2 Review Solutions
Math 95 Exam ~~1 No Calculator~~ Name: Solutions

1. The functions f, g, and h are described below. Use them to answer the following questions.

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x	$f(x)$
-3	$\frac{3}{2}$
-2	$\frac{4}{3}$
-1	1
0	0
1	und.
2	4
3	3

- $g(x) = \dfrac{3x}{x+2}$

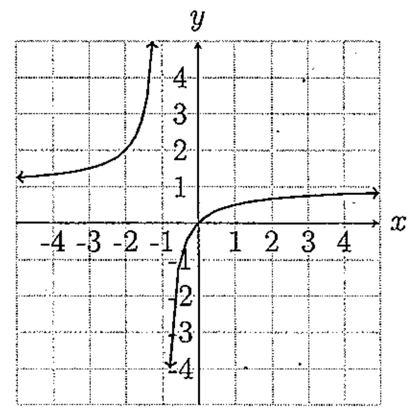
- 

Figure 1: $y = h(x)$

a. Evaluate $f(-1)$

$f(-1)=1$

b. Evaluate $g(0)$

$g(0)=0$

c. Evaluate $h(-2)$

$h(-2)=2$

d. What is the domain and range of g?

$D=\{x \mid x\neq -2\}$

$R=\{y \mid y\neq 3\}$

e. Solve $f(x) = \frac{4}{3}$. The set of solutions is $\{-2\}$

f. Solve $h(x) = 0$. The set of solutions is $\{0\}$

g. Solve $g(x) = 0$. The set of solutions is $\{0\}$.

h. What is the domain and range of h?

$D=\{x \mid x\neq -1\}$

$R=\{y \mid y\neq 1\}$

2 pts 2. Suppose you have a function M with $M(2) = -1$ and $M(-3) = 5$. What are two points on the graph of M?

$(2,-1)$ & $(-3,5)$

3 pts 3. What is the definition of a solution to an equation in one variable?

A number which, when plugged in for the variable, allows for the #s on the left & right to be equal.

3ca 4. Determine which of the following relationships between inputs and outputs can be categorized as a function and justify your response. State the domain and range of the relations.

a. $\{(-3,4),(1,-4),(0,0),(2,4),(-3,5)\}$

This is not a function since -3 has more than one output.

$D=\{-3,0,1,2\}$

$R=\{-4,0,4,5\}$

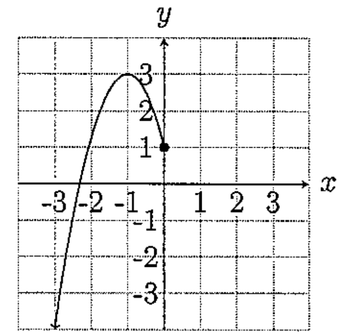
b. $\{(.5,.5),(3,.5),(5,1),(7.5,.5),(-11.25,1)\}$

This is a function since each input has exactly 1 output.

$D=\{-11.25, .5, 3, 5, 7.5\}$

$R=\{.5, 1\}$

c.



This is a function since each input (x-value) has only one output (y-value).

$D=(-\infty,0]$

$R=(-\infty,3]$

3pts 5. What is the one requirement for a relationship between inputs and outputs to be considered a function? For an individual ~~for any~~ input the function will have at most one output.

4 pts 6. Solve the equation $\frac{2}{3}x-1=-\frac{1}{3}x+2$ numerically. Define a function to represent each side of the equation and use function notation in your table headings as appropriate. State a conclusion using set notation in a complete sentence. **Note: You will receive zero points for solving this equation symbolically.**

Let $f(x)=\frac{2}{3}x-1$

& $g(x)=-\frac{1}{3}x+2$

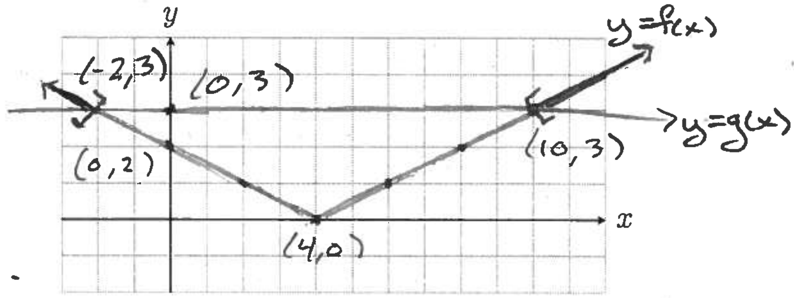
The set of solutions is $\{3\}$.

	x	$f(x)$	$g(x)$
	-3	-3	3
	0	-1	2
✓*	3	1	1

16

7 pts 7. Solve the equation $\left|-\frac{1}{2}x+2\right| \geq 3$ graphically. Define a function to represent each side of the equation and label each functions graph appropriately. Label any x or y intercepts along with any intersection points. State a conclusion using set notation in a complete sentence. **Note: You will receive zero points for solving this equation symbolically.**

Let $f(x) = \left|-\frac{1}{2}x+2\right|$
+ $g(x) = 3$
The set of solutions is $\{x \mid x \leq -2 \text{ or } x \geq 10\}$.



7 each 8. Solve the following equations symbolically. **Part of your grade for these problems is to *justify* breaking up the absolute value inequalities.** State a conclusion using interval notation within a complete sentence.

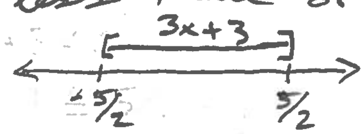
a. $|3x-3| > -1$

We need the size of $3x-3$ to be bigger than -1. but this is always True so the interval of solutions is $(-\infty, \infty)$.

b. $\frac{-2|3x+3|}{2} \leq \frac{5}{2}$

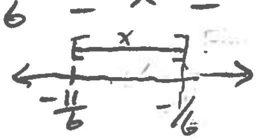
$|3x+3| \leq 5/2$

First isolate the absolute value. So the size of $3x+3$ needs to be less than or equal to $5/2$.



Thus we set up

$$-5/2 \leq 3x+3 \leq 5/2$$
$$-3 \qquad -3 \qquad -3$$
$$\frac{-\frac{11}{2}}{3} \leq \frac{3x}{3} \leq \frac{-1/2}{3}$$
$$-\frac{11}{6} \leq x \leq -1/6$$



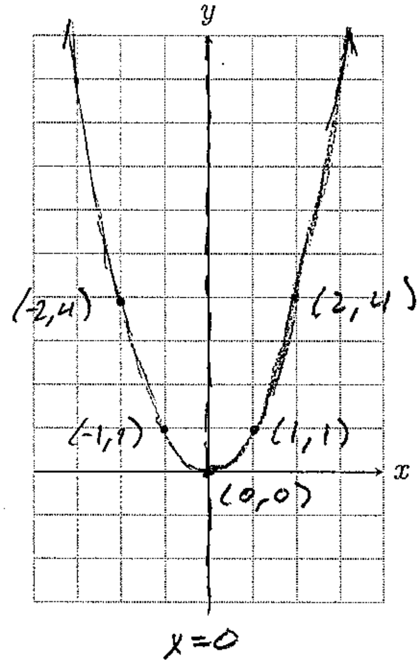
Thus if $-11/6 \leq x \leq -1/6$ then the size of $3x+3$ is less than or equal to $5/2$. So the interval of solutions is $[-11/6, -1/6]$.

16

3 pts

9. a. Make a table for $sqr(x) = x^2$ then graph $sqr(x) = x^2$ in the space provided.

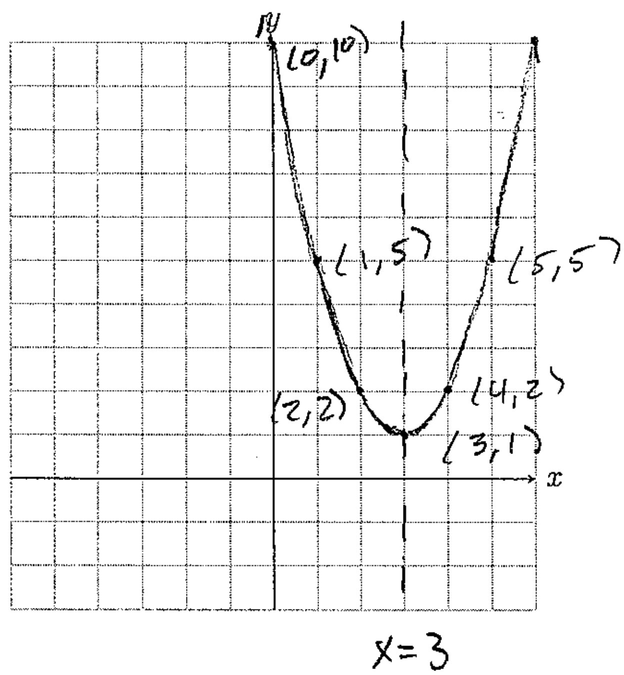
x	$sqr(x)$
-2	4
-1	1
0	0
1	1
2	4



6pts

b. Make a table for $f(x) = (x-3)^2 + 1$ then graph $f(x) = (x-3)^2 + 1$ in the space provided.

x	$f(x)$
1	5
2	2
3	1
4	2
5	5



3pts c. Describe using a complete sentence how the function *sqr* is translated to become f.

f is the translation of sqr by 3 units to the right & one unit up.

12

6ea

10. Complete the square of the following functions to put them into vertex form. Then state the vertex.

a. $f(x) = 2x^2 - 8x + 13$

$= 2(x^2 - 4x) + 13$

$= 2(x^2 - 4x + 4 - 4) + 13$

$= 2(x-2)^2 - 8 + 13$

$= 2(x-2)^2 + 5$

The vertex is $(2, 5)$.

b. $g(x) = 4x^2 + 12x + 10$

$= 4(x^2 + 3x) + 10$

$= 4(x^2 + 3x + \frac{9}{4} - \frac{9}{4}) + 10$

$= 4(x + \frac{3}{2})^2 - 9 + 10$

$= 4(x + \frac{3}{2})^2 + 1$

The vertex is $(-3/2, 1)$.

3ea 11. Evaluate the discriminant of the following functions and the state how many x-intercepts the function has based upon what number you found when evaluating the discriminant.

a. $f(x) = x^2 - 10x + 25$

$100 - 4(1)(25) = 0$

There is one x-int.

b. $g(x) = -2x^2 - 4x + 1$

$16 - 4(-2)(1) = 16 + 8 = 24$

There are 2 x-ints.

6ea 12. Solve the following quadratic equations using the quadratic formula. If your answer is complex, write it in standard complex number form and state your conclusion using set notation in a complete sentence.

a. Solve $f(x) = 0$ where $f(x) = -3x^2 - 5$.

$x = \frac{0 \pm \sqrt{0 - 4(-3)(-5)}}{-6}$

$= \frac{\pm\sqrt{-60}}{-6} = \frac{\pm 2\sqrt{15}\, i}{-6}$

$= \pm \frac{1}{3}\sqrt{15}\, i$

The set of solutions is $\{\frac{1}{3}\sqrt{15}\, i, -\frac{1}{3}\sqrt{15}\, i\}$.

b. Solve $g(x) = 0$ where $g(x) = 2x^2 - 12x + 19$.

7 19 8 152

$x = \frac{12 \pm \sqrt{144 - 4(2)(19)}}{4}$

$= \frac{12 \pm \sqrt{-8}}{4}$

$= 3 \pm \frac{2\sqrt{2}\, i}{4} = 3 \pm \frac{1}{2}\sqrt{2}\, i$

The set of solutions is $\{3 + \frac{1}{2}\sqrt{2}\, i, 3 - \frac{1}{2}\sqrt{2}\, i\}$.

30

13. You realize how awesome a trebuchet is and decide to build your own. It's a fantasticly huge wonder which you launch multiple pianos with. During the destroying of many a good piano you take measurements and determine the piano's flight path can be modeled by the function

$$f(d) = -0.031d^2 + 1.653d + 39.680,$$

where $h = f(d)$ is the height of the piano (in meters) at the given horizontal distance d (in meters) from the trebuchet.

Graph $h = f(d)$ on your calculator and use it to answer the following questions.

You need to use complete sentence conclusions to answer each of the following questions.

3pts a. Use the graph on your graphing calculator to determine the horizontal distance(s) of the piano from the trebuchet when it is at a height of 55 meters.

The piano is at a height of 55m when it is about 11.94 m out or again when it is about 41.38 meters from the trebuchet.

3pts b. Use the graph on your graphing calculator to determine at what horizontal distance from the trebuchet the piano will hit the ground.

The piano will hit the ground about 71.28 meters from the trebuchet.

3pts c. Use the graph on your graphing calculator to determine *the maximum height* the piano reaches and *the horizontal distance of the piano from the trebuchet* when it reaches this maximum height.

The piano reaches a max height of about 61.72 meters when it is about 26.66 meters from the trebuchet.

2pts d. From what height is the piano released?

The piano's height upon release is 39.680 meters.

3pts e. State the domain and range of f *in context* of this story problem.

$D \approx [0, 71.28]$

$R \approx [0, 61.72]$.

15

14. Determine the domain of the following rational functions. Then graph the function on your calculator to determine its range.

a. $f(x) = \frac{1}{x+2}$

$D = \{x \mid x \neq -2\}$

$R = \{y \mid y \neq 0\}$

b. $g(x) = \frac{2x}{x^2 - 3x + 2}$

$(x-2)(x-1)$

$D = \{x \mid x \neq 2, 1\}$

$R = \{y \mid y \geq -0.34\}$

c. $h(t) = \frac{4t^3}{t^3 - t}$

$D = \{t \mid t \neq -1, 0, 1\}$

$R = \{y \mid y > 4 \text{ or } y < 0\}$.

15. Multiply or divide and simplify the following rational products. Be sure to identify any places where two expressions are not equivalent despite being equivalent for all other inputs.

a. $\frac{2x^2y^3}{3xy^2} \cdot \frac{(2x^3y)^2}{2(xy)^3} = \frac{\cancel{2}x^2y^3 \cdot 4x^6y^2}{3xy^2 \cdot \cancel{2}x^3y^3}$

$= \frac{4x^{8}y^{5}}{3x^{4}y^{5}}$

$= \frac{4}{3}x^4$, provided $x \neq 0$ or $y \neq 0$

c. $\frac{x^2-1}{x^2+x-6} \div \frac{x-1}{x+3} = \frac{\cancel{(x-1)}(x+1)\cancel{(x+3)}}{\cancel{(x+3)}(x-2)\cancel{(x-1)}}$

$= \frac{x+1}{x-2}$, provided $x \neq 1, -3$

b. $\frac{x^3-x}{x-1} \cdot \frac{x+1}{x} = \frac{\cancel{x}(x^2-1)(x+1)}{(x-1)(\cancel{x})}$

$= \frac{\cancel{(x-1)}(x+1)(x+1)}{\cancel{x-1}}$, provided $x \neq 0$

$= (x+1)^2$, provided $x \neq 0, 1$

d. $\frac{x^2-25}{x^2+5x+4} \div \frac{x^2-10x+25}{2x^2+8x}$

$= \frac{\cancel{(x-5)}(x+5) \cdot 2x\cancel{(x+4)}}{\cancel{(x+4)}(x+1)\cancel{(x-5)}(x-5)}$

$= \frac{2x^2+10x}{(x+1)(x-5)}$, provided $x \neq -4$

(The $x \neq 5$ condition isn't necessary since 5 is still a domain restriction)

16. Add and simplify the following rational sums and differences.

g. $\frac{3}{x^2} - \frac{x+3}{x^2} = \frac{3-x-3}{x^2}$

$= \frac{-x}{x^2}$

$= -\frac{1}{x}$

j. $\frac{x-1}{x} - \frac{5}{x+5} = \frac{x-1}{x} \cdot \frac{x+5}{x+5} - \frac{5}{x+5} \cdot \frac{x}{x}$

$= \frac{x^2+4x-5}{x(x+5)} - \frac{5x}{x(x+5)}$

$= \frac{x^2-x-5}{x(x+5)}$

$=$

h. $\frac{2x}{x^2-1} - \frac{x+1}{x^2-1} = \frac{2x-x-1}{(x-1)(x+1)}$

$= \frac{\cancel{x-1}}{(\cancel{x-1})(x+1)}$

$= \frac{1}{x+1}, x \neq 1$

k. $\frac{1}{x^2-3x+2} - \frac{1}{x^2-x-2} = \frac{1}{(x-2)(x-1)} - \frac{1}{(x-2)(x+1)}$

$= \frac{1}{(x-2)(x-1)} \cdot \frac{x+1}{x+1} - \frac{1}{(x-2)(x+1)} \cdot \frac{x-1}{x-1}$

$= \frac{x+1}{(x-2)(x-1)(x+1)} - \frac{x-1}{(x-2)(x+1)(x-1)}$

$= \frac{2}{(x-2)(x-1)(x+1)}$

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i. $\frac{5a}{b^2} - \frac{4b}{a^2} = \frac{5a}{b^2} \cdot \frac{a^2}{a^2} - \frac{4b}{a^2} \cdot \frac{b^2}{b^2}$

$= \frac{5a^3}{a^2b^2} - \frac{4b^3}{a^2b^2}$

$= \frac{5a^3-4b^3}{a^2b^2}$

l. $\frac{1}{x-3} - \frac{2}{x+3} + \frac{x}{x^2-9}$

$= \frac{1}{(x-3)} \cdot \frac{x+3}{x+3} - \frac{2}{x+3} \cdot \frac{x-3}{x-3} + \frac{x}{(x+3)(x-3)}$

$= \frac{x+3}{(x-3)(x+3)} - \frac{2x-6}{(x+3)(x-3)} + \frac{x}{(x+3)(x-3)}$

$= \frac{x+3-2x+6+x}{(x-3)(x+3)}$

$= \frac{9}{(x-3)(x+3)}$

17. Simplify the following complex fractions.

a. $$\frac{1+\frac{1}{x}}{1+\frac{1}{y}} \cdot \frac{xy/1}{xy/1}$$

$$= \frac{xy+y}{xy+x}$$

c. $$\frac{n^{-2}+m^{-2}}{1+(nm)^{-2}} = \frac{\frac{1}{n^2}+\frac{1}{m^2}}{1+\frac{1}{n^2m^2}} \cdot \frac{n^2m^2/1}{n^2m^2/1}$$

$$= \frac{m^2+n^2}{n^2m^2+1}$$

b. $$\frac{1-\frac{1}{x}}{1+\frac{1}{2x}} \cdot \frac{2x/1}{2x/1} = \frac{2x-2}{2x+1}$$

d. $$\frac{\frac{1}{x^2+2x+1}-\frac{1}{x^2-2x+1}}{(x+1)(x-1)}$$

$$= \frac{\frac{1}{(x+1)(x+1)}-\frac{1}{(x-1)(x-1)}}{(x+1)(x-1)} \cdot \frac{(x+1)^2(x-1)^2}{(x+1)^2(x-1)^2}$$

$$= \frac{(x-1)^2-(x+1)^2}{(x+1)^3(x-1)^3}$$

$$= \frac{x^2-2x+1-(x^2+2x+1)}{(x+1)^3(x-1)^3}$$

$$= \frac{-4x}{(x+1)^3(x-1)^3}$$

18. Solve the following rational equations.

a. $12x \cdot \left(\frac{x+1}{2x} - \frac{x-1}{4x}\right) = \left(\frac{1}{3}\right) \cdot 12x$

$6(x+1) - 3(x-1) = 4x$

$6x + 6 - 3x + 3 = 4x$

$3x + 9 = 4x$

$9 = x$

The sol set is $\{9\}$.

c. $4x^2\left(\frac{1}{x} + \frac{1}{x^2}\right) = \frac{3}{4} \cdot 4x^2$

$4x + 4 = 3x^2$

$0 = 3x^2 - 4x - 4$

-12

$2 \quad -6 = -4$

$0 = 3x^2 + 2x - 6x - 4$

$0 = x(3x+2) - 2(3x+2)$

$0 = (3x+2)(x-2)$

The sol set is $\{-\frac{2}{3}, 2\}$.

b. $(x-2)(x+2)\left(\frac{3}{x-2} + \frac{5}{x+2}\right) = \left(\frac{12}{x^2-4}\right) \cdot (x-2)(x+2)$

$(x-2)(x+2)$

$3(x+2) + 5(x-2) = 12$

$3x + 6 + 5x - 10 = 12$

$8x - 4 = 12$

$8x = 16$

$x = 2$

The sol set is $\{\}$ since 2 is not in the domain of either side of the equation.

d. $(x+2)(x-1)\left(\frac{2x}{x+2} + \frac{3x}{x-1}\right) = 7 \cdot (x+2)(x-1)$

$2x(x-1) + 3x(x+2) = 7(x^2 + x - 2)$

$2x^2 - 2x + 3x^2 + 6x = 7x^2 + 7x - 14$

$5x^2 + 4x = 7x^2 + 7x - 14$

$0 = 2x^2 + 3x - 14$

-28

$1 \quad 28$

$2 \quad 14$

$-4 + 7$

$0 = 2x^2 - 4x + 7x - 14$

$0 = 2x(x-2) + 7(x-2)$

$0 = (x-2)(2x+7)$

The sol set is $\{2, -\frac{7}{2}\}$.

2. Solve the following equation numerically, symbolically, and graphically.

$$\frac{1}{x-2} + \frac{1}{x+2} = -\frac{2}{3}$$

Let $f(x) = \frac{1}{x-2} + \frac{1}{x+2}$ & $g(x) = -2/3$

- Numerically:

	x	f(x)	g(x)
	0	0	-2/3
*	1	-2/3	-2/3
	3	6/5	-2/3
	-1	2/3	-2/3
	-3	-6/5	-2/3
*	-4	-2/3	-2/3

- Symbolically

$$3(x-2)(x+2)\left(\frac{1}{x-2} + \frac{1}{x+2}\right) = \left(-\frac{2}{3}\right)(x-2)(x+2)\cdot 3$$

$$3(x+2) + 3(x-2) = -2(x-2)(x+2)$$

$$3x+6+3x-6 = -2(x^2-4)$$

$$6x = -2x^2+8$$

$$\frac{0}{-2} = \frac{-2x^2-6x+8}{-2}$$

$$0 = x^2+3x-4$$

$$0 = (x-1)(x+4)$$

$$x-1=0 \quad \text{or} \quad x+4=0$$

$$x=1 \quad \text{or} \quad x=-4$$

The solution set is $\{1, -4\}$.

- Graphically: