

Section 3.6

Linear Inequalities in Two Variables

How much cholesterol?



An egg provides 165 milligrams of cholesterol and an ounce of meat provides 110 milligrams of cholesterol.

In this section of the textbook, we will explore an application of a linear inequality in two variables to help us to keep a patient from exceeding her daily limit.



First Steps:

- Take comprehensive notes** from your instructor’s lecture and insert your notes into this section of the *Learning Guide*. Be sure to write down all examples, definitions, and other key concepts. Additional learning resources include the *Lecture Series on DVD*, the *PowerPoints*, and Section 3.6 of your textbook which begins on page 262.
- Complete the *Concept and Vocabulary Check* on page 268 of the textbook.

Guided Practice:

- Review each of the following *Solved Problems* and complete each *Pencil Problem*.

Objective #1: Determine whether an ordered pair is a solution of an inequality.

Objective #2: Graph a linear inequality in two variables.

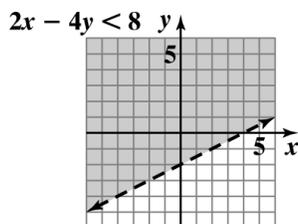
✓ Solved Problem #2

2a. Graph: $2x - 4y < 8$

Step 1. Replace $<$ with $=$ and graph the linear equation $2x - 4y = 8$. Draw a dashed line because the inequality contains a $<$ symbol.

Step 2. Use $(0,0)$ as a test point. $2x - 4y < 8$
 $2(0) - 4(0) < 8$
 $0 < 8$, true

Step 3. The test point $(0,0)$ is part of the solution set, so shade the half-plane containing $(0,0)$.



✎ Pencil Problem #2 ✎

2a. Graph: $x + 2y > 4$

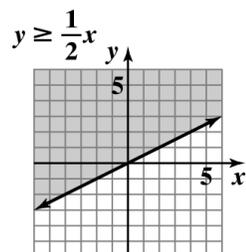
2b. Graph: $y \geq \frac{1}{2}x$

Step 1. Replace \geq with $=$ and graph the linear equation $y = \frac{1}{2}x$. The line passes through the origin so we must find an additional point. For example, using the slope we get the point $(2,1)$. Draw a solid line because the inequality contains a \geq symbol.

Step 2. We cannot use $(0,0)$ as a test point because the line passes through it.

Use $(0,10)$ as a test point $y \geq \frac{1}{2}x$
 $10 \geq \frac{1}{2}(0)$
 $10 \geq 0$, true

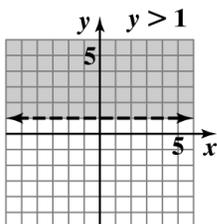
Step 3. The test point $(0,10)$ is part of the solution set, so shade the half-plane containing $(0,10)$.



2b. Graph: $y \leq \frac{1}{3}x$

2c. Graph the inequality in a rectangular coordinate system: $y > 1$

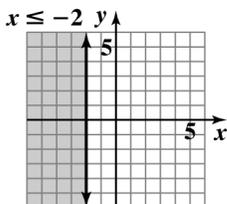
Graph the horizontal line, $y = 1$, with a dashed line and shade the half-plane above the line.



2c. Graph the inequality in a rectangular coordinate system: $y \leq -1$

2d. Graph the inequality in a rectangular coordinate system: $x \leq -2$

Graph the vertical line, $x = -2$, with a solid line and shade the half-plane to the left of the line.

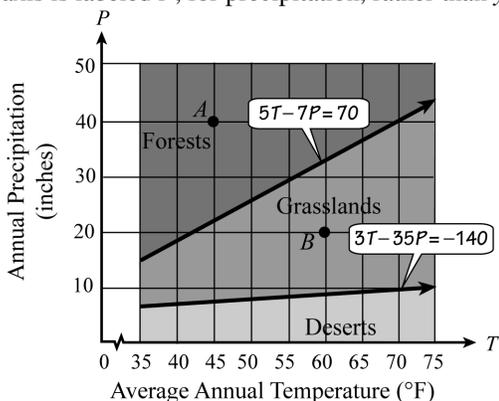


2d. Graph the inequality in a rectangular coordinate system: $x \geq 0$

Objective #3: Solve applied problems involving linear inequalities in two variables.

✓ Solved Problem #3

3. Temperature and precipitation affect whether or not trees and forests can grow. At certain levels of precipitation and temperature, only grasslands and deserts will exist. The figure shows three kinds of regions—deserts, grasslands, and forests—that result from various ranges of temperature and precipitation. Notice that the horizontal axis is labeled T , for temperature, rather than x . The vertical axis is labeled P , for precipitation, rather than y .



✎ Pencil Problem #3 ✎

3. Bottled water and medical supplies are to be shipped to survivors of a hurricane by plane. Each plane can carry no more than 80,000 pounds. The bottled water weighs 20 pounds per container and each medical kit weighs 10 pounds.

Let x represent the number of bottles of water to be shipped. Let y represent the number of medical kits.

The plane's weight limitations can be described by the following inequality:

$$\underbrace{\text{The total weight of the water bottles}}_{20x} \underbrace{\text{plus}}_{+} \underbrace{\text{the total weight of the medical kits}}_{10y} \underbrace{\text{must be less than or equal to}}_{\leq} \underbrace{80,000 \text{ pounds.}}_{80,000}$$

Graph the inequality. Because x and y must be nonnegative, limit the graph to quadrant I and its boundary only.

We can use inequalities in two variables, T and P , to describe the regions in the figure.

For average annual temperatures that exceed 35°F , the inequalities $5T - 7P \geq 70$ and $3T - 35P \leq -140$ model where grasslands occur. Show that the coordinates of point B satisfy both of these inequalities.

Substitute the coordinates of B , $(60, 20)$, into each inequality.

$$5T - 7P \geq 70$$

$$5(60) - 7(20) \geq 70$$

$$300 - 140 \geq 70$$

$$160 \geq 70, \text{ true}$$

$$3T - 35P \leq -140$$

$$3(60) - 35(20) \leq -140$$

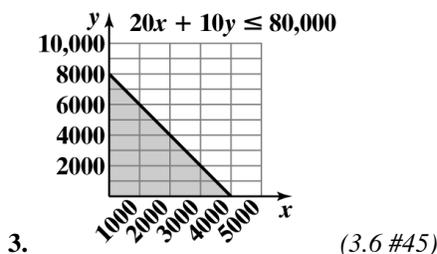
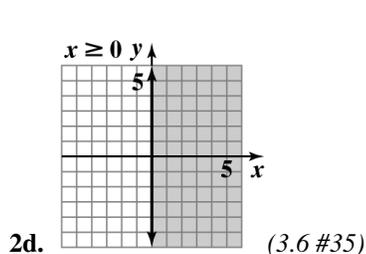
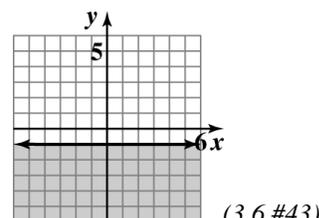
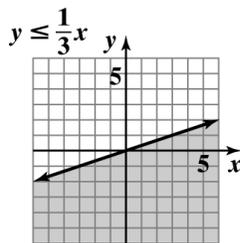
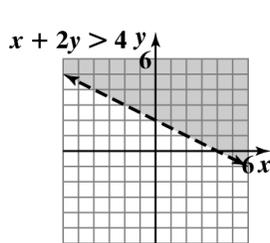
$$180 - 700 \leq -140$$

$$-520 \leq -140, \text{ true}$$

Graph $20x + 10y \leq 80,000$

Answers for Pencil Problems (Textbook Exercise references in parentheses):

1a. not a solution (3.6 #3) 1b. not a solution (3.6 #5)



Homework:

- Review the Section 3.6 summary on page 274 of the textbook.
- Insert your homework into this section of the *Learning Guide*. Show all work neatly and check your answers. Strive to work through difficulties when possible, making note of any exercises where you need additional help. Remember, even if your instructor assigns homework through *MyMathLab*, you should still write out your work.