Math 95 Final Review No Calculator

Name: Solutions

1. The functions f, g, and h are described below. Use them to answer the following questions.

x	$f(x)$
-2	$\frac{4}{3}$
-1	2
0	0
1	und.
2	4
3	3
4	$\frac{8}{3}$

$$g(x) = \frac{1}{x-3}$$

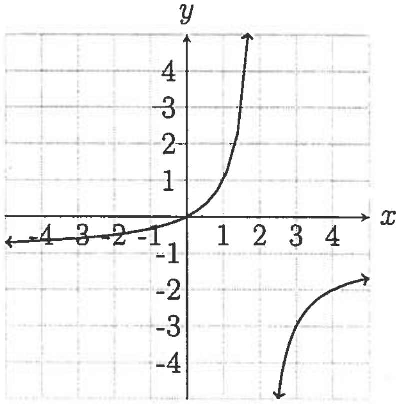


Figure 1: $y = h(x)$

a. Evaluate $f(1)$.

$f(1)$ is undefined

b. Evaluate $g(5)$.

$g(5) = \frac{1}{5-3} = \frac{1}{2}$

c. Evaluate $h(1)$.

$h(1) = 1$

d. What is the domain of g?

$D = \{x \mid x \neq 3\}$

e. Solve $f(x) = \frac{8}{3}$.

The sol set is $\{4\}$.

f. Solve $h(x) = -3$.

The sol set is $\{3\}$.

g. Solve $g(x) = 1$. Work on scratch, no need to show it.

The sol set is $\{4\}$.

h. What is the domain and range of h?

$D = \{x \mid x \neq 2\}$

$R = \{y \mid y \neq -1\}$.

2. What is the one requirement for a relationship between inputs and outputs to be considered a function?

Each valid input has exactly one output.

3. Solve the following equation symbolically. **Part of your grade for these problems is to *justify* breaking up the absolute value inequalities by translating the inequality and showing the numberline which motivates the inequality set up.** State a conclusion using interval notation within a complete sentence.

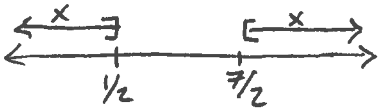
4. $|2x - 4| \geq 3$

The size of 2x-4 must be bigger than or equal to 3.

$2x-4 \leq -3$ or $3 \leq 2x-4$

$2x \leq 1$ $\quad$ $7 \leq 2x$

$x \leq 1/2$ $\quad$ $7/2 \leq x$

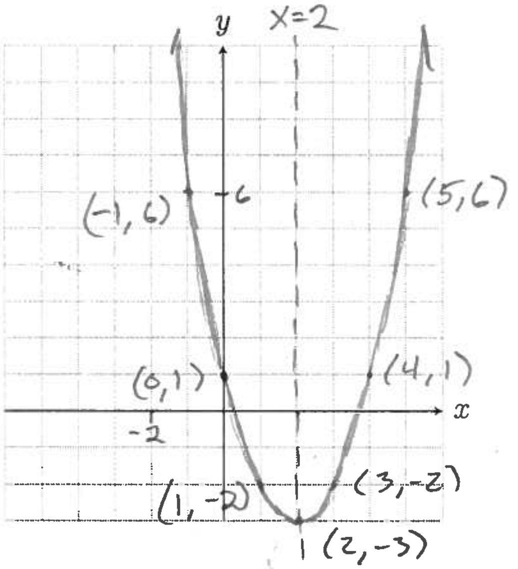


The sol interval is $(-\infty, 1/2] \cup [7/2, \infty)$

5.

a. Make a table of points for $f(x) = (x-2)^2 - 3$ using at least 5 points with the vertex in the middle (so at least 2 points on each side of the vertex). Then graph $y = f(x)$ in the space provided. Label all points from the table along with the axis of symmetry.

x	$f(x)$
-1	6
0	1
1	-2
2	-3 ← vertex
3	-2
4	1
5	6



b. Describe using a complete sentence how the function $sqr(x) = x^2$ is translated to become f.

It is shifted 2 units right & 3 units down.

6. In terms of inputs and outputs, describe what the points on the graph of a function represent.

Given a point (x, y) on the graph of a function, the x-value is the input & the y-value is the corresponding output.

7. Complete the square of the following function to put it into vertex form. Then state the vertex.

$f(x) = 3x^2 - 15x + 1$

$= 3[x^2 - 5x] + 1$

$= 3[(x - 5/2)^2 - \frac{25}{4}] + 1$

$= 3(x - 5/2)^2 - \frac{75}{4} + \frac{4}{4}$

$= 3(x - 5/2)^2 - \frac{71}{4}$

$x^2 - 5x = x^2 - 5x + \frac{25}{4} - \frac{25}{4}$ $\quad (\frac{-5}{2})^2 = \frac{25}{4}$

$= (x - \frac{5}{2})^2 - \frac{25}{4}$

The vertex is $(5/2, -71/4)$.

8. Solve $0 = 2x^2 - 4x + 5$ using the quadratic formula. If your answer is complex, write it in standard complex number form and state your conclusion using set notation in a complete sentence.

$x = \frac{4 \pm \sqrt{16 - 4(2)(5)}}{2(2)}$

$= \frac{4 \pm \sqrt{16 - 40}}{4}$

$= \frac{4}{4} \pm \frac{\sqrt{-24}}{4}$

$= 1 \pm \frac{2\sqrt{6}}{4} i$

$= 1 \pm \frac{\sqrt{6}}{2} i$

The sol set is $\{1 \pm \frac{\sqrt{6}}{2} i\}$.

9. Simplify the following expressions. Be sure to identify any inputs where two expressions are not equivalent despite being equivalent for all other inputs.

a. $\dfrac{x-1}{x^2+x-6} \div \dfrac{x-1}{x+3}$

$= \dfrac{x-1}{(x+3)(x-2)} \cdot \dfrac{x+3}{x-1}$

$= \dfrac{1}{x-2}$ unless $x=1$ or $x=-3$

b. $\dfrac{3}{x-1} - \dfrac{x}{x+4}$

$= \dfrac{3(x+4)}{(x-1)(x+4)} - \dfrac{x(x-1)}{(x-1)(x+4)}$

$= \dfrac{3x+12-x^2+x}{(x-1)(x+4)}$

$= \dfrac{-x^2+4x+12}{(x-1)(x+4)}$

$= -\dfrac{x^2-4x-12}{(x-1)(x+4)} = -\dfrac{(x-6)(x+2)}{(x-1)(x+4)}$

10. Simplify the following complex fraction:

$$\dfrac{\dfrac{1}{x^2-1} - \dfrac{2}{x+1}}{\dfrac{3}{x+1} - \dfrac{2}{x-1}}$$

$= \dfrac{\dfrac{1}{(x-1)(x+1)} - \dfrac{2}{x+1}}{\dfrac{3}{x+1} - \dfrac{2}{x-1}} \cdot \dfrac{(x-1)(x+1)/1}{(x-1)(x+1)/1}$

$= \dfrac{1-2(x-1)}{3(x-1)-2(x+1)}$

$= \dfrac{1-2x+2}{3x-3-2x-2}$

$= \dfrac{-2x+3}{x-5}$

11. Solve the following rational equation. State your conclusion using set notation in a complete sentence.

$(x-1)(x+3)\left(\dfrac{2}{x-1} - \dfrac{5}{x+3}\right) = 1(x-1)(x+3)$

$2(x+3) - 5(x-1) = x^2+2x-3$

$2x+6-5x+5 = x^2+2x-3$

$-3x+11 = x^2+2x-3$

$0 = x^2+5x-14$

$0 = (x+7)(x-2)$

The sol set is $\{-7, 2\}$.

12. Solve the following equations.

a. $(4 - x)^3 = -1$

$4 - x = -1$

$-x = -5$

$x = 5$

The sol set is $\{5\}$.

b. $\frac{1}{2}(x-1)^5 = 16$

$(x-1)^5 = 32$

$x - 1 = 2$

$x = 3$

The sol set is $\{3\}$.

13. Solve the following radical equations. **Check your solutions always!**

a. $\sqrt{2x-1} = 3$

$2x - 1 = 9$

$2x = 10$

$x = 5$

Check:

$\sqrt{2(5)-1} = 3$

c. $\sqrt{4-x} + 5 = 8$

$\sqrt{4-x} = 3$

$4 - x = 9$

$-x = 5$

$x = -5$

Check:

$\sqrt{4+5} + 5 = 8$

$\sqrt{9} + 5 = 8$ ✓

The sol set is $\{-5\}$.

b. $\sqrt{3x+3} = 2x - 1$

$3x + 3 = (2x-1)^2$

$3x + 3 = 4x^2 - 4x + 1$

$4(-2) = -8$

$1 \quad -8$

$0 = 4x^2 - 7x - 2$

$0 = \underline{4x^2 - 8x} + \underline{x - 2}$

$0 = 4x(x-2) + (x-2)$

$0 = (x-2)(4x+1)$

The sol set is $\{2\}$.

Check:

$\sqrt{3(2)+3} = 2(2) - 1$

$\sqrt{9} = 3$ ✓

$\sqrt{3(-\frac{1}{4})+3} = 2(-\frac{1}{4}) - 1$

$\cancel{\sqrt{9/4} = -3/2}$

d. $\sqrt{2x} - 1 = \sqrt{x+1}$

$(\sqrt{2x} - 1)^2 = x + 1$

$2x - 2\sqrt{2x} + 1 = x + 1$

$-2x \qquad -1 \quad -2x \quad -1$

$-2\sqrt{2x} = -x$

$(-2\sqrt{2x})^2 = (-x)^2$

$4(2x) = x^2$

$8x = x^2$

$0 = x^2 - 8x$

$0 = x(x-8)$

Check:

$\sqrt{2(0)} - 1 = \sqrt{0+1}$

$0 \cancel{-1} = 1$

$\sqrt{2(8)} - 1 = \sqrt{8+1}$

$4 - 1 = 3$ ✓

The sol set is $\{8\}$.

14. Determine the domain and range (calculator okay for range) of the following functions. State your conclusions using both set and interval notation.

a. $f(x) = \sqrt{1+2x^2}$

$D = \mathbb{R} = (-\infty, \infty)$

$R = \{y \mid y \geq 1\} = [1, \infty)$

d. $f(x) = \sqrt[3]{2x-1}$

$D = \mathbb{R} = (-\infty, \infty)$

$R = \mathbb{R} = (-\infty, \infty)$

b. $f(x) = \dfrac{1}{\sqrt{x-1}}$

$D = \{x \mid x > 1\} = (1, \infty)$

$R = \{y \mid y > 0\} = (0, \infty)$

e. $f(x) = \sqrt{3x+5} - 2$

$D = \{x \mid x \geq -5/3\} = [-5/3, \infty)$

$R = \{y \mid y \geq -2\} = [-2, \infty)$

c. $f(x) = \sqrt[4]{x+2}$

$D = \{x \mid x \geq -2\} = [-2, \infty)$

$R = \{y \mid y \geq 0\} = [0, \infty)$

f. $f(x) = \sqrt[7]{x-8} + 3$

$D = \mathbb{R} = (-\infty, \infty)$

$R = \mathbb{R} = (-\infty, \infty)$

15. Simplify and/or rationalize the following radical expressions:

a. $\sqrt{5} \cdot \sqrt{20} = \sqrt{100}$

$= 10$

b. $\sqrt[4]{\frac{1}{3}} \cdot \sqrt[4]{\frac{1}{9}} \cdot \sqrt[4]{\frac{1}{3}} = \sqrt[4]{\frac{1}{81}}$

$= \frac{1}{3}$

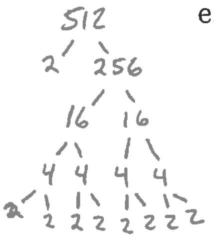
c. $\sqrt{x} \cdot \sqrt{x^3} = \sqrt{x^4}$

$= x^2$

d. $\sqrt[5]{\frac{2x}{y}} \cdot \sqrt[5]{\frac{16y}{x}} = \sqrt[5]{\frac{32xy}{xy}}$

$= \sqrt[5]{32}$

$= 2$



e. $\sqrt[4]{512} = \sqrt[4]{2^9}$

$= 2^2 \cdot \sqrt[4]{2}$

$= 4 \cdot \sqrt[4]{2}$

OR

$\sqrt[4]{512} = \sqrt[4]{256 \cdot 2}$

$= 4 \cdot \sqrt[4]{2}$

Since $4^4 = 256$

f. $\sqrt{32n^3} = 4n\sqrt{2n}$

g. $\sqrt[3]{-16x^3y^5} = -2xy \cdot \sqrt[3]{2y^2}$

h. $\sqrt[3]{2a} \cdot \sqrt[3]{4a^2b} = \sqrt[3]{8a^3b}$

$= 2a \cdot \sqrt[3]{b}$

i. $\sqrt{5}\cdot\sqrt[4]{5} = 5^{1/2}\cdot 5^{1/4}$

$= 5^{3/4}$

$= \sqrt[4]{5^3}$

$= \sqrt[4]{125}$

j. $\sqrt[3]{x}\cdot\sqrt[4]{x} = x^{1/3}\cdot x^{1/4}$

$= x^{7/12}$

$= \sqrt[12]{x^7}$

k. $\sqrt[4]{\frac{16x^3}{y^4}} = \frac{2\cdot\sqrt[4]{x^3}}{y}$

l. $\sqrt{\frac{5a^2}{8}}\cdot\sqrt{\frac{5a^3}{2}} = \sqrt{\frac{25a^5}{16}}$

$= \frac{5a^2\sqrt{a}}{4}$

$= \frac{5}{4}a^2\sqrt{a}$

m. $5\sqrt[3]{6}+\sqrt[3]{6} = 6\cdot\sqrt[3]{6}$

n. $\sqrt{12}+7\sqrt{3} = 2\sqrt{3}+7\sqrt{3}$

$= 9\sqrt{3}$

o. $3\sqrt{3k}+5\sqrt{12k}+9\sqrt{48k}$

$=3\sqrt{3k}+5\cdot 2\sqrt{3k}+9\cdot 4\sqrt{3k}$

$=3\sqrt{3k}+10\sqrt{3k}+36\sqrt{3k}$

$=49\sqrt{3k}$

p. $5\sqrt{z}+\sqrt[3]{z}-2\sqrt{z}$

$=3\sqrt{z}+\sqrt[3]{z}$

q. $3\sqrt[3]{xy^2}-2\sqrt[3]{xy^2}$

$=\sqrt[3]{xy^2}$

r. $\sqrt{16x^3}-\sqrt{x^3}=4x\sqrt{x}-x\sqrt{x}$

$=3x\sqrt{x}$

s. $\frac{3}{5\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{3\sqrt{3}}{5\cdot 3}$

$=\frac{\sqrt{3}}{5}$

t. $\sqrt{\frac{x}{24}}\cdot\frac{\sqrt{24}}{\sqrt{24}}=\frac{\sqrt{24x}}{24}$

$=\frac{2\sqrt{6x}}{24}$

$=\frac{\sqrt{6x}}{12}$

u. $\frac{xy}{\sqrt{y^3}} = \frac{xy}{y\sqrt{y}}$

$= \frac{x}{\sqrt{y}} \cdot \frac{\sqrt{y}}{\sqrt{y}}$

$= \frac{x\sqrt{y}}{y}$

v. $\frac{1}{1+\sqrt{2}} \cdot \frac{1-\sqrt{2}}{1-\sqrt{2}}$

$= \frac{1-\sqrt{2}}{1-2}$

$= \frac{1-\sqrt{2}}{-1}$

$= \frac{1}{-1} - \frac{\sqrt{2}}{-1}$

$= -1 + \sqrt{2}$

w. $\frac{3+\sqrt{5}}{2-\sqrt{5}} \cdot \frac{2+\sqrt{5}}{2+\sqrt{5}}$

$= \frac{6 + 3\sqrt{5} + 2\sqrt{5} + 5}{4-5}$

$= \frac{11 + 5\sqrt{5}}{-1}$

$= -11 - 5\sqrt{5}$

x. $\frac{\sqrt{x}}{\sqrt{x}-2} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2}$

$= \frac{x + 2\sqrt{x}}{x-4}$

16. Divide (Rationalize) the following complex numbers.

a. $\frac{2+3i}{3i} \cdot \frac{i}{i} = \frac{2i-3}{-3}$

$= -\frac{2}{3}i + 1$

$= 1 - \frac{2}{3}i$

b. $\frac{3-i}{5+i} \cdot \frac{5-i}{5-i} = \frac{15-3i-5i-1}{25+1}$

$= \frac{14-8i}{26}$

$= \frac{7}{13} - \frac{4}{13}i$

17. You realize how awesome a trebuchet is and decide to build your own. It's a fantasticly huge wonder which you launch multiple pianos with. During the destroying of many a good piano you take measurements and determine the piano's flight path can be modeled by the function

$$f(d) = -0.04d^2 + 1.431d + 32.5,$$

where $h = f(d)$ is the height of the piano (in meters) at the given horizontal distance d (in meters) from the trebuchet.

Graph $h = f(d)$ on your calculator and use it to answer the following questions.

You need to use complete sentence conclusions to answer each of the following questions. Any rounding necessary should be to the second decimal place.

a. Use the graph on your graphing calculator to determine the horizontal distance(s) of the piano from the trebuchet when it is at a height of 40 meters.

The piano is 40 meters up when it is about 6.38 m & 29.40m from the trebuchet.

b. Use the graph on your graphing calculator to determine at what horizontal distance from the trebuchet the piano will hit the ground.

The piano will hit the ground about 51.54m from the trebuchet.

c. Use the graph on your graphing calculator to determine *the maximum height* the piano reaches **and** *the horizontal distance of the piano from the trebuchet* when it reaches this maximum height.

The piano will reach a max height of about 45.30m when it is about 17.89m from the trebuchet.

d. From what height is the piano released?

The piano is released from a height of 32.5m.

e. State the domain and range of f *in context* of this story problem.

$D = [0, 51.54]$

$R = [0, 45.30]$

18. Solve the following equation numerically. Remember to define a function to represent each side of the equation. State a conclusion using set notation in a complete sentence. You will receive zero points for doing this symbolically so show some work to prove to me you're doing it numerically! **There are two solutions to this equation.**

$$\frac{3}{x+2} = x$$

Let $f(x) = \frac{3}{x+2}$ + $g(x) = x$

	x	$f(x)$	$g(x)$
	-1	3	-1
	-0	$3/2$	0
✓	1	1	1
✓	-3	-3	-3
	-4	$-3/2$	-4

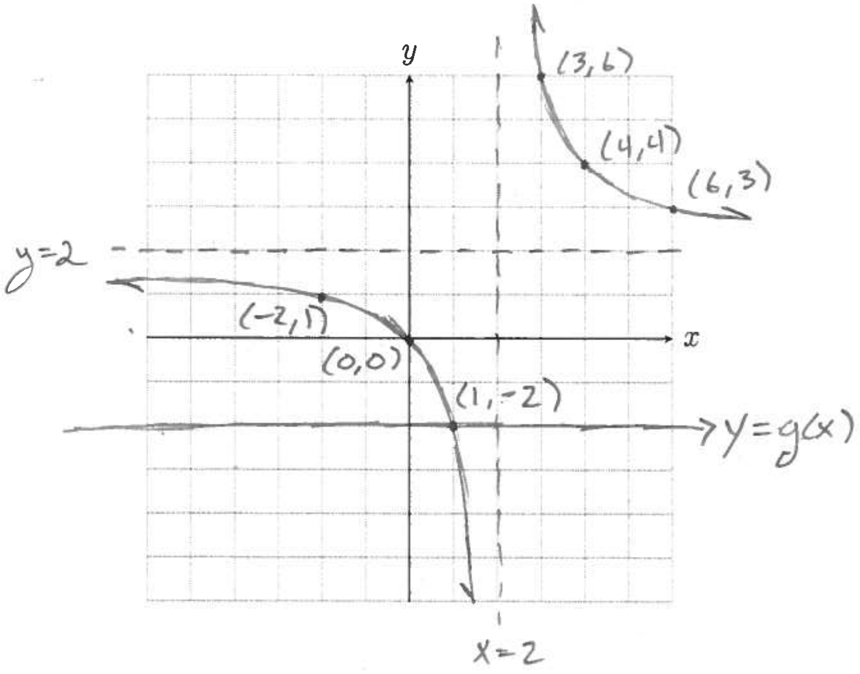
The sol set is $\{1, -3\}$.

19. Solve the following equation graphically. Define a function to represent each side of the equation. Use your calculator to graph both functions and then **nicely** copy the graphs into the space provided. Label a few key points on each graph and any asymptotes. State a conclusion using set notation in a complete sentence.

$$\frac{2x}{x-2} = -2$$

Let $f(x) = \frac{2x}{x-2}$

+ $g(x) = -2$



The sol set is $\{1\}$.

20. You own a pool and an old pump to empty the pool. The old pump takes 12 hours to empty the pool on its own. You decide you want an upgrade and buy a new pump. You hook them both up to the pool and, viola, it only takes 3 hours to empty the pool! How long would the new pump take to empty the pool if it was hooked up all by itself?

Let x = time (in hours) for the new pump to empty the pool.

$$\frac{12x}{1}\cdot\left(\frac{1}{12}+\frac{1}{x}\right)=\left(\frac{1}{3}\right)\cdot\frac{12x}{1}$$

$$x+12=4x$$

$$12=3x$$

$$x=4$$

The sol set
is $\{4\}$.

21. Without any wind you can ride your bike at 15 miles per hour. You ride your bike 6 miles into the wind on your way to work and then return home with the wind with a total travel time of 50 minutes (Hint: Units). Find the average speed of the wind.

Let x = the average speed of the wind in mph.

$D \div R = T$

To Work	6	$15-x$	$6/15-x$
From Work	6	$15+x$	$6/15+x$

$50\text{ min} = \frac{5}{6}\text{ hrs}$

$$6(15-x)(15+x)\left(\frac{6}{15-x}+\frac{6}{15+x}\right)=\left(\frac{5}{6}\right)\cdot 6(15-x)(15+x)$$

$$36(15+x)+36(15-x)=5(225-x^2)$$

$$540+36x+540-36x=1125-5x^2$$

$$1080=1125-5x^2$$

$$0=45-5x^2$$

$$0=9-x^2$$

$$x=3$$

The average
wind speed is
3 mph.