

## Section 1.1

### Introduction to Algebra: Variables and Mathematical Models

## *Where Goes the Bride????*



American's attitudes about marriage are changing. More and more people are saying that marriage is optional and obsolete.

In this section of your textbook we explore how mathematics can be used to see how these attitudes vary by age group.



#### **First Steps:**

- Take comprehensive notes** from your instructor's lecture and insert your notes into this section of the *Learning Guide*. Be sure to write down all examples, definitions, and other key concepts. Additional learning resources include the *Lecture Series on DVD*, the *PowerPoints*, and Section 1.1 of your textbook which begins on page 2.
- Complete the *Concept and Vocabulary Check* on page 10 of the textbook.

#### **Guided Practice:**

- Review each of the following *Solved Problems* and complete each *Pencil Problem*.

#### **Objective #1:** Evaluate algebraic expressions.

##### ✓ *Solved Problem #1*

- 1a.** Evaluate the expression  $2(x+6)$  for  $x=10$ .

$$\begin{aligned} 2(x+6) &= 2(\overset{x}{10}+6) \\ &= 2(16) \\ &= 32 \end{aligned}$$

##### ✎ *Pencil Problem #1* ✎

- 1a.** Evaluate the expression  $5+3x$  for  $x=4$ .

- 1b.** Evaluate the expression  $\frac{6x-y}{2y-x-8}$   
for  $x=3$  and  $y=8$ .

$$\begin{aligned} \frac{6x-y}{2y-x-8} &= \frac{6 \cdot \overset{x}{3} - \overset{y}{8}}{2 \cdot \underset{y}{8} - \underset{x}{3} - 8} \\ &= \frac{18-8}{16-3-8} \\ &= \frac{10}{5} \\ &= 2 \end{aligned}$$

- 1b.** Evaluate the expression  $\frac{2x-y+6}{2y-x}$   
for  $x=7$  and  $y=5$ .

**Objective #2:** Translate English phrases into algebraic expressions.

 **Solved Problem #2**

2. Write each English phrase as an algebraic expression. Let the variable  $x$  represent the number.

2a. the product of 6 and a number

$$6x$$

2b. a number added to 4

$$4 + x$$

2c. three times a number, increased by 5

$$3x + 5$$

2d. twice a number subtracted from 12

$$12 - 2x$$

2e. the quotient of 15 and a number

$$\frac{15}{x}$$

 **Pencil Problem #2** 

2. Write each English phrase as an algebraic expression. Let the variable  $x$  represent the number.

2a. four more than a number

2b. nine subtracted from a number

2c. three times a number, decreased by 5

2d. one less than the product of 12 and a number

2e. six more than the quotient of a number and 30

**Objective #3:** Determine whether a number is a solution of an equation.

 **Solved Problem #3**

3a. Determine whether the given number is a solution of the equation.

$$9x - 3 = 42; 6$$

$$9x - 3 = 42$$

$$9(6) - 3 = 42$$

$$54 - 3 = 42$$

$$51 = 42, \text{ false}$$

6 is not a solution.

3b. Determine whether the given number is a solution of the equation.

$$2(y + 3) = 5y - 3; 3$$

$$2(y + 3) = 5y - 3$$

$$2(3 + 3) = 5(3) - 3$$

$$2(6) = 15 - 3$$

$$12 = 12, \text{ true}$$

3 is a solution.

 **Pencil Problem #3** 

3a. Determine whether the given number is a solution of the equation.

$$5a - 4 = 2a + 5; 3$$

3b. Determine whether the given number is a solution of the equation.

$$2(w + 1) = 3(w - 1); 7$$

**Objective #4:** Translate English sentences into algebraic equations.**✓ Solved Problem #4**

- 4a. Write the sentence as an equation. Let the variable  $x$  represent the number.

The quotient of a number and 6 is 5.

$$\frac{x}{6} = 5$$

**✎ Pencil Problem #4 ✎**

- 4a. Write the sentence as an equation. Let the variable  $x$  represent the number.

Four times a number is 28.

- 4b. Write the sentence as an equation. Let the variable  $x$  represent the number.

Seven decreased by twice a number yields 1.

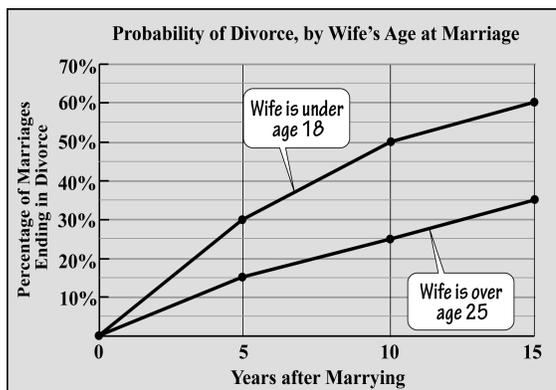
$$7 - 2x = 1$$

- 4b. Write the sentence as an equation. Let the variable  $x$  represent the number.

Five times a number is equal to 24 decreased by the number.

**Objective #5:** Evaluate formulas.**✓ Solved Problem #5**

5. Divorce rates are considerably higher for couples who marry in their teens. The line graphs in the figure show the percentages of marriages ending in divorce based on the wife's age at marriage.



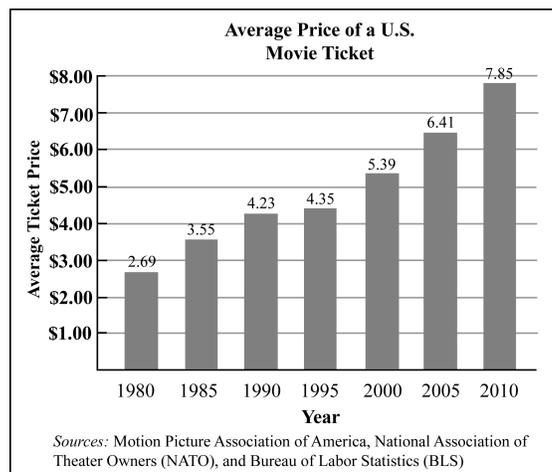
The following mathematical models approximate the data displayed by the line graphs.

Wife is under 18 at time of marriage:  
 $d = 4n + 5$

Wife is over 25 at time of marriage:  
 $d = 2.3n + 1.5$

**✎ Pencil Problem #5 ✎**

5. The bar graph shows average price of a movie ticket for selected years from 1980 through 2010.



A mathematical model that approximates the data displayed by the bar graph is

$$T = 0.15n + 2.72$$

where  $T$  represents the average movie ticket price and  $n$  is number of years after 1980.

**5a.** Use the appropriate formula to determine the percentage of marriages ending in divorce after 15 years when the wife is under 18 at the time of marriage.

$$d = 4n + 5$$

$$d = 4(15) + 5 = 65$$

65% of marriages end in divorce after 15 years when the wife is under 18 at the time of marriage

**5b.** Use the appropriate line graph in the figure to determine the percentage of marriages ending in divorce after 15 years when the wife is under 18 at the time of marriage.

According to the line graph, 60% of marriages end in divorce after 15 years when the wife is under 18 at the time of marriage.

**5c.** Does the value given by the mathematical model underestimate or overestimate the actual percentage of marriages ending in divorce after 15 years as shown by the graph? By how much?

$$65\% - 60\% = 5\%$$

The mathematical model overestimates the actual percentage shown in the graph by 5%.

**5a.** Use the formula to find the average ticket price 10 years after 1980, or in 1990. Does the mathematical model underestimate or overestimate the average ticket price shown by the bar graph for 1990? By how much?

**5b.** Does the mathematical model underestimate or overestimate the average ticket price shown by the bar graph for 2010? By how much?

**Answers for Pencil Problems (Textbook Exercise references in parentheses):**

**1a.** 17 (1.1 #9)   **1b.** 5 (1.1 #23)   **2a.**  $x + 4$  (1.1 #25)   **2b.**  $x - 9$  (1.1 #31)   **2c.**  $3x - 5$  (1.1 #35)

**2d.**  $12x - 1$  (1.1 #37)   **2e.**  $\frac{x}{30} + 6$  (1.1 #41)   **3a.** solution (1.1 #53)   **3b.** not a solution (1.1 #57)

**4a.**  $4x = 28$  (1.1 #59)   **4b.**  $5x = 24 - x$  (1.1 #73)

**5a.** \$4.22; underestimates by \$0.01 (1.1 #83a)   **5b.** underestimates by \$0.63 (1.1 #83b)

**Homework:**

- Review the Section 1.1 summary** on page 102 of the textbook.
- Insert your homework** into this section of the *Learning Guide*. Show all work neatly and check your answers. Strive to work through difficulties when possible, making note of any exercises where you need additional help. Remember, even if your instructor assigns homework through *MyMathLab*, you should still write out your work.