Section 2.6
Problem Solving in Geometry

Are they telling you the truth?

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Did you know that graphs can be used to distort data, making it difficult for the viewer to learn the truth?

One potential source of misunderstanding involves geometric figures whose lengths are in the correct ratios for the displayed data, but whose areas or volumes are then varied to create a misimpression about how the data are changing over time.

Two such examples of misleading visual displays are given in the text and are related to the objectives of this section.

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First Steps:

☐ **Take comprehensive notes** from your instructor's lecture and insert your notes into this section of the *Learning Guide*. Be sure to write down all examples, definitions, and other key concepts. Additional learning resources include the *Lecture Series on DVD*, the *PowerPoints*, and Section 2.6 of your textbook which begins on page 168.

☐ Complete the *Concept and Vocabulary Check* on page 176 of the textbook.

Guided Practice:

☐ Review each of the following *Solved Problems* and complete each *Pencil Problem*.

Objective #1: Solve problems using formulas for perimeter and area.

✔ *Solved Problem #1*	✎ *Pencil Problem #1* ✎
1. A sailboat has a triangular sail with an area of 24 square feet and a base that is 4 feet long. Find the height of the sail.	**1.** A triangle has a base of 5 feet and an area of 20 square feet. Find the triangle's height.

Use the formula $A = \dfrac{1}{2}bh$, where $A = 24$ and $b = 4$.

$A = \dfrac{1}{2}bh$

$24 = \dfrac{1}{2} \cdot 4 \cdot h$

$24 = 2h$

$12 = h$

The height of the sail is 12 ft.

Objective #2: Solve problems using formulas for a circle's area and circumference.

✔ *Solved Problem #2*	✎ *Pencil Problem #2*✎

2a. The diameter of a circular landing pad for helicopters is 40 feet. Find the area and circumference of the landing pad. Express answers in terms of π. Then round answers to the nearest square foot and foot, respectively.

Use the formulas for the area and circumference of a circle. The radius is 20 ft.

Area: $A = \pi r^2$

$\qquad A = \pi(20)^2$

$\qquad\quad = 400\pi$

$\qquad\quad \approx 1256 \text{ or } 1257$

The area is 400π ft^2
or approximately 1256 ft^2 or 1257 ft^2.

Circumference: $C = 2\pi r$

$\qquad\qquad C = 2\pi(20)$

$\qquad\qquad\quad = 40\pi$

$\qquad\qquad\quad \approx 126$

The circumference is 40π ft or approximately 126 ft.

2a. The radius of a circle is 4 cm. Find the area and circumference of the circle. Express answers in terms of π. Then round answers to the nearest whole number.

2b. Which one of the following is the better buy: a large pizza with an 18-inch diameter for $20.00 or a medium pizza with a 14-inch diameter for $14.00?

The radius of the large pizza is 9 inches, and the radius of the medium pizza is 7 inches.

large pizza: $A = \pi r^2 = \pi(9 \text{ in.})^2 = 81\pi \text{ in.}^2 \approx 254 \text{ in.}^2$

medium pizza: $A = \pi r^2 = \pi(7 \text{ in.})^2 = 49\pi \text{ in.}^2 \approx 154 \text{ in.}^2$

For each pizza, find the price per square inch by dividing the price by the area.

Price per square inch for the large pizza:
$$\frac{\$20.00}{81\pi \text{ in.}^2} \approx \frac{\$20.00}{254 \text{ in.}^2} \approx \frac{\$0.08}{\text{in.}^2}$$

Price per square inch for the medium pizza:
$$\frac{\$14.00}{49\pi \text{ in.}^2} \approx \frac{\$14.00}{154 \text{ in.}^2} \approx \frac{\$0.09}{\text{in.}^2}.$$

The large pizza is the better buy.

2b. Which one of the following is the better buy: a large pizza with an 14-inch diameter for $12.00 or a medium pizza with a 7-inch diameter for $5.00?

Objective #3: Solve problems using formulas for volume.	
✔ **Solved Problem #3**	✎ **Pencil Problem #3**✎

3. A cylinder with a radius of 3 inches and a height of 5 inches has its height doubled. How many times greater is the volume of the larger cylinder than the volume of the smaller cylinder?

Smaller cylinder: $r = 3$ in., $h = 5$ in.

$V = \pi r^2 h$

$V = \pi(3)^2 \cdot 5$

$\quad = 45\pi$

The volume of the smaller cylinder is 45π in.^{3}.

Larger cylinder: $r = 3$ in., $h = 10$ in.

$V = \pi r^2 h$

$V = \pi(3)^2 \cdot 10$

$\quad = 90\pi$

The volume of the smaller cylinder is 90π in.^{3}.

The ratio of the volumes of the two cylinders is

$\dfrac{V_{\text{larger}}}{V_{\text{smaller}}} = \dfrac{90\pi \text{ in.}^3}{45\pi \text{ in.}^3} = \dfrac{2}{1}$.

Thus, the volume of the larger cylinder is 2 times the volume of the smaller cylinder.

3. A cylinder with a radius of 3 inches and a height of 4 inches has its radius tripled. How many times greater is the volume of the larger cylinder than the volume of the smaller cylinder?

Objective #4: Solve problems involving the angles of a triangle.	
✔ **Solved Problem #4**	✎ **Pencil Problem #4**✎

4. In a triangle, the measure of the first angle is three times the measure of the second angle. The measure of the third angle is $20°$ less than the second angle. What is the measure of each angle?

Let $3x =$ the measure of the first angle.
Let $x =$ the measure of the second angle.
Let $x - 20 =$ the measure of the third angle.

$3x + x + (x - 20) = 180$

$\qquad\qquad 5x - 20 = 180$

$\qquad\qquad\quad\ \ 5x = 200$

$\qquad\qquad\qquad x = 40$

$\qquad\qquad\quad 3x = 120$

$\qquad\qquad x - 20 = 20$

The three angle measures are 120°, 40°, and 20°.

4. One angle of a triangle is twice as large as another. The measure of the third angle is $20°$ more than that of the smallest angle. What is the measure of each angle?

Objective #5: Solve problems involving complementary and supplementary angles.	

✔ *Solved Problem #5*	✎ *Pencil Problem #5*✎
5. The measure of an angle is twice the measure of its complement. What is the angle's measure?	**5.** An angle's measure is $60°$ more than that of its complement. What is the angle's measure?

Step 1
Let x = the measure of the angle.

Step 2
Let $90 - x$ = the measure of its complement.

Step 3
The angle's measure is twice that of its complement, so the equation is $x = 2 \cdot (90 - x)$.

Step 4
Solve this equation
$$x = 2 \cdot (90 - x)$$
$$x = 180 - 2x$$
$$x + 2x = 180 - 2x + 2x$$
$$3x = 180$$
$$x = 60$$
The measure of the angle is 60°.

Step 5
The complement of the angle is $90° - 60° = 30°$, and 60° is indeed twice 30°.

Answers for Pencil Problems *(Textbook Exercise references in parentheses)*:

1. 8 feet *(2.6 #9)*

2a. Area is 16π cm^2 (approximately 50 cm^2) and circumference is 8π cm (approximately 25 cm). *(2.6 #13)*

2b. large pizza is better buy *(2.6 #59)*

3. 9 times *(2.6 #29)*

4. The three angle measures are $40°$, $80°$, and $60°$. *(2.6 #35)*   **5.** $75°$ *(2.6 #45)*

Homework:

☐ **Review the Section 2.6 summary** that begins on page 201 of the textbook.

☐ **Insert your homework** into this section of the *Learning Guide*. Show all work neatly and check your answers. Strive to work through difficulties when possible, making note of any exercises where you need additional help. Remember, even if your instructor assigns homework through *MyMathLab*, you should still write out your work.