Accelerated 65 - 95 Final Review No Calculator   Name: Solutions

1. Identify the slope and y-intercept of the following equations and then use them to solve the system of equations by graphing. Remember to label the y-intercepts and the intersection of the lines. Write your solution in set notation.

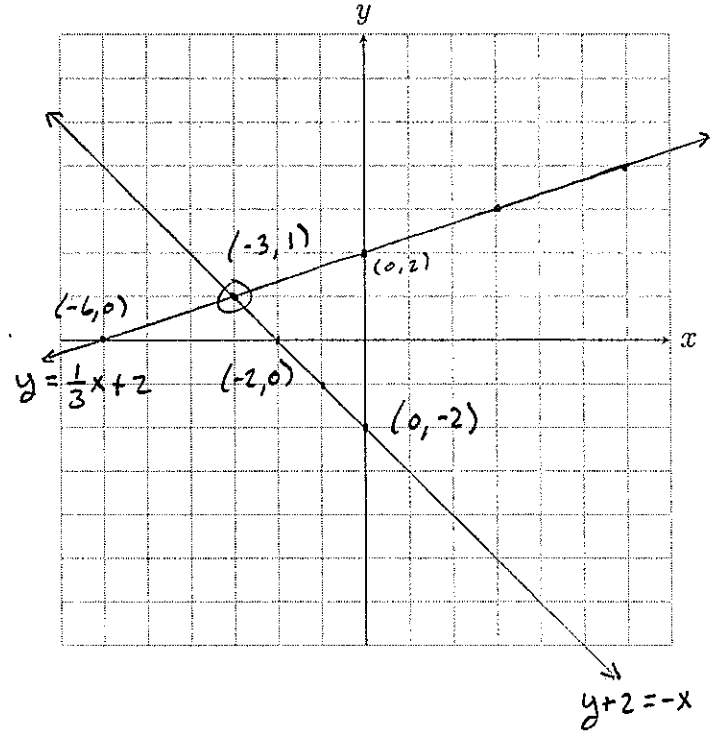
$y = \frac{1}{3}x + 2 \rightarrow m = \frac{1}{3} \quad y\text{-int} = (0, 2)$

$y + 2 = -x$

$y = -x - 2$

$m = -1 \quad y\text{-int} = (0, -2)$

The solution set is $\{(-3, 1)\}$.



2. Solve the following equation symbolically. **Part of your grade for these problems is to *justify* breaking up the absolute value inequalities.** State a conclusion using interval notation within a complete sentence.

a. $\dfrac{-2|2x - 4|}{-2} \geq \dfrac{-6}{-2}$

$|2x - 4| \leq 3$ ← AND

The interval of solutions is $[1/2, 7/2]$

If $2x - 4 \geq 0$

Then $|2x-4| \leq 3$

becomes $2x - 4 \leq 3$

$2x \leq 7$

$x \leq 7/2$

AND

If $2x - 4 < 0$

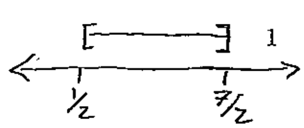
Then $|2x-4| \leq 3$

becomes $-(2x-4) \leq 3$

$-2x + 4 \leq 3$

$-2x \leq -1$

$x \geq 1/2$



3. Simplify the expression assuming that all variables are positive.

a. $\sqrt[3]{-64x^7y^3} = -4x^2y \cdot \sqrt[3]{x}$

c. $\sqrt[5]{96} + 6\sqrt[5]{3} = 2\sqrt[5]{3} + 6\sqrt[5]{3}$

32 3

$= 8\sqrt[5]{3}$

b. $\sqrt{\frac{5x^3}{8}} \cdot \sqrt{\frac{5x^2}{2}} = \sqrt{\frac{25x^5}{16}}$

$= \frac{5x^2\sqrt{x}}{4}$

d. $-3\sqrt{2y} + \sqrt{32y} - \sqrt{50y^3}$

$= -3\sqrt{2y} + 4\sqrt{2y} - 5y\sqrt{2y}$

$= \sqrt{2y} - 5y\sqrt{2y}$

4. Rationalize the denominator.

a. $\frac{x}{\sqrt{12}} \cdot \frac{\sqrt{12}}{\sqrt{12}} = \frac{x\sqrt{12}}{12}$

$= \frac{2x\sqrt{3}}{12}$

$= \frac{x\sqrt{3}}{4}$

b. $\frac{1}{5+\sqrt{2}} \cdot \frac{5-\sqrt{2}}{5-\sqrt{2}} = \frac{5-\sqrt{2}}{5-2}$

$= \frac{5-\sqrt{2}}{3}$

5. Use positive rational exponents to simplify the expression. Assume that all variables are positive.

a. $\sqrt{x^5} \cdot \sqrt[4]{x^3} = x^{5/2} \cdot x^{3/4}$

$= x^{13/4}$

$= \sqrt[4]{x^{13}}$

$= x^3 \cdot \sqrt[4]{x}$

b. $p^{1/3}\left(p^{2/3} + p^{5/3}\right) = p^{\frac{1}{3}+\frac{2}{3}} + p^{\frac{1}{3}+\frac{5}{3}}$

$= p + p^2$

6. Factor the following polynomials.

a. $45x^5 - 5x^7 = 5x^5(9 - x^2)$

$= 5x^5(3-x)(3+x)$

b. $z^3 - 125 = (z-5)(z^2+5z+25)$

7. Solve the equation symbolically. State your conclusion using set notation in a complete sentence.

a. $2\sqrt{x-5}+3=7$

$2\sqrt{x-5} = 4$

$\sqrt{x-5} = 2$

$x-5=4$

$x=9$

Check:

$2\sqrt{9-5}+3 \stackrel{?}{=} 7$

$2\cdot 2+3=7\ \checkmark$

The solution set is $\{9\}$.

b. $\left(\sqrt{x-1}\right)^2 = \left(\sqrt{x+4}-1\right)^2$

$x-1 = (\cancel{x}+4) - 2\sqrt{x+4} + 1$

$-5 \qquad -5$

$-6 = -2\sqrt{x+4}$

$3 = \sqrt{x+4}$

$9 = x+4$

$5 = x$

Check:

$\sqrt{5-1} \stackrel{?}{=} \sqrt{5+4} - 1$

$2 = 3-1\ \checkmark$

The solution set is $\{5\}$.

8. Complete the square of the following function to put it into vertex form. Then state the vertex.

$f(x) = 2x^2 - 10x + 2$

$= 2(x^2 - 5x) + 2$

$= 2\left(x^2 - 5x + \frac{25}{4} - \frac{25}{4}\right) + 2$

$= 2(x - 5/2)^2 - \frac{25}{2} + 2$

$= 2(x - 5/2)^2 - 21/4$

The vertex is $(5/2, -21/4)$.

9. Solve

$g(x) = 0$, where $g(x) = 2x^2 - 3x + 6$,

by completing the square. If your answer is complex, write it in standard complex number form and state your conclusion using set notation in a complete sentence.

$0 = 2x^2 - 3x + 6$

$-6 = 2x^2 - 3x$

$-3 = x^2 - \frac{3}{2}x$

$-3 + \frac{9}{16} = x^2 - \frac{3}{2}x + \frac{9}{16}$

$\frac{-39}{16} = (x - 3/4)^2$

$\pm\sqrt{\frac{-39}{16}} = x - 3/4$

$\frac{3}{4} \pm \frac{\sqrt{39}\,i}{4} = x$

$\left(\frac{1}{2}\left(-\frac{3}{2}\right)\right)^2 = 9/16$

$-3 + \frac{9}{16} = -\frac{48}{16} + \frac{9}{16}$

$= -\frac{39}{16}$

The solution set is $\left\{\frac{3}{4} \pm \frac{\sqrt{39}}{4} i\right\}$.

10. Suppose $f(x) = x^2 + 2x - 3$. Answer the following questions. Remember to label all of the points shown in the table and also the equation of the axis of symmetry when you graph the function.

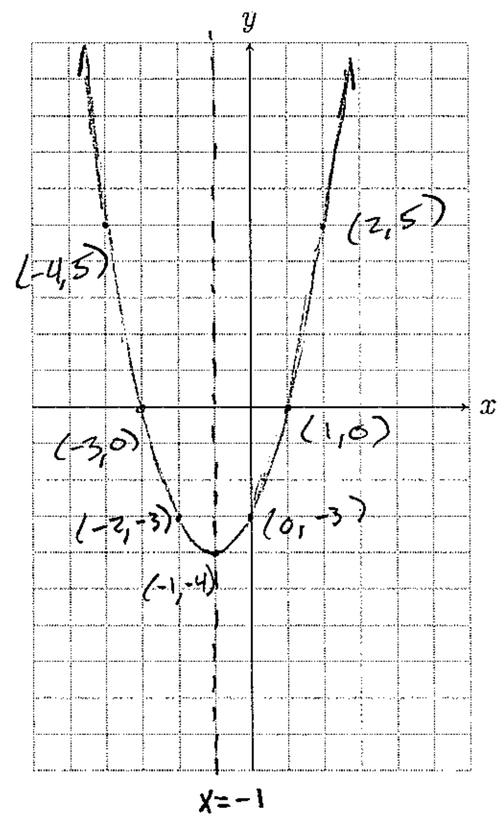
a. Does the graph open up or down?

Up

b. Complete the table with the points needed to sketch a graph of $f(x)$. Write the name of each key point in the blanks provided. Show your work in the space below.

Name of point	x	$f(x)$
vertex	-1	-4
y-int	0	-3
y-int mirror	-2	-3
x-ints {	1	0
	-3	0

c. Graph the function.



$x_v = \frac{-2}{2(1)} = -1$

$f(-1) = 1 - 2 - 3 = -4$

$0 = x^2 + 2x - 3$

$0 = (x-1)(x+3)$

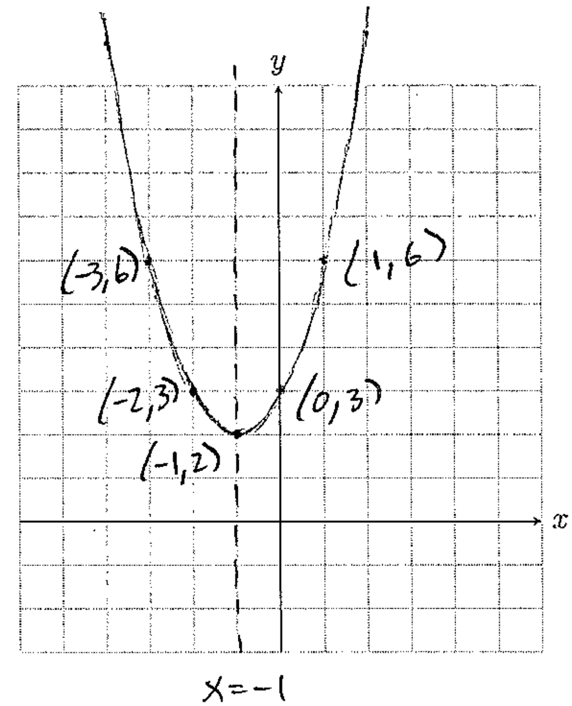
$x = 1 , x = -3$

11.

a. Make a table for $f(x) = (x+1)^2 + 2$ then graph $y = f(x)$ in the space provided with the vertex in the middle. Label all points from the table along with the axis of symmetry.

Symmetry

x	$f(x)$	
-4	11	
-3	6	
-2	3	
-1	2	vertex
0	3	
1	6	
2	11	



b. Describe using a complete sentence how the function $sqr(x) = x^2$ is translated to become f.

If you translate sqr left 1 & up 2 you will get f.

12. Determine the domain of the following rational functions. Then graph the function on your calculator to determine its range.

a. $f(x) = \frac{1}{x+2}$

$D = \{x \mid x \neq -2\}$

$R = \{y \mid y \neq 0\}$

b. $g(x) = \frac{2x}{x^2 - 3x + 2}$

$(x-2)(x-1)$

$D = \{x \mid x \neq 2, 1\}$

$R = \{y \mid y \leq -11.\bar{6} \text{ or } y \geq -0.34315\}$

c. $h(t) = \frac{4t^3}{t^3 - t}$

$t(t^2-1)$

$D = \{t \mid t \neq 0, -1, 1\}$

$R = \{y \mid y < 0 \text{ or } y > 4\}$.

13. Multiply or divide and simplify the following rational products. Be sure to identify any places where two expressions are not equivalent despite being equivalent for all other inputs.

a. $\frac{\cancel{2}x^2y^3}{3xy^2} \cdot \frac{(2x^3y)^2}{\cancel{2}(xy)^3} = \frac{x^2y^3 \cdot 4x^6y^2}{3xy^2 \cdot x^3y^3}$

$= \frac{4x^8\cancel{y^5}}{3x^4\cancel{y^5}}$

$= \frac{4}{3}x^4$ provided $x \neq 0$ & $y \neq 0$

b. $\frac{x^3 - x}{x-1} \cdot \frac{x+1}{x} = \frac{\cancel{x}(x^2-1)}{x-1} \cdot \frac{x+1}{\cancel{x}}$ provided $x \neq 0$

$= \frac{\cancel{(x-1)}(x+1)}{\cancel{x-1}} \cdot \frac{x+1}{1}$

$= (x+1)^2$ provided $x \neq 1$

c. $\frac{x^2-1}{x^2+x-6} \div \frac{x-1}{x+3} = \frac{\cancel{(x-1)}(x+1)}{\cancel{(x+3)}(x-2)} \cdot \frac{\cancel{x+3}}{\cancel{x-1}}$

$= \frac{x+1}{x-2}$

provided $x \neq 1$ or -3

d. $\frac{x^2-25}{x^2+5x+4} \div \frac{x^2-10x+25}{2x^2+8x}$

$= \frac{\cancel{(x-5)}(x+5)}{\cancel{(x+4)}(x+1)} \cdot \frac{2x\cancel{(x+4)}}{\cancel{(x-5)}(x-5)}$

$= \frac{2x(x+5)}{(x+1)(x-5)}$ provided $x \neq -4$.

14. Add and simplify the following rational sums and differences.

a. $\frac{3}{x^2} - \frac{x+3}{x^2} = \frac{-x}{x^2}$

$= -\frac{1}{x}$

b. $\frac{2x}{x^2-1} - \frac{x+1}{x^2-1} = \frac{\cancel{x-1}}{\cancel{(x-1)}(x+1)}$

$= \frac{1}{x+1}$ provided $x \neq 1$

c. $\frac{5a}{b^2} - \frac{4b}{a^2} = \frac{5a^3 - 4b^3}{a^2b^2}$

d. $\frac{x-1}{x} - \frac{5}{x+5} = \frac{(x+5)(x-1)}{(x+5)x} - \frac{5x}{(x+5)x}$

$= \frac{x^2+4x-5-5x}{(x+5)x}$

$= \frac{x^2-x-5}{x(x+5)}$

e. $\frac{1}{x^2-3x+2} - \frac{1}{x^2-x-2}$

$(x-2)(x-1) \qquad (x-2)(x+1)$

$= \frac{x+1}{(x-2)(x-1)(x+1)} - \frac{x-1}{(x-2)(x+1)(x-1)}$

$= \frac{2}{(x-2)(x-1)(x+1)}$

f. $\frac{1}{x-3} - \frac{2}{x+3} + \frac{x}{x^2-9}$

$= \frac{x+3}{(x-3)(x+3)} - \frac{2(x-3)}{(x-3)(x+3)} + \frac{x}{(x-3)(x+3)}$

$= \frac{x+3-2x+6+x}{(x-3)(x+3)}$

$= \frac{9}{(x-3)(x+3)}$

15. Simplify the following complex fractions.

a. $$\frac{1+\frac{1}{x}}{1+\frac{1}{y}} \cdot \frac{xy}{xy} = \frac{xy+y}{xy+x}$$

$$= \frac{y(x+1)}{x(y+1)}$$

c. $$\frac{n^{-2}+m^{-2}}{1+(nm)^{-2}} = \frac{\frac{1}{n^2}+\frac{1}{m^2}}{1+\frac{1}{n^2m^2}} \cdot \frac{n^2m^2}{n^2m^2}$$

$$= \frac{m^2+n^2}{n^2m^2+1}$$

b. $$\frac{1-\frac{1}{x}}{1+\frac{1}{2x}} \cdot \frac{2x}{2x} = \frac{2x-2}{2x+1}$$

d. $$\frac{\frac{1}{x^2+2x+1}-\frac{1}{x^2-2x+1}}{(x+1)(x-1)}$$

$$= \frac{\frac{1}{(x+1)(x+1)}-\frac{1}{(x-1)(x-1)}}{(x+1)(x-1)} \cdot \frac{(x+1)(x+1)(x-1)(x-1)}{(x+1)(x+1)(x-1)(x-1)}$$

$$= \frac{(x-1)(x-1)-(x+1)(x+1)}{(x+1)^3(x-1)^3}$$

$$= \frac{x^2-2x+1-(x^2+2x+1)}{(x+1)^3(x-1)^3}$$

$$= \frac{-4x}{(x+1)^3(x-1)^3}$$

16. Solve the following rational equations.

a. $12x \cdot \left(\frac{x+1}{2x} - \frac{x-1}{4x}\right) = \frac{1}{3} \cdot 12x$

$6(x+1) - 3(x-1) = 4x$

$6x+6-3x+3 = 4x$

$3x+9 = 4x$

$9 = x$

The solution set is $\{9\}$.

c. $4x^2 \cdot \left(\frac{1}{x} + \frac{1}{x^2}\right) = \frac{3}{4} \cdot 4x^2$

$4x+4 = 3x^2$

$0 = 3x^2 - 4x - 4$

-12; $2 - 6 = -4$

$0 = 3x^2 - 6x + 2x - 4$

$0 = 3x(x-2) + 2(x-2)$

$0 = (x-2)(3x+2)$

$x = 2$ or $x = -2/3$

The solution set is $\{2, -2/3\}$.

b. $(x-2)(x+2)\left(\frac{3}{x-2} + \frac{5}{x+2}\right) = \frac{12}{x^2-4}(x-2)(x+2)$

$3(x+2) + 5(x-2) = 12$

$3x+6+5x-10 = 12$

$8x - 4 = 12$

$8x = 16$

$x = 2$

The solution set is $\{\}$ since 2 isn't in the domain of either side

d. $(x+2)(x-1)\left(\frac{2x}{x+2} + \frac{3x}{x-1}\right) = 7(x+2)(x-1)$

$2x(x-1) + 3x(x+2) = 7(x^2+x-2)$

$2x^2 - 2x + 3x^2 + 6x = 7x^2 + 7x - 14$

$5x^2 + 4x = 7x^2 + 7x - 14$

$0 = 2x^2 + 3x - 14$

-28; $-4 + 7 = 3$

$0 = 2x^2 - 4x + 7x - 14$

$0 = 2x(x-2) + 7(x-2)$

$0 = (x-2)(2x+7)$

$x = 2$ or $x = -7/2$

The solution set is $\{2, -7/2\}$.

(a) The functions f, g, and h are described below. Use them to answer the following questions.

x	$f(x)$
-3	$\frac{3}{2}$
-2	$\frac{4}{3}$
-1	1
0	0
1	und.
2	4
3	3

- $g(x) = \dfrac{3x}{x+2}$

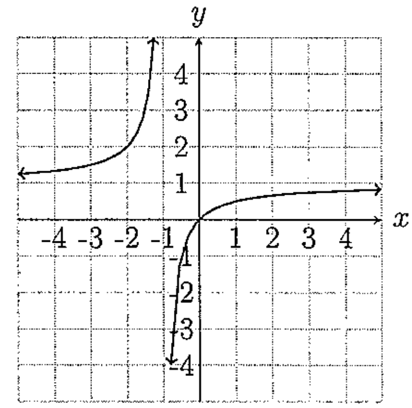
- 

Figure 1: $y = h(x)$

a. Evaluate $f(-1)$

$f(-1) = 1$

b. Evaluate $g(0)$

$g(0) = 0$

c. Evaluate $h(-2)$

$h(-2) = 2$

d. What is the domain and range of g?

$D = \{x \mid x \neq -2\}$

$R = \{y \mid y \neq 3\}$.

e. Solve $f(x) = \frac{4}{3}$.

The solution set is $\{-2\}$.

f. Solve $h(x) = 0$.

The solution set is $\{0\}$.

g. Solve $g(x) = 0$.

The solution set is $\{0\}$.

h. What is the domain and range of h?

$D = \{x \mid x \neq -1\}$

$R = \{y \mid y \neq 1\}$.

17. Solve the following equation numerically, symbolically, and graphically.

$$\frac{1}{x-2}+\frac{1}{x+2}=-\frac{2}{3}$$

Let $f(x)=\frac{1}{x-2}+\frac{1}{x+2}$ $\quad g(x)=-2/3$

- Numerically:

	x	f(x)	g(x)
	0	0	-2/3
	-1	2/3	-2/3
*	1	-2/3	-2/3
	3	1 1/5	-2/3
	-3	-1 1/5	-2/3
*	-4	-2/3	-2/3

- Symbolically

$$3(x-2)(x+2)\left(\frac{1}{x-2}+\frac{1}{x+2}\right)=-\frac{2}{3}(x-2)(x+2)\cdot 3$$

$$3(x+2)+3(x-2)=-2(x^2-4)$$

$$3x+6+3x-6=-2x^2+8$$

$$6x=-2x^2+8$$

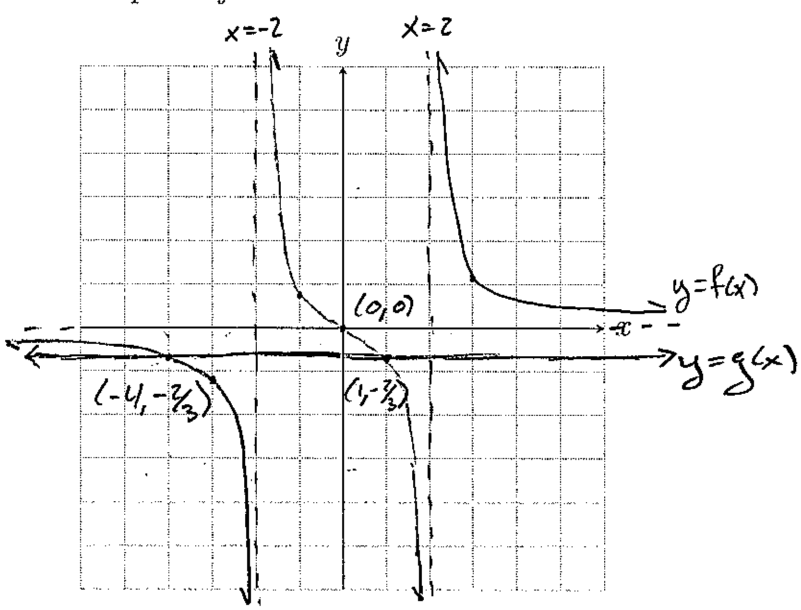
$$2x^2+6x-8=0$$

$$x^2+3x-4=0$$

$$(x+4)(x-1)=0$$

$$x=-4 \quad x=1$$

- Graphically:



The solution set is $\{-4,1\}$.

18. Light traveling from the Messier 83 Galaxy takes approximately 4.73×10^{14} seconds to reach earth. The speed of light is approximately 3.00×10^{8} meters per second. Use this information to determine approximately how many meters the Messier 83 Galaxy is from earth. Do your calculations in scientific notation and give your conclusion using scientific notation.

$D = R \times T$

$D = 3.00 \times 10^{8} \cdot 4.73 \times 10^{14}$

$= 14.19 \times 10^{22}$

$\approx 1.42 \times 10^{23}$

It is approximately 1.42×10^{23} meters to the Messier 83 Galaxy from Earth.

Solve the following story problem involving systems of equations by following these steps:

Step 1: Define variables to be the unknown quantities.

Step 2: Write a system of equations that model's the problem's conditions.

Step 3: Solve the system of equations.

Step 4: Write a conclusion which answers the story problem using complete sentences.

19. In a discount clothing store, all jeans are sold at one fixed price and all t-shirts are sold at another fixed price. If 6 pairs of jeans and 4 t-shirts cost \$180, while 2 pairs of jeans and 7 t-shirts cost \$128, find the price of one pair of jeans and the price of one t-shirt.

Let x = price of 1 pair of jeans
& y = price of 1 t-shirt

$6x + 4y = 180 \rightarrow 6x + 4y = 180$

$-3(2x + 7y) = (128)(-3) \rightarrow -6x - 21y = -384$

$-17y = -204$

$y = 12$

$2x + 7(12) = 128$

$2x + 84 = 128$

$2x = 44$

$x = 22$

It is \$12 for a t-shirt & \$22 for a pair of jeans.

20. You realize how awesome a trebuchet is and decide to build your own. It's a fantasticly huge wonder which you launch multiple pianos with. During the destroying of many a good piano you take measurements and determine the piano's flight path can be modeled by the function

$$f(d) = -0.031d^2 + 1.653d + 39.680,$$

where $h = f(d)$ is the height of the piano (in meters) at the given horizontal distance d (in meters) from the trebuchet.

Graph $h = f(d)$ on your calculator and use it to answer the following questions.

You need to use complete sentence conclusions to answer each of the following questions. Any rounding necessary should be to the second decimal place.

a. Use the graph on your graphing calculator to determine the horizontal distance(s) of the piano from the trebuchet when it is at a height of 55 meters.

The piano is 55 meters high when it is about 11.94 ~~&~~ again at about 41.38 meters from the trebuchet.

b. Use the graph on your graphing calculator to determine at what horizontal distance from the trebuchet the piano will hit the ground.

The piano will hit the ground about 71.28 meters from the trebuchet.

c. Use the graph on your graphing calculator to determine *the maximum height* the piano reaches and *the horizontal distance of the piano from the trebuchet* when it reaches this maximum height.

The piano will reach a max height of about 61.72 meters when it is about 26.66 meters from the trebuchet.

d. From what height is the piano released?

The piano is released from a height of 39.68 meters

e. State the domain and range of f *in context* of this story problem.

D = [0, 71.28]

R = [0, 61.72]