Accelerated 65 - 95 Exam 2 Review Name: Solution

3 pts 1. What is the definition of a solution to a system of equations in two variables?

It is a point which satisfies every equation in the system.

7 pts 2. Identify the slope and y-intercept of the following equations and then use them to solve the system of equations by graphing. Remember to label the y-intercepts and the intersection of the lines. Write your solution in set notation.

$3y - x = 6 \rightarrow 3y = x + 6$

$y = \frac{1}{3}x + 2$

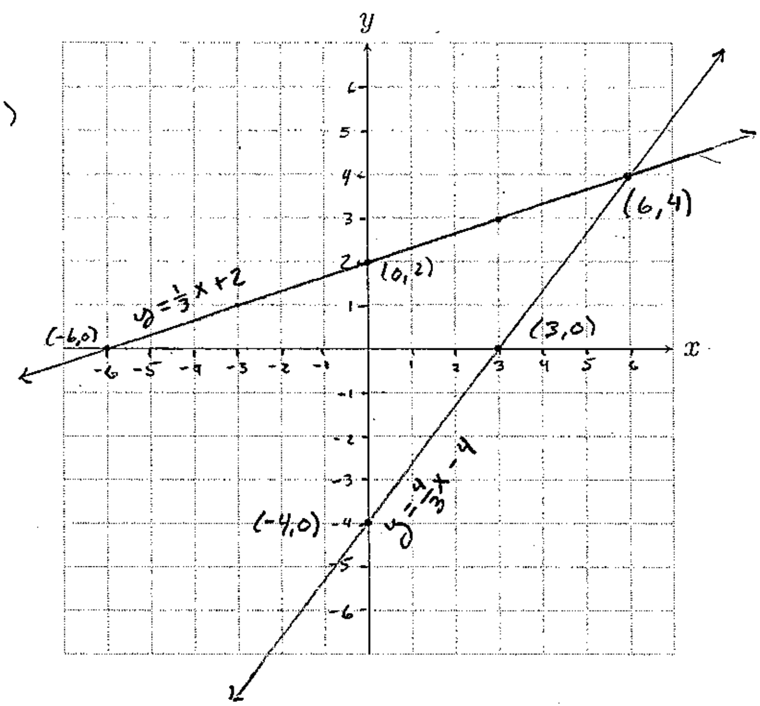
$m = 1/3$ y-int $= (0, 2)$

$3y = 4x - 12$

$y = \frac{4}{3}x - 4$

$m = 4/3$

y-int $= (0, -4)$



The solution set is $\{(6,4)\}$.

6 pts 3. Let x represent the first number and let y represent the second number. Suppose that twice the first number, increased by 5 times the second number results in 12. Further assume the first number equals $-\frac{5}{2}$ of the second number, increased by 6. Use the given conditions to write a system of equations and then solve that system using the substitution method. Write the solution in set notation.

$2x + 5y = 12$ $\quad x = -\frac{5}{2}y + 6$

$2(-\frac{5}{2}y + 6) + 5y = 12$

$-5y + 12 + 5y = 12$

$12 = 12$

The solution set is $\{(x,y) \mid 2x + 5y = 12\}$

2 ea 4. Identify each polynomial as a monomial, a binomial or a trinomial. Give the degree of the polynomial.

a. 15

Monomial of degree 0

b. $324x^3 - 9y^{192} + 13$

Trinomial of degree 192

3 pts 5. Add or subtract the following polynomials:

a. $\left(\frac{2}{3}x^3 + \frac{1}{5}x - \frac{3}{4}\right) - \left(-\frac{2}{3}x^3 - \frac{3}{4}x + \frac{3}{4}\right)$

$= \frac{4}{3}x^3 + \frac{19}{20}x - \frac{3}{2}$

3 ea 6. Multiply the following expressions.

a. $(-3x^2y^6)(5x^2y^{15}) = -15x^4y^{21}$

b. $\left(3x - \frac{1}{6}\right)(12x^3 - 18x^2 + 30x)$

$= 36x^4 - 56x^3 + 93x^2 - 5x$

3 ea 7. Simplify the following expressions by using the exponent rules gone over in class. Final forms should have only positive exponents.

a. $(-9xyz)^0 = 1$

b. $\frac{12z^3 - 6z^2 + 24z}{6z} = 2z^2 - z + 4$

c. $\left(\frac{15y^{-4}}{25x^2}\right)^{-3} = \left(\frac{3}{5x^2y^4}\right)^{-3}$

$= \left(\frac{5x^2y^4}{3}\right)^3$

$= \frac{125x^6y^{12}}{27}$

3 ea 8. Simplify the following expressions using scientific notation. Write the simplified form in scientific notation.

a. $(3 \times 10^7)(5 \times 10^{-23})$ $= 15 \times 10^{-16}$

$= 1.5 \times 10^{-15}$

b. $(2 \times 10^3)^5$ $= 32 \times 10^{15}$

$= 3.2 \times 10^{16}$

2 ea 9. The functions f, g, and h are given below. Use the appropriate function to evaluate or solve as requested below. State solutions to equations you are solving using set notation in a complete sentence.

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x	-5	-1	0	2	8	13
$f(x)$	7	-4	5	1	0	-16

- $h(x) = 3x - 1$

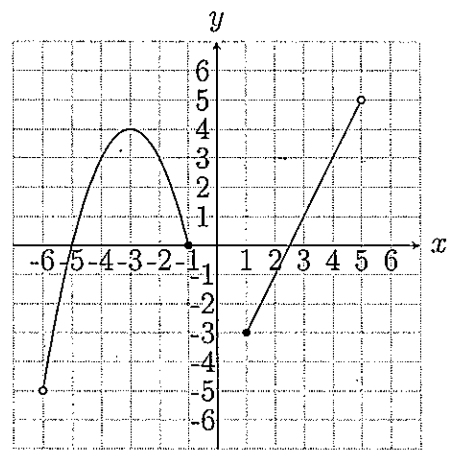
- 

Figure 1: $y = g(x)$

a. $f(0) = 5$

b. $h(-2) = -7$

c. $g(2) = -1$

d. Solve $f(x) = -16$.

The solution set is $\{13\}$.

e. Solve $g(x) = 3$.

The solution set is $\{-4, -2, 4\}$.

f. Solve $h(x) = -1$.

The solution set is $\{0\}$.

2 pts 10. Suppose you have a function M with $M(2) = -1$ and $M(-3) = 5$. What are two points on the graph of M?

The points $(2,-1)$ & $(-3,5)$ are on the graph of M.

3 pts 11. What is the definition of a solution to an equation in one variable?

A number which, when plugged in for the variable, results in the left & right side of the equal sign having the same quantity.

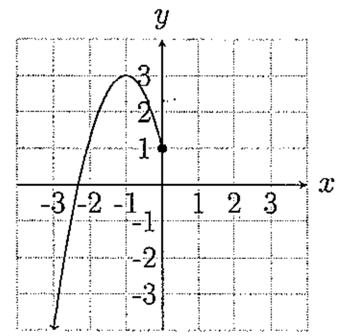
3 ea 12. Determine which of the following relationships between inputs and outputs can be categorized as a function and justify your response. State the domain and range of the relations.

a. $\{(-3,4),(1,-4),(0,0),(2,4),(-3,5)\}$ b.

This is not a function since the input −3 has more than one output.

$D=\{-3,1,0,2\}$

$R=\{4,-4,0,5\}$



This is a function since every input only has one output.

$D=(-\infty,0]$ $R=(-\infty,3]$

3 ea 13. What is the one requirement for a relationship between inputs and outputs to be considered a function?

Each input has, at most, one output.

4 pts 14. Solve the equation $\frac{2}{3}x - 1 = -\frac{1}{3}x + 2$ numerically. Define a function to represent each side of the equation and use function notation in your table headings as appropriate. State a conclusion using set notation in a complete sentence. **Note: You will receive zero points for solving this equation symbolically.**

Let $f(x) = \frac{2}{3}x - 1$

& $g(x) = -\frac{1}{3}x + 2$

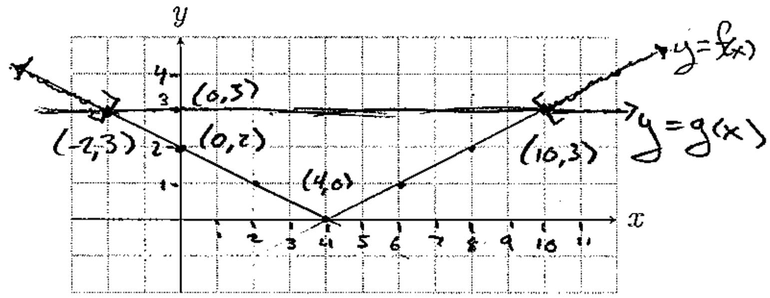
The solution set is $\{3\}$.

	x	f(x)	g(x)
	0	−1	2
*	3	1	1

7 pts 15. Solve the equation $\left|-\frac{1}{2}x+2\right| \geq 3$ graphically. Define a function to represent each side of the equation and label each functions graph appropriately. Label any x or y intercepts along with any intersection points. State a conclusion using set notation in a complete sentence. **Note: You will receive zero points for solving this equation symbolically.**

Let $f(x) = |-\frac{1}{2}x+2|$
& $g(x) = 3$

The solution set is
$\{x \mid x \leq -2 \text{ or } x \geq 10\}$.



7 pts 16. Solve the following equations symbolically. **Part of your grade for these problems is to *justify* breaking up the absolute value inequalities.** State a conclusion using interval notation within a complete sentence.

a. $2|3x+3| \leq 5$

$\frac{2|3x+3|}{2} \leq \frac{5}{2}$ ← And

$|3x+3| \leq 5/2$

Case 1: If 3x+3 (the inside) is positive or zero, then

$|3x+3| \leq 5/2$

becomes $3x+3 \leq 5/2$

$-3 \quad -3$

$\frac{3x}{3} \leq \frac{-1/2}{3}$

$x \leq -1/6$

Case 2: If (3x+3) (the inside) is negative, Then

$|3x+3| \leq 5/2$

becomes $-(3x+3) \leq 5/2$

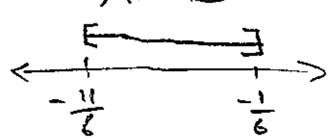
$-3x-3 \leq 5/2$

$+3 \quad +3$

$\frac{-3x}{-3} \leq \frac{11/2}{-3}$

$x \geq -\frac{11}{6}$

AND



The solution interval is $[-11/6, -1/6]$.

17. Determine the domain and range of the following functions. State your conclusions using both set and interval notation.

a. $f(x) = \sqrt{1 + 2x^2}$

$D = \mathbb{R} = (-\infty, \infty)$

$R = \{y \mid y \geq 1\} = [1, \infty)$

b. $f(x) = \dfrac{1}{\sqrt{x - 1}}$

$D = \{x \mid x > 1\} = (1, \infty)$

$R = \{y \mid y > 0\} = (0, \infty)$

18. Determine the domain and range of the following functions. State your answers in both set and interval notation.

a. $f(x) = \sqrt[4]{x + 2}$

$D = \{x \mid x \geq -2\} = [-2, \infty)$

$R = \{y \mid y \geq 0\} = [0, \infty)$

b. $f(x) = \sqrt[3]{x - 2} + 3.$

$D = \mathbb{R} = (-\infty, \infty)$

$R = \mathbb{R} = (-\infty, \infty)$

19. Use positive rational exponents to simplify the expression. Assume that all variables are positive.

a. $(x^2)^{3/2} = x^3$

b. $\sqrt[3]{x^3 y^6} = xy^2$

c. $\sqrt{y^3} \cdot \sqrt[3]{y^2} = y^{3/2} \cdot y^{2/3}$

$= y^{\frac{3}{2} + 2/3}$

$= y^{\frac{9}{6} + \frac{4}{6}}$

$= y^{13/6}$

d. $\left(\dfrac{x^6}{27}\right)^{2/3} = \dfrac{(x^6)^{2/3}}{(27)^{2/3}} = \dfrac{x^4}{3^2} = \dfrac{x^4}{9}$

↑ cube root first is easier!

e. $\sqrt{\sqrt{y}} = (y^{1/2})^{1/2}$
$= y^{1/4}$ } either/or
$= \sqrt[4]{y}$

g. $p^{1/2}\left(p^{3/2} + p^{1/2}\right) = p^2 + p$

f. $\sqrt{b} \cdot \sqrt[4]{b} = b^{1/2} \cdot b^{1/4}$
$= b^{3/4}$
$= \sqrt[4]{b^3}$ ← any
$= (\sqrt[4]{b})^3$

h. $\sqrt[3]{x}\left(\sqrt{x} - \sqrt[3]{x^2}\right) = x^{1/3}(x^{1/2} - x^{2/3})$
$= x^{5/6} - x$
$= \sqrt[6]{x^5} - x$ ← any
$= (\sqrt[6]{x})^5 - x$

20. Simplify the expression assuming that all variables are positive.

a. $\sqrt{\frac{1}{2}} \cdot \sqrt{\frac{1}{8}} = \sqrt{\frac{1}{16}}$
$= \frac{1}{4}$

c. $\sqrt{\frac{x}{2}} \cdot \sqrt{\frac{x}{8}} = \sqrt{\frac{x^2}{16}} = \frac{x}{4}$

b. $\sqrt[4]{\frac{x}{81}} = \frac{\sqrt[4]{x}}{3}$

d. $\sqrt{200} = \sqrt{100 \cdot 2}$
$= 10\sqrt{2}$

e. $\sqrt[5]{-64} = -2\cdot\sqrt[5]{2}$

f. $\sqrt{12a^2b^5} = \sqrt{4a^2b^4\cdot 3b}$

$= 2ab^2\sqrt{3b}$

g. $\sqrt[3]{-125x^4y^5} = -5xy\sqrt[3]{xy^2}$

h. $\sqrt{\frac{7a^2}{27}}\cdot\sqrt{\frac{7a}{3}} = \sqrt{\frac{49a^3}{81}}$

$= \frac{7a\sqrt{a}}{9}$

i. $\sqrt[4]{rt}\cdot\sqrt[3]{r^2t} = r^{1/4}t^{1/4}\cdot r^{2/3}\cdot t^{1/3}$

$= r^{\frac{1}{4}+\frac{2}{3}}\cdot t^{\frac{1}{4}+\frac{1}{3}}$

$= r^{11/12}\cdot t^{7/12}$

j. $\sqrt[3]{16}+3\sqrt[3]{2} = 2\sqrt[3]{2}+3\sqrt[3]{2}$

$= 5\cdot\sqrt[3]{2}$

k. $\sqrt{2}+\sqrt{18}+\sqrt{32} = \sqrt{2}+3\sqrt{2}+4\sqrt{2}$

$9\cdot 2$ $\quad 16\cdot 2$

$= 8\sqrt{2}$

l. $\sqrt[4]{48}+4\sqrt[4]{3} = 2\cdot\sqrt[4]{3}+4\cdot\sqrt[4]{3}$

$16\cdot 3$

$= 6\cdot\sqrt[4]{3}$

m. $3\sqrt{2k}+\sqrt{8k}+\sqrt{18k} = 3\sqrt{2k}+2\sqrt{2k}+3\sqrt{2k}$

$4\cdot 2$ $\quad 9\cdot 2$

$= 8\sqrt{2k}$

n. $2\sqrt[4]{64}-\sqrt[4]{324}+\sqrt[4]{4} = 2\cdot 2\cdot\sqrt[4]{4}-3\cdot\sqrt[4]{4}+\sqrt[4]{4}$

$16\cdot 4$ $\quad 81\cdot 4$

$= 2\sqrt[4]{4}$

324
2 162
2 81
9 9
3 3 3 3

21. Rationalize the denominator.

a. $\frac{1}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{7}}{7}$

FOIL!

c. $\frac{\sqrt{7}-2}{\sqrt{7}+2} \cdot \frac{\sqrt{7}-2}{\sqrt{7}-2} = \frac{7-4\sqrt{7}+4}{7-4}$

$= \frac{11-4\sqrt{7}}{3}$

b. $\frac{1}{3-\sqrt{2}} \cdot \frac{3+\sqrt{2}}{3+\sqrt{2}} = \frac{3+\sqrt{2}}{9-2}$

$= \frac{3+\sqrt{2}}{7}$

d. $\frac{\sqrt{z}}{\sqrt{z}-3} \cdot \frac{\sqrt{z}+3}{\sqrt{z}+3} = \frac{z+3\sqrt{z}}{z-9}$

22. Factor the following polynomials.

a. $36x^4 - 4x^6 = 4x^4(9-x^2)$

$= 4x^4(3-x)(3+x)$

b. $z^3 + 64 = (z+4)(z^2-4z+16)$

23. Solve each of the following quadratic equations. Describe the set of solutions in set notation.

a. $(3x-5)^2 = 64$

$3x-5 = \pm 8$

$3x-5=8$ or $3x-5=-8$

$3x=13$ $\quad$ $3x=-3$

$x=13/3$ $\quad$ $x=-1$

The solution set is $\{13/3, -1\}$

b. $3x^2 + 11x = 4$

$3x^2+11x-4=0$

$3x^2-x+12x-4=0$

$x(3x-1)+4(3x-1)=0$

$(3x-1)(x+4)=0$

$x=1/3$ or $x=-4$

The solution set is $\{1/3, -4\}$.

-12

$-1 + 12 = 11$

c. $x^2 - 6x - 11 = 0$ Not factorable

$x = \frac{6 \pm \sqrt{36 - 4(-11)}}{2}$

$= \frac{6 \pm \sqrt{80}}{2}$

$= \frac{6 \pm 4\sqrt{5}}{2} = 3 \pm 2\sqrt{5}$

80 / 2 40 / 4 20 / 5 (16)

The solution set is $\{3+2\sqrt{5}, 3-2\sqrt{5}\}$.

24. Solve the equation symbolically. State your conclusion using set notation in a complete sentence.

a. $2\sqrt{x-2}+1=5$

$-1 \quad -1$

$2\sqrt{x-2} = 4$

$\div 2 \quad \div 2$

$\sqrt{x-2} = 2$

$x - 2 = 4$

$+2 \quad +2$

$x = 6$

Check: $2\sqrt{6-2} + 1 \stackrel{?}{=} 5$

$2\sqrt{4} + 1 \stackrel{?}{=} 5$

$2 \cdot 2 + 1 \stackrel{?}{=} 5$

$5 = 5$ ✓

The solution set is $\{6\}$.

b. $(\sqrt{x})^2 = (\sqrt{x-5}+1)^2$ FOIL!

$x = (x-5) + 2\sqrt{x-5} + 1$

$-x \quad -x$

$0 = -4 + 2\sqrt{x-5}$

$4 = 2\sqrt{x-5}$

$2 = \sqrt{x-5}$

$4 = x - 5$

$9 = x$

Check: $\sqrt{9} \stackrel{?}{=} \sqrt{9-5} + 1$

$3 \stackrel{?}{=} \sqrt{4} + 1$

$3 = 3$ ✓

Graphical Solutions:

25. Solve the equations graphically using your calculators graphing capabilities. Define $y_1(x)$ and $y_2(x)$ for each and then state the solution(s) in set notation. Approximate solutions to the nearest hundredth when appropriate.

(a) $\sqrt[3]{x+5} = 2$

$y_1(x) = \sqrt[3]{x+5}$

$y_2(x) = 2$

The set of solutions is:

$\{3\}$.

(b) $\sqrt{x+2} + \sqrt{3x+2} = 2$

$y_1(x) = \sqrt{x+2} + \sqrt{3x+2}$

$y_2(x) = 2$

The set of solutions is: $\{x \mid x \approx -0.47\}$.

26. Suppose $f(x) = x^2 - 2x - 8$. Answer the following questions. Remember to label all of the points shown in the table and also the equation of the axis of symmetry when you graph the function.

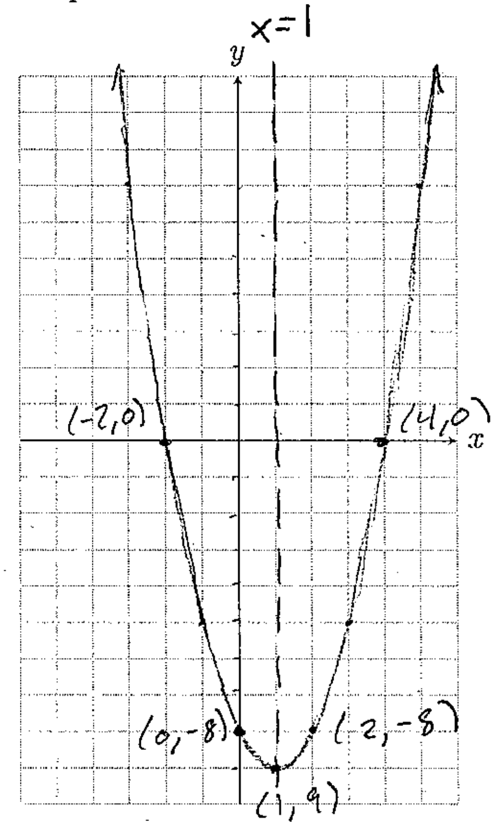
a. Does the graph open up or down?

Up

b. Complete the table with the points needed to sketch a graph of $f(x)$. Write the name of each key point in the blanks provided. Show your work in the space below.

Name of point	x	$f(x)$
y-int	0	-8
vertex	1	-9
y-int mirror	2	-8
x-ints {	4	0
	-2	0

c. Graph the function.



y-int: $f(0) = -8$

Vertex: $\frac{-(-2)}{2(1)} = 1$

$f(1) = 1 - 2 - 8$
$= -9$

x-ints: $0 = x^2 - 2x - 8$
$0 = (x-4)(x+2)$
$x = 4,\ x = -2$
$(4,0)$ & $(-2,0)$

Axis is symmetry: $x = 1$

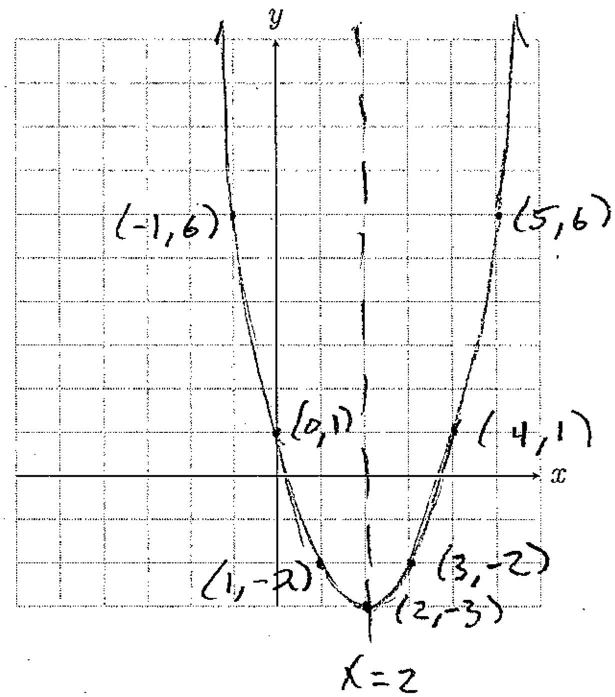
27.

10 pts a. Make a table for $f(x) = (x-2)^2 - 3$ then graph $y = f(x)$ in the space provided with the vertex in the middle. Label all points from the table along with the axis of symmetry.

x	$f(x)$
-1	6
0	1
1	-2
2	-3
3	-2
4	1

← vertex

5 6



4 pts b. Describe using a complete sentence how the function $sqr(x) = x^2$ is translated to become f.

The function sqr is shifted 2 units right & 3 units down to obtain f.

10 pts 28. Complete the square of the following function to put it into vertex form. Then state the vertex.

$f(x) = 3x^2 - 12x + 1$

$= 3(x^2 - 4x) + 1$

$= 3(x^2 - 4x + 4 - 4) + 1$

$= 3(x-2)^2 - 12 + 1$

$= 3(x-2)^2 - 11$

The vertex is $(2, -11)$.

10 pts 29. Solve

$g(x) = 0$, where $g(x) = 2x^2 - 4x + 5$,

by completing the square. If your answer is complex, write it in standard complex number form and state your conclusion using set notation in a complete sentence.

$0 = 2x^2 - 4x + 5$

$-5 = 2x^2 - 4x$

$-\frac{5}{2} = x^2 - 2x$

$+1 \qquad +1$

$-\frac{3}{2} = x^2 - 2x + 1$

$-\frac{3}{2} = (x-1)^2$

$\pm\sqrt{\frac{-3}{2}} = x - 1$

$+1 \qquad +1$

$1 \pm \sqrt{\frac{3}{2}}\, i = x$

The solution set is $\{1 + \sqrt{\frac{3}{2}}\, i, 1 - \sqrt{\frac{3}{2}}\, i$

Acc 65-95 Exam 2 Review Calculator Okay   Name: ________________

6 pts 1. The half-life of Uranium 234 is 2.5×10^5 years and the half-life of Plutonium is 8.0×10^7 years. How many times greater is the half-life of Plutonium than Uranium 234?

$\frac{8.0 \times 10^7}{2.5 \times 10^5} = 3.2 \times 10^2$

The half life of Plotonium is 3.2×10^2 times greater than the half life of Uranium.

Solve the following story problems involving systems of equations by following these steps:

Step 1: Define variables to be the unknown quantities.

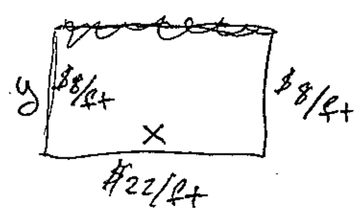
Step 2: Write a system of equations that model's the problem's conditions.

Step 3: Solve the system of equations.

Step 4: Write a conclusion which answers the story problem using complete sentences.

8 pts 2. A rectangular lot is being fenced on three sides. The fencing along the lot's length costs \$22 per foot while the fencing along the two side widths costs \$8 per foot. The total fencing comes out to 170ft while the cost of the fencing along the three sides comes to \$2340. What are the lot's dimensions?

x = length (ft) of lot
y = width (ft) of lot



Total fencing is 170ft. Total cost: $234[0]

$22x + 8y + 8y = 2340$

$22x + 16y = 2340$

$-22(x + 2y) = (170)(-22)$

$-22x - 44y = -3740$

$22x + 16y = 2340$

$-28y = -1400$

$y = 50$

$x + 2(50) = 170$

$x = 70$

The length is 70 ft & the width is 50ft.

8 pts 3. You are choosing between memberships between Outdoor Store and Adventure Warehouse. Outdoor Store offers an annual membership fee of \$111 which allows you to pay only 80% of the retail price. Adventure Warehouse offers an annual membership fee of \$75 but you pay only 85% of the retail price. What is the retail price of merchandise which would allow you to end up spending the same total amount (including membership fee) at either store? What would that total amount be?

Let x = retail value of merch. (\$)
& y = total cost after discount & fee.

O.S.: $y = .8x + 111$

A.W.: $y = .85x + 75$

$.8x + 111 = .85x + 75$

$111 = .05x + 75$

$\frac{36}{.05} = \frac{.05x}{.05}$

$x = 720$

$y = .8(720) + 111$
$= 687$

If you buy \$720 worth of merch @ either store you will pay \$687 after discount & fee.

4. You realize how awesome a trebuchet is and decide to build your own. It's a fantasticly huge wonder which you launch multiple pianos with. During the destroying of many a good piano you take measurements and determine the piano's flight path can be modeled by the function

$$f(d) = -0.04d^2 + 1.431d + 32.5,$$

where $h = f(d)$ is the height of the piano (in meters) at the given horizontal distance d (in meters) from the trebuchet.

Graph $h = f(d)$ on your calculator and use it to answer the following questions.

You need to use complete sentence conclusions to answer each of the following questions. Any rounding necessary should be to the second decimal place.

4 pts a. Use the graph on your graphing calculator to determine the horizontal distance(s) of the piano from the trebuchet when it is at a height of 40 meters.

The piano is 40 meters up when it is about 6.38 meters & 29.48 meters from the trebuchet.

4 pts b. Use the graph on your graphing calculator to determine at what horizontal distance from the trebuchet the piano will hit the ground.

The piano will hit the ground about 51.54 meters from the trebuchet.

4 pts c. Use the graph on your graphing calculator to determine *the maximum height* the piano reaches **and** *the horizontal distance of the piano from the trebuchet* when it reaches this maximum height.

The piano will reach a max height of about 45.30 meters when it is about 17.89 meters from the trebuchet.

d. From what height is the piano released?

The piano is released from a height of 32.5 meters

e. State the domain and range of f *in context* of this story problem.

D≉[0, 51.54]

R≉[0, 45.30]