

Section 2.3

Solving Linear Equations



Slow Down or Pay the fine!

In Massachusetts, speeding fines are determined by the mathematical formula $F = 10(x - 65) + 50$, where F is the cost, in dollars, of the fine if a person is caught driving x miles per hour.

In this section, you will learn to apply the formula to determine how fast a person was driving if you are told the amount of their fine.



First Steps:

- Take comprehensive notes** from your instructor's lecture and insert your notes into this section of the *Learning Guide*. Be sure to write down all examples, definitions, and other key concepts. Additional learning resources include the *Lecture Series on DVD*, the *PowerPoints*, and Section 2.3 of your textbook which begins on page 132.
- Complete the *Concept and Vocabulary Check* on page 141 of the textbook.

Guided Practice:

- Review each of the following *Solved Problems* and complete each *Pencil Problem*.

Objective #1: Solve linear equations.

✓ Solved Problem #1

1a. Solve: $-7x + 25 + 3x = 16 - 2x - 3$

Simplify the algebraic expression on each side.

$$-7x + 25 + 3x = 16 - 2x - 3$$

$$-4x + 25 = 13 - 2x$$

Collect variable terms on one side and constant terms on the other side.

$$-4x + 25 = 13 - 2x$$

$$-4x + 25 + 2x = 13 - 2x + 2x$$

$$-2x + 25 = 13$$

$$-2x + 25 - 25 = 13 - 25$$

$$-2x = -12$$

Isolate the variable and solve.

$$\frac{-2x}{-2} = \frac{-12}{-2}$$

$$x = 6$$

The solution set is $\{6\}$.

✎ Pencil Problem #1 ✎

1a. Solve: $4x - 9x + 22 = 3x + 30$

1b. Solve: $4(2x+1) - 29 = 3(2x-5)$

Simplify the algebraic expression on each side.

$$4(2x+1) - 29 = 3(2x-5)$$

$$8x + 4 - 29 = 6x - 15$$

$$8x - 25 = 6x - 15$$

Collect variable terms on one side and constant terms on the other side.

$$8x - 6x - 25 = 6x - 6x - 15$$

$$2x - 25 = -15$$

$$2x - 25 + 25 = -15 + 25$$

$$2x = 10$$

Isolate the variable and solve.

$$\frac{2x}{2} = \frac{10}{2}$$

$$x = 5$$

The solution set is $\{5\}$.

1b. Solve: $5(2x-8) - 2 = 5(x-3) + 3$

Objective #2: Solve linear equations containing fractions.

 **Solved Problem #2**

2. Solve: $\frac{x}{4} = \frac{2x}{3} + \frac{5}{6}$

Begin by multiplying both sides of the equation by 12, the least common denominator.

$$\frac{x}{4} = \frac{2x}{3} + \frac{5}{6}$$

$$12 \cdot \frac{x}{4} = 12 \left(\frac{2x}{3} + \frac{5}{6} \right)$$

$$12 \cdot \frac{x}{4} = 12 \cdot \frac{2x}{3} + 12 \cdot \frac{5}{6}$$

$$3x = 8x + 10$$

$$3x - 8x = 8x - 8x + 10$$

$$-5x = 10$$

$$\frac{-5x}{-5} = \frac{10}{-5}$$

$$x = -2$$

The solution set is $\{-2\}$.

 **Pencil Problem #2** 

2. Solve: $\frac{y}{3} + \frac{2}{5} = \frac{y}{5} - \frac{2}{5}$

Objective #3: Solve linear equations containing decimals.
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 Solved Problem #3	 Pencil Problem #3 
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3. Solve: $0.48x + 3 = 0.2(x - 6)$

First apply the distributive property to remove the parentheses, and then multiply both sides by 100 to clear the decimals.

$$0.48x + 3 = 0.2(x - 6)$$

$$0.48x + 3 = 0.2x - 1.2$$

$$100(0.48x + 3) = 100(0.2x - 1.2)$$

$$48x + 300 = 20x - 120$$

$$48x + 300 - 300 = 20x - 120 - 300$$

$$48x = 20x - 420$$

$$48x - 20x = 20x - 20x - 420$$

$$28x = -420$$

$$\frac{28x}{28} = \frac{-420}{28}$$

$$x = -15$$

The solution set is $\{-15\}$.

3. Solve: $0.3x - 4 = 0.1(x + 10)$

Objective #4: Identify equations with no solution or infinitely many solutions.
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 Solved Problem #4	 Pencil Problem #4 
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4a. Solve: $3x + 7 = 3(x + 1)$

$$3x + 7 = 3(x + 1)$$

$$3x + 7 = 3x + 3$$

$$3x - 3x + 7 = 3x - 3x + 3$$

$$7 = 3$$

The original equation is equivalent to the false statement $7 = 3$. Thus, the equation has no solution.

The solution set is $\{ \}$.

4a. Solve: $4(x + 2) + 1 = 7x - 3(x - 2)$

4b. Solve: $3(x - 1) + 9 = 8x + 6 - 5x$

$$3(x - 1) + 9 = 8x + 6 - 5x$$

$$3x - 3 + 9 = 3x + 6$$

$$3x + 6 = 3x + 6$$

$$3x - 3x + 6 = 3x - 3x + 6$$

$$6 = 6$$

The original equation is equivalent to $6 = 6$, which is true for every value of x . The equation's solution is all real numbers or $\{x \mid x \text{ is a real number}\}$.

4b. Solve: $2(x + 4) = 4x + 5 - 2x + 3$

Objective #5: Solve applied problems using formulas. **Solved Problem #5**

5. It has been shown that persons with a low sense of humor have higher levels of depression in response to negative life events than those with a high sense of humor. This can be modeled by the following formulas:

$$\text{Low-Humor Group: } D = \frac{10}{9}x + \frac{53}{9}$$

$$\text{High-Humor Group: } D = \frac{1}{9}x + \frac{26}{9}$$

where x represents the intensity of a negative life event (from a low of 1 to a high of 10) and D is the level of depression in response to that event.

If the low-humor group averages a level of depression of 10 in response to a negative life event, what is the intensity of that event?

$$\text{Low-Humor Group: } D = \frac{10}{9}x + \frac{53}{9}$$

$$10 = \frac{10}{9}x + \frac{53}{9}$$

$$9 \cdot 10 = 9 \left(\frac{10}{9}x + \frac{53}{9} \right)$$

$$90 = 10x + 53$$

$$90 - 53 = 10x + 53 - 53$$

$$37 = 10x$$

$$\frac{37}{10} = \frac{10x}{10}$$

$$3.7 = x$$

$$x = 3.7$$

The formula indicates that if the low-humor group averages a level of depression of 10 in response to a negative life event, the intensity of that event is 3.7.

 **Pencil Problem #5**

5. The formula $p = 15 + \frac{5d}{11}$ describes the pressure of sea water, p , in pounds per square foot, at a depth of d feet below the surface.

The record depth for breath-held diving, by Francisco Ferreras (Cuba) off Grand Bahama Island, on November 14, 1993, involved pressure of 201 pounds per square foot. Use the formula to determine the depth to which Ferreras descended?

Answers for Pencil Problems (Textbook Exercise references in parentheses):

1a. $\{-1\}$ (2.3 #3) 1b. $\{6\}$ (2.3 #25) 2. $\{-6\}$ (2.3 #41) 3. $\{25\}$ (2.3 #51)

4a. \emptyset (2.3 #67) 4b. $\{x \mid x \text{ is a real number}\}$ (2.3 #61) 5. 409.2 feet (2.3 #91)

Homework:

- Review the Section 2.3 summary** that begins on page 199 of the textbook.
- Insert your homework** into this section of the *Learning Guide*. Show all work neatly and check your answers. Strive to work through difficulties when possible, making note of any exercises where you need additional help. Remember, even if your instructor assigns homework through *MyMathLab*, you should still write out your work.