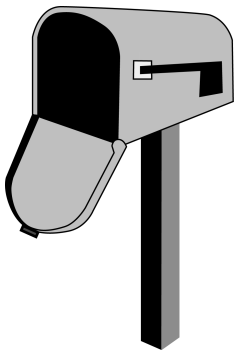
Section 2.7
Solving Linear Inequalities

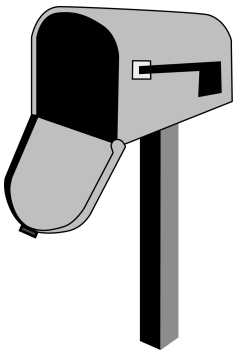


THE CHECK IS NOT IN THE MAIL!

There has been an incredible decline in the number of stamped letters mailed in the United States.

This drop was from a huge 55 billion in 2000 to 30 billion in 2010.

We will learn to apply the concept of linear inequalities to discover the number of years it will take before that value is expected to be no more than 15 billion.



First Steps:

- ☐ **Take comprehensive notes** from your instructor's lecture and insert your notes into this section of the *Learning Guide*. Be sure to write down all examples, definitions, and other key concepts. Additional learning resources include the *Lecture Series on DVD*, the *PowerPoints*, and Section 2.7 of your textbook which begins on page 182.
- ☐ Complete the ***Concept and Vocabulary Check*** on page 194 of the textbook.

Guided Practice:

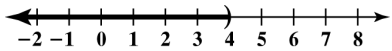
- ☐ Review each of the following ***Solved Problems*** and complete each ***Pencil Problem***.

Objective #1: Graph the solutions of an inequality on a number line.

✔ *Solved Problem #1*

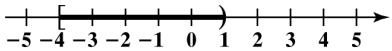
1a. Graph the solution of the inequality: $x < 4$

The solutions of $x < 4$ are all real numbers that are less than 4. They are graphed on a number line by shading all points to the left of 4. The parenthesis at 4 indicates that 4 is not a solution. The arrow shows that the graph extends indefinitely to the left.



1b. Graph the solution of the inequality: $-4 \le x < 1$

The solutions of $-4 \le x < 1$ are all real numbers between –4 and 1, not including 1 but including –4. The square bracket at –4 shows that –4 is a solution. The parenthesis at 1 indicates that 1 is not a solution. Shading indicates the other solutions.



✎ *Pencil Problem #1* ✎

1a. Graph the solution of the inequality: $x > 5$

1b. Graph the solution of the inequality: $-2 < x \le 6$

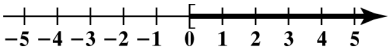
Objective #2: Use interval notation.

✔ *Solved Problem #2*

2a. Express the solution set of the inequality in interval notation and graph the interval: $x \geq 0$

The solutions of $x \geq 0$ are all real numbers greater than or equal to 0.
Use a square bracket at 0 because 0 is a solution.
Since the solution extends indefinitely to the right, use a parenthesis at ∞.
$[0, \infty)$

Graph:



✎ *Pencil Problem #2*

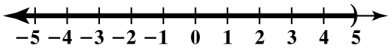
2a. Express the solution set of the inequality in interval notation and graph the interval: $x \leq 3$

2b. Express the solution set of the inequality in interval notation and graph the interval: $x < 5$

The solutions of $x < 5$ are all real numbers less than 5.
Since the solution extends indefinitely to the left, use a parenthesis at $-\infty$.
Use a parenthesis at 5 because 5 is not a solution.

$(-\infty, 5)$

Graph:



2b. Express the solution set of the inequality in interval notation and graph the interval: $x > \frac{5}{2}$

Objective #3: Understand properties used to solve linear inequalities.

✔ *Solved Problem #3*

3. *True* or *False*: When we add (or subtract) a negative number to (or from) both sides of an inequality, the direction of the inequality symbol is reversed.

False; This rule applies to multiplication and division.

✎ *Pencil Problem #3*

3. *True* or *False*: When we multiply or divide both sides of an inequality by a negative number, the direction of the inequality symbol is reversed.

Objective #4: Solve linear inequalities.

✔ *Solved Problem #4*

4a. Solve: $x + 6 < 9$

$$x + 6 < 9$$
$$x + 6 - 6 < 9 - 6$$
$$x < 3$$

The solution set is $(-\infty, 3)$ or $\{x | x < 3\}$.

✎ *Pencil Problem #4*

4a. Solve: $x - 3 > 4$

4b. Solve: $\frac{1}{4}x < 2$

$$\frac{1}{4}x < 2$$
$$4 \cdot \frac{1}{4}x < 4 \cdot 2$$
$$x < 8$$

The solution set is $(-\infty, 8)$ or $\{x | x < 8\}$.

4b. Solve: $-4y \le \frac{1}{2}$

4c. Solve: $-6x < 18$

$$-6x < 18$$
$$\frac{-6x}{-6} > \frac{18}{-6}$$
$$x > -3$$

The solution set is $(-3, \infty)$ or $\{x | x > -3\}$.

4c. Solve: $4x < 20$

Objective #5: Identify inequalities with no solution or true for all real numbers.

✔ *Solved Problem #5*

5a. Solve: $4(x+2) > 4x + 15$

$$4(x+2) > 4x + 15$$
$$4x + 8 > 4x + 15$$
$$4x - 4x + 8 > 4x - 4x + 15$$
$$8 > 15, \text{ false}$$

There is no solution or $\{\ \}$.

✎ *Pencil Problem #5* ✎

5a. Solve: $4x - 4 < 4(x - 5)$

5b. Solve: $3(x+1) \ge 2x + 1 + x$

$$3(x+1) \ge 2x + 1 + x$$
$$3x + 3 \ge 3x + 1$$
$$3x - 3x + 3 \ge 3x - 3x + 1$$
$$3 \ge 1, \text{ true}$$

The solution is $(-\infty, \infty)$ or $\{x | x \text{ is a real number}\}$.

5b. Solve: $2(x+3) > 2x + 1$

Objective #6: Solve problems using linear inequalities.

✔ *Solved Problem #6*

6. To earn a B in a course, you must have a final average of at least 80%. On the first three examinations, you have grades of 82%, 74%, and 78%.
If the final examination counts as two grades, what must you get on the final to earn a B in the course?

Let x = your grade on the final examination.

$$\frac{82+74+78+x+x}{5} \geq 80$$

$$\frac{234+2x}{5} \geq 80$$

$$5\left(\frac{234+2x}{5}\right) \geq 5 \cdot 80$$

$$234+2x \geq 400$$

$$234-234+2x \geq 400-234$$

$$2x \geq 166$$

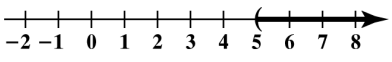
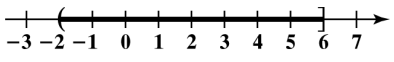
$$x \geq 83$$

To earn a B you must get at least an 83% on the final examination.

✎ *Pencil Problem #6* ✎

6. An elevator at a construction site has a maximum capacity of 3000 pounds. If the elevator operator weighs 245 pounds and each cement bag weighs 95 pounds, how many bags of cement can be safely lifted on the elevator in one trip?

Answers for Pencil Problems *(Textbook Exercise references in parentheses)*:

1a.  *(2.7 #1)* **1b.**  *(2.7 #9)*

2a. $(-\infty, 3]$ *(2.7 #13)* **2b.** $\left(\frac{5}{2}, \infty\right)$ *(2.7 #15)*

3. True *(2.7 #45,47)*

4a. $(7, \infty)$ *(2.7 #21)* **4b.** $\left[-\frac{1}{8}, \infty\right)$ *(2.7 #53)* **4c.** $(-\infty, 5)$ *(2.7 #43)*

5a. There is no solution or $\{\ \}$. *(2.7 #81)* **5b.** The solution is $(-\infty, \infty)$ or $\{x \mid x \text{ is a real number}\}$. *(2.7 #87)*

6. up to 29 bags of cement *(2.7 #111)*

Homework:

- ☐ **Review the Section 2.7 summary** that begins on page 202 of the textbook.
- ☐ **Insert your homework** into this section of the *Learning Guide*. Show all work neatly and check your answers. Strive to work through difficulties when possible, making note of any exercises where you need additional help. Remember, even if your instructor assigns homework through *MyMathLab*, you should still write out your work.