

Section 3.3

Slope

Are You Stressed Out?

If you are, you might be happy to learn that as we age,
daily stress and worry decrease,
but happiness increases!

For example,
49% of 22-year-olds reported having “a lot of stress,”
but only
29% of 62-year olds reported the same high stress level.

An application exercise in this section will utilize the
concept of slope to mathematically analyze this research.

First Steps:

- ❑ **Take comprehensive notes** from your instructor’s lecture and insert your notes into this section of the *Learning Guide*. Be sure to write down all examples, definitions, and other key concepts. Additional learning resources include the *Lecture Series on DVD*, the *PowerPoints*, and Section 3.3 of your textbook which begins on page 234.
- ❑ Complete the *Concept and Vocabulary Check* on page 241 of the textbook.

Guided Practice:

- ❑ Review each of the following *Solved Problems* and complete each *Pencil Problem*.

Objective #1: Compute a line’s slope.

✓ *Solved Problem #1*

1a. Find the slope of the line passing through the pair of points: $(4, -2)$ and $(-1, 5)$.

Let $(x_1, y_1) = (4, -2)$ and $(x_2, y_2) = (-1, 5)$.

$$m = \frac{\text{Change in } y}{\text{Change in } x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - (-2)}{-1 - 4} = \frac{7}{-5} = -\frac{7}{5}$$

The slope is $-\frac{7}{5}$.

Since the slope is negative, the line falls from left to right.

Pencil Problem #1

1a. Find the slope of the line passing through the pair of points: $(4, 7)$ and $(8, 10)$.

- 1b.** Find the slope of the line passing through $(6, 5)$ and $(2, 5)$ or state that the slope is undefined. Indicate if the line is horizontal or vertical.

Let $(x_1, y_1) = (6, 5)$ and $(x_2, y_2) = (2, 5)$.

$$\begin{aligned}
 m &= \frac{\text{Change in } y}{\text{Change in } x} = \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{5 - 5}{2 - 6} \\
 &= \frac{0}{-4} \\
 &= 0
 \end{aligned}$$

Since the slope is 0, the line is horizontal.

- 1b.** Find the slope of the line passing through $(4, -2)$ and $(3, -2)$ or state that the slope is undefined. Indicate if the line is horizontal or vertical.

- 1c.** Find the slope of the line passing through $(1, 6)$ and $(1, 4)$ or state that the slope is undefined. Indicate if the line is horizontal or vertical.

Let $(x_1, y_1) = (1, 6)$ and $(x_2, y_2) = (1, 4)$.

$$\begin{aligned}
 m &= \frac{\text{Change in } y}{\text{Change in } x} = \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{4 - 6}{1 - 1} \\
 &= \frac{-2}{0}
 \end{aligned}$$

Because division by 0 is undefined the slope is undefined.

Since the slope is undefined, the line is vertical.

- 1c.** Find the slope of the line passing through $(5, 3)$ and $(5, -2)$ or state that the slope is undefined. Indicate if the line is horizontal or vertical.

Objective #2: Use slope to show that lines are parallel.

 **Solved Problem #2**

2. Show that the line passing through (4, 2) and (6, 6) is parallel to the line passing through (0, -2) and (1, 0).

Slope of line through (4,2) and (6,6):

$$m = \frac{\text{Change in } y}{\text{Change in } x} = \frac{6-2}{6-4} = \frac{4}{2} = 2$$

Slope of line through (0,-2) and (1,0):

$$m = \frac{\text{Change in } y}{\text{Change in } x} = \frac{0-(-2)}{1-0} = \frac{2}{1} = 2$$

Since their slopes are equal, the lines are parallel.

 **Pencil Problem #2**

2. Determine if the line passing through (-2, 0) and (0, 6) is parallel to the line passing through (1, 8) and (0, 5).

Objective #3: Use slope to show that lines are perpendicular.

 **Solved Problem #3**

3. Show that the line passing through (-1, 4) and (3, 2) is perpendicular to the line passing through (-2, -1) and (2, 7).

Line through (-1,4) and (3,2):

$$\begin{aligned} m &= \frac{\text{Change in } y}{\text{Change in } x} = \frac{2-4}{3-(-1)} \\ &= \frac{-2}{4} \\ &= -\frac{1}{2} \end{aligned}$$

Line through (-2,-1) and (2,7):

$$\begin{aligned} m &= \frac{\text{Change in } y}{\text{Change in } x} = \frac{7-(-1)}{2-(-2)} \\ &= \frac{8}{4} \\ &= 2 \end{aligned}$$

Since the product of their slopes is $-\frac{1}{2}(2) = -1$, the lines are perpendicular.

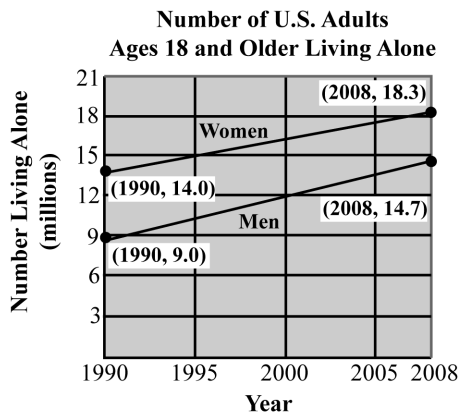
 **Pencil Problem #3**

3. Determine if the line passing through (1, 5) and (0, 3) is perpendicular to the line passing through (-2, 8) and (2, 6).

Objective #4: Calculate rate of change in applied situations.

✓ Solved Problem #4

4. Use the ordered pairs in the figure shown to find the slope of the line segment that represents men. Express the slope correctly to two decimal places and describe what it represents.



Source: U.S. Census Bureau

Let $(x_1, y_1) = (1990, 9.0)$ and $(x_2, y_2) = (2008, 14.7)$.

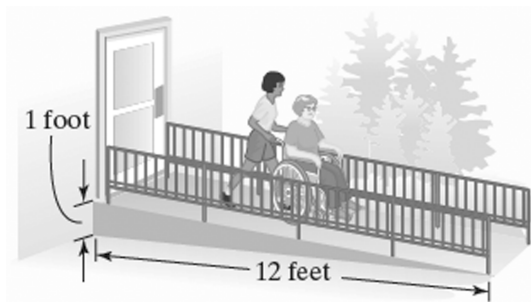
$$m = \frac{\text{Change in } y}{\text{Change in } x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{14.7 - 9.0}{2008 - 1990} = \frac{5.7}{18} \approx 0.32$$

The number of men living alone increased at a rate of 0.32 million per year.

The rate of change is 0.32 million men per year.

✎ Pencil Problem #4 ✎

4. Construction laws are very specific when it comes to access ramps for the disabled. The grade of a ramp refers to its slope expressed as a percent. Every vertical rise of 1 foot requires a horizontal run of 12 feet.



What is the grade of such a ramp?
Round to the nearest tenth of a percent.

Answers for Pencil Problems (Textbook Exercise references in parentheses):

- 1a. $m = \frac{3}{4}$ (3.3 #1) 1b. $m = 0$; the line is horizontal (3.3 #5)
 1c. the slope is undefined; the line is vertical (3.3 #9) 2. parallel (3.3 #23)
 3. perpendicular (3.3 #27) 4. 8.3% (3.3 #49)

Homework:

- Review the Section 3.3 summary on page 273 of the textbook.
- Insert your homework into this section of the *Learning Guide*. Show all work neatly and check your answers. Strive to work through difficulties when possible, making note of any exercises where you need additional help. Remember, even if your instructor assigns homework through *MyMathLab*, you should still write out your work.