3		1		3		6
$j(x)$	170		50		-10		-100
rise		-120		-60		-90	

- slope between first pair of points: $m = \dfrac{-120}{4}$

$$= -30$$

- slope between second pair of points: $m = \dfrac{-60}{2}$

$$= -30$$

- slope between third pair of points: $m = \dfrac{-90}{3}$

$$= -30$$

Since all the slopes are identical, j is a linear function with slope -30.

d. Since we know that the slope is -30, we know that the formula for the function must look like $y = -30x + b$. Now we have to find b by using one of the points given in the table. Let's use $(1, 50)$:

$$\begin{aligned} (1, 50) \Rightarrow \quad y &= -30x + b \\ 50 &= -30(1) + b \\ 50 &= -30 + b \\ b &= 80 \end{aligned}$$

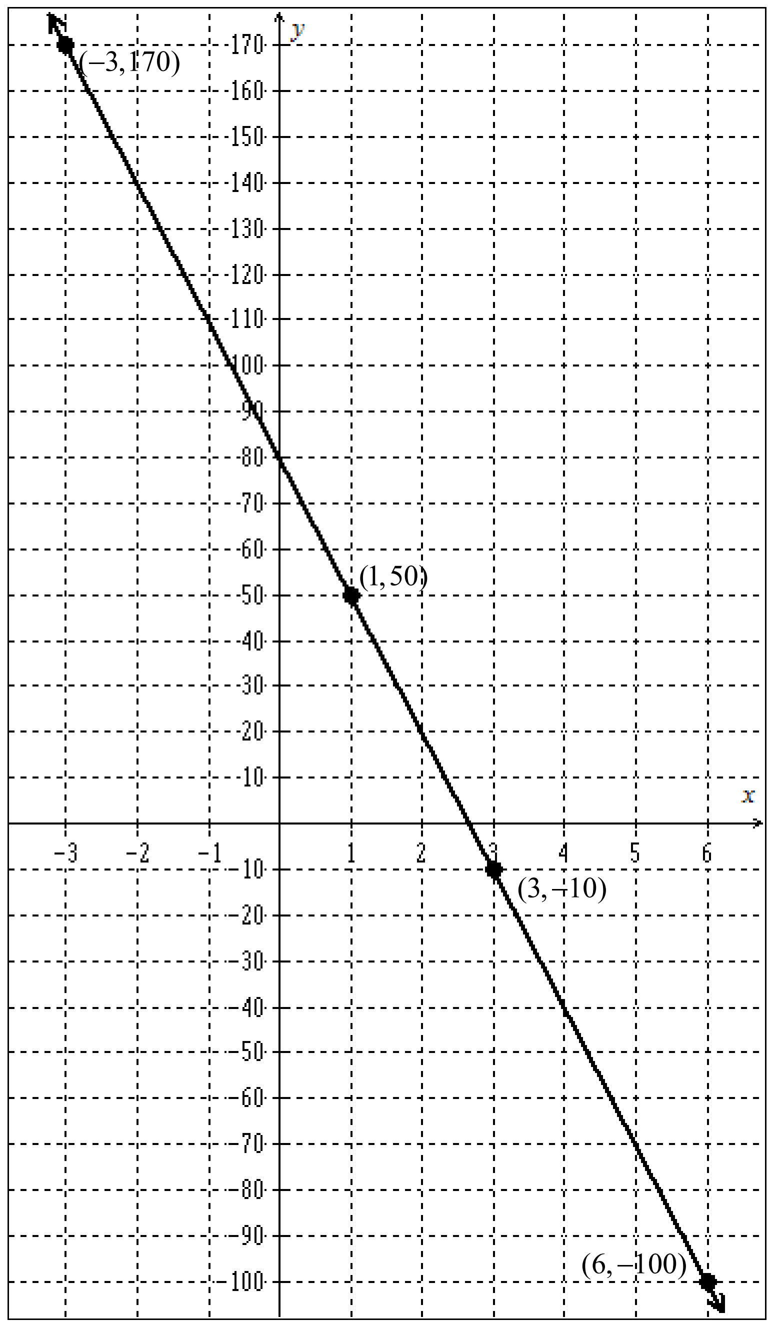
So, the formula must be $y = j(x) = -30x + 80$.

e. $j(-8) = -30(-8) + 80$

$= 240 + 80$

$= 320$

f. Below is the graph of $y = j(x)$. Points from the table are plotted and labeled for clarity.



The graph of $y = j(x)$.

6. **a.** $P(25) = 20.8(25) - 262$

$= 258$

This means that when the wind blows at 25 miles per hour, the turbine generates 258 Watts of power.

b.
$$\begin{aligned} P(x) &= 450 \\ 20.8x - 262 &= 450 \\ 20.8x &= 712 \\ x &\approx 34 \end{aligned}$$

This means that in order to generate 450 Watts of power, there needs to be a wind-speed of about 34 miles per hour.

c.
$$\begin{aligned} P(x) &= 300 \\ 20.8x - 262 &= 300 \\ 20.8x &= 562 \\ x &\approx 27 \end{aligned}$$

So the wind needs to blow at about 27 miles per hour in order for the turbine to generate 300 Watts of power.

d. $P(10) = 20.8(10) - 262$

$= -54$

A wind-speed of 10 miles per hour must not be fast enough to make the turbine spin because we cannot generate a *negative* quantity of power.

e.
$$\begin{aligned} P(x) &= 0 \\ 20.8x - 262 &= 0 \\ 20.8x &= 262 \\ x &\approx 13 \end{aligned}$$

This means that when the wind blows at about 13 miles per hour, the turbine just barely starts to spin fast enough to generate power. Thus, the cut-in speed for this turbine is about 13 miles per hour.