11. The functions f and g are given below in a table and a graph. Answer the questions below using these functions.

x	-4	-2	0	1	3	5
$f(x)$	8	5	3	0	-2	-6

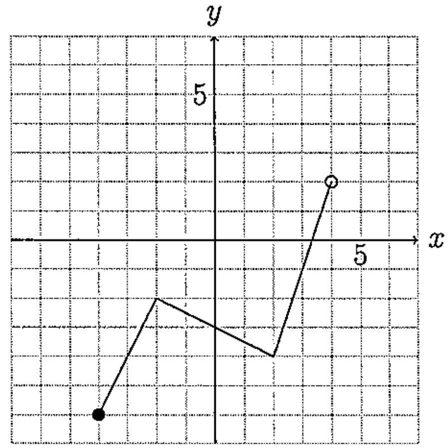


Figure 13: $g(x)$

a. Find the following

i. $f(3) =$ -2

ii. $f(-2) =$ 5

iii. $g(0) =$ -3

iv. $g(2) =$ -4

v. When $f(x) = 5$, then $x =$ -2.

vi. When $g(x) = -1$, then $x =$ 3.

vii. When $g(x) = -4$, then $x =$ 2 or $x=-3$.

b. What is the domain of g?

$[-4, 4) = \{x \mid -4 \le x < 4\}$

c. What is the range of g?

$[-6, 2) = \{y \mid -6 \le y < 2\}$

d. Find all x for which $g(x) > -1$

$g(x) > -1$ when

$3 < x < 4$.

The set of solutions is $\{x \mid 3 < x < 4\}$

$= \underbrace{(3,4)}_{\text{interval notation}}$

2. Given $g(x) = \frac{x}{x+2}$, find each of the following and write the corresponding ordered pair:

a. $g(2) = \frac{2}{2+2} = \frac{1}{2}$

c. $g(0) = 0$

b. $g(-3) = 3$

d. $g(-1) = -1$

3. Determine whether the following relationships between inputs and outputs can be categorized as a function and justify your response. State the domain and range of both relations.

a. $\{(-2,7),(3,4),(5,9),(2,4),(3,-7)\}$

This is not a function since the input of 3 has 2 outputs.

$D = \{-2, 2, 3, 5\}$

$R = \{-7, 4, 7, 9\}$

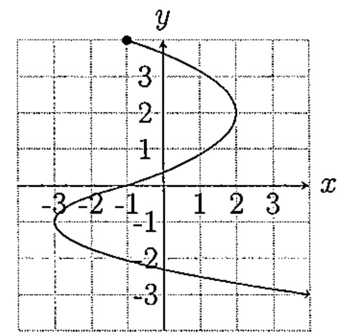
b. $\{(2,.5),(-7,.5),(-1,.5),(3,2),(4,2)\}$

This is a function since each input has a unique output.

$D = \{-7,-1,2,3,4\}$

$R = \{0.5, 2\}$

c.



This is not a function since 0 (along with many other inputs) has more than one output.

$D = [-3, \infty)$

$R = (-\infty, 4]$

4. What is the one requirement for a relationship between inputs and outputs to be considered a function?

Each input in the domain of the function has just one output.

5. What is the definition of a solution to an equation in one variable?

A number which, when plugged in for the variable, results in the expression on the left holding the same value as the expression on the right.

6. For the next two problems, set up two functions, one to represent each side of the equation then solve the equation numerically (using the function name in your header), symbolically, and graphically (labeling both graphs using the correct function name). State a conclusion using a full sentence and proper set notation.

a. $2x + 3 = 5$ Let $f(x) = 2x+3$ & $g(x) = 5$

i. Numerically:

x	$f(x)$	$g(x)$
0	3	5
3	9	5
2	7	5
1	5	5

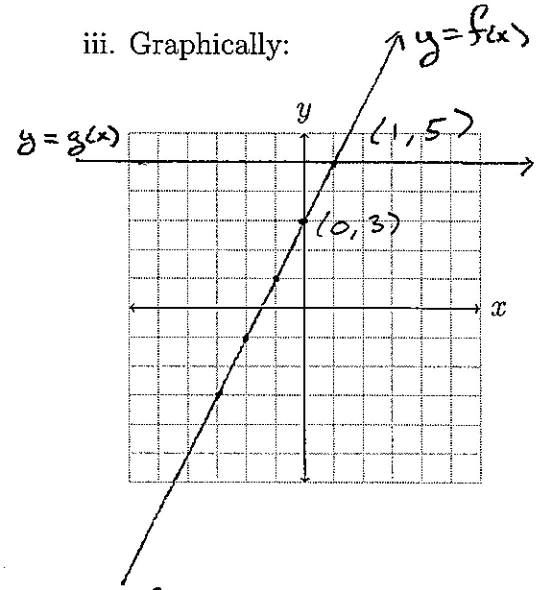
ii. Symbolically:

$$2x + 3 = 5$$

$$\frac{2x}{2} = \frac{2}{2}$$

$$x = 1$$

iii. Graphically:



The set of solutions is $\{1\}$.

b. $x - 2 = -2x + 1$ Let $f(x) = x-2$ & $g(x) = -2x+1$

i. Numerically:

x	$f(x)$	$g(x)$
0	−2	1
5	3	−9
3	1	−5
1	−1	−1

ii. Symbolically:

$$x - 2 = -2x + 1$$

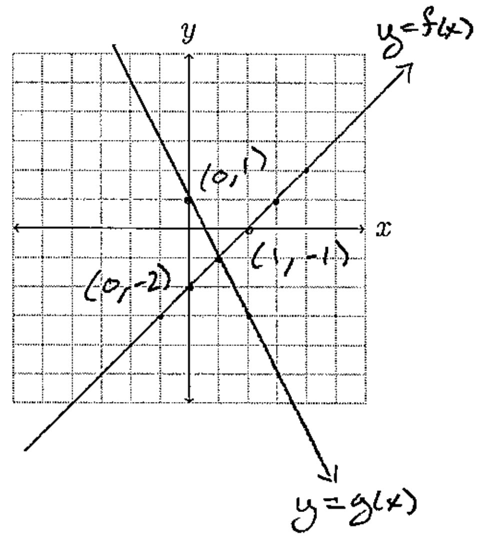
$$+2x \qquad +2x$$

$$3x - 2 = 1$$

$$\frac{3x}{3} = \frac{3}{3}$$

$$x = 1$$

iii. Graphically:



The set of solutions is $\{1\}$.

5. For the next two problems, set up two or three functions, one to represent each side of the equation then solve the equation numerically (using the function name in your header), symbolically, and graphically (labeling both graphs using the correct function name). State a conclusion using a full sentence and proper set notation.

a. $2x - 4 < -2$ or $2x - 4 \geq 6$

Let $f(x) = -2$, $g(x) = 2x-4$
& $h(x) = 6$

a. Numerically:

x	f(x)	g(x)	h(x)
0	-2	-4	6
* 1	-2	-2	6
3	-2	2	6
7	-2	10	6
6	-2	8	6
* 5	-2	6	6

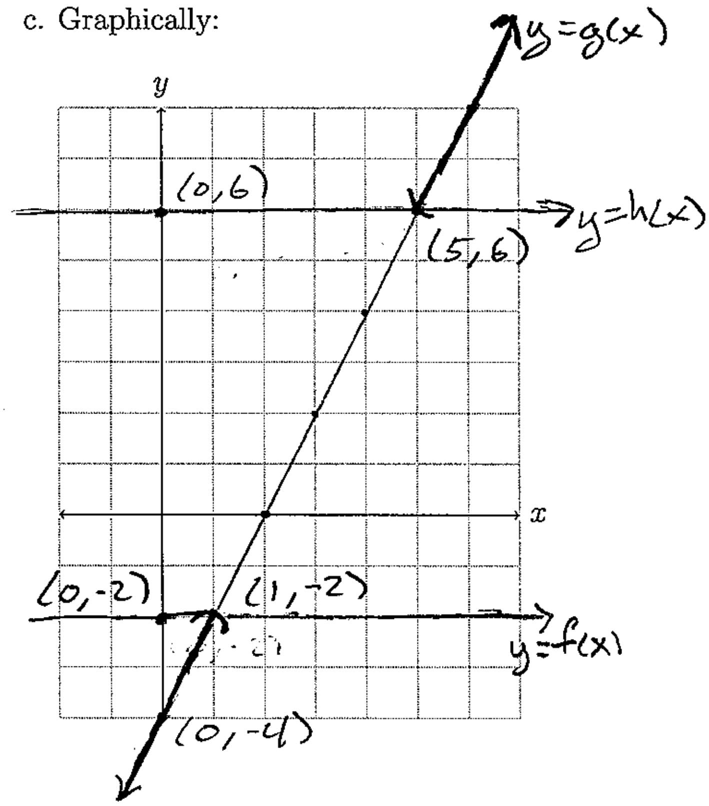
b. Symbolically:

$2x-4 < -2$ or $2x-4 \geq 6$

$2x < 2$ $\qquad$ $2x \geq 10$

$x < 1$ or $x \geq 5$

c. Graphically:



The solutions are in the interval $(-\infty, 1) \cup (5, \infty)$. The set of solutions is $\{x \mid x < 1 \text{ or } x > 5\}$.

b. $-2 \leq -\frac{1}{2}x + 2 < 6$

Let $f(x) = -2$, $g(x) = -\frac{1}{2}x + 2$, $h(x) = 6$

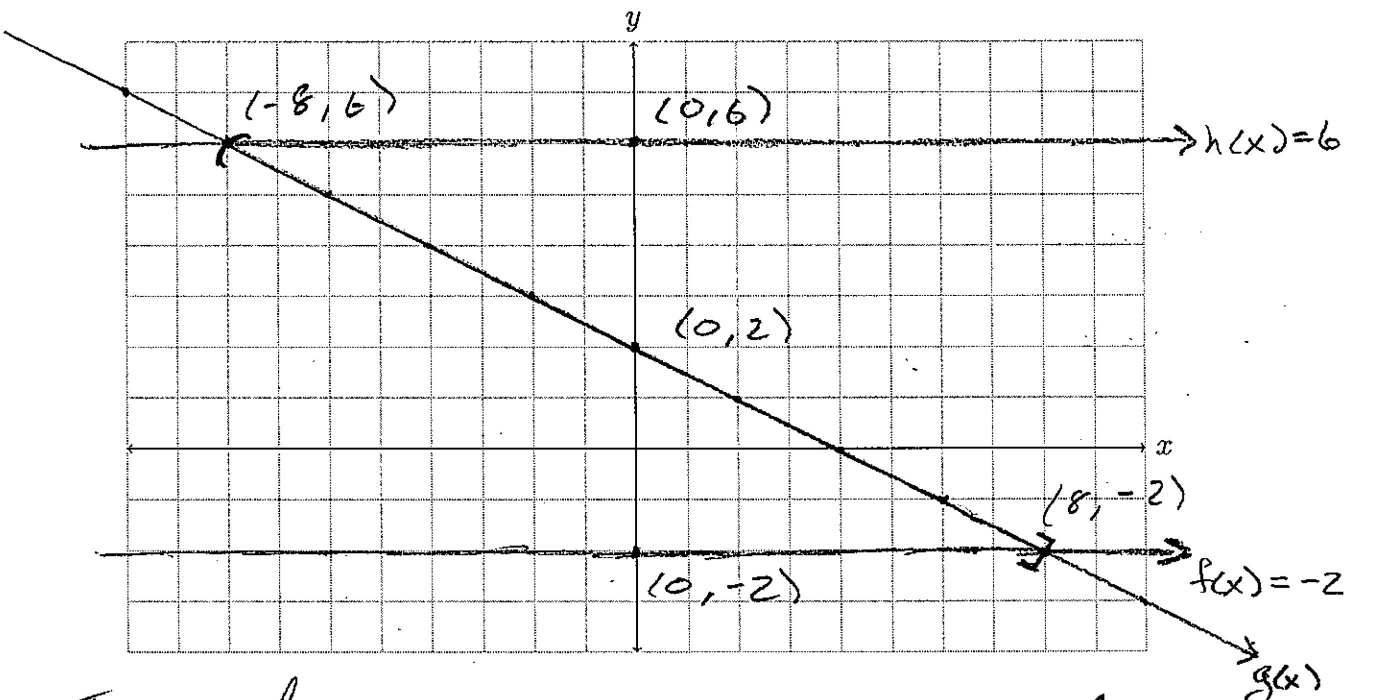
a. Numerically:

x	f(x)	g(x)	h(x)
0	-2	2	6
-4	-2	4	6
* -8	-2	6	6
4	-2	0	6
* 8	-2	-2	6

b. Symbolically:

$$-2 \leq -\frac{1}{2}x + 2 < 6$$
$$-4 \leq -\frac{1}{2}x < 4$$
$$8 \geq x > -8$$
$$-8 < x \leq 8$$

c. Graphically:



The solutions are in the interval $(-8, 8]$. The set of solutions is $\{x \mid -8 < x \leq 8\}$.

2. Solve the following equations and inequalities graphically.

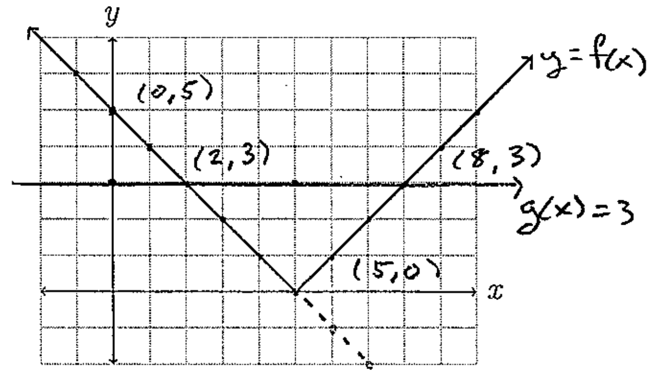
a. $|5 - x| = 3$

Let $f(x) = |5-x| = |-x+5|$

$m = -1$

y-int $= (0,5)$

& $g(x) = 3$



The set of solutions is $\{2, 8\}$

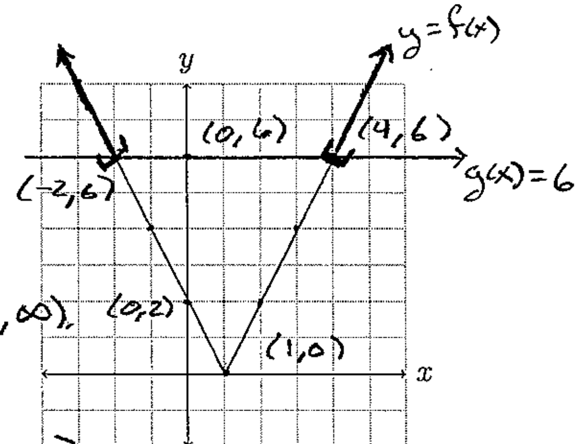
b. $|-2x+2| \geq 6$

Let $f(x) = |-2x+2|$

$m = -2$

y-int $= (0,2)$

& $g(x) = 6$



The interval of solutions is $(-\infty,-2] \cup [4,\infty)$.

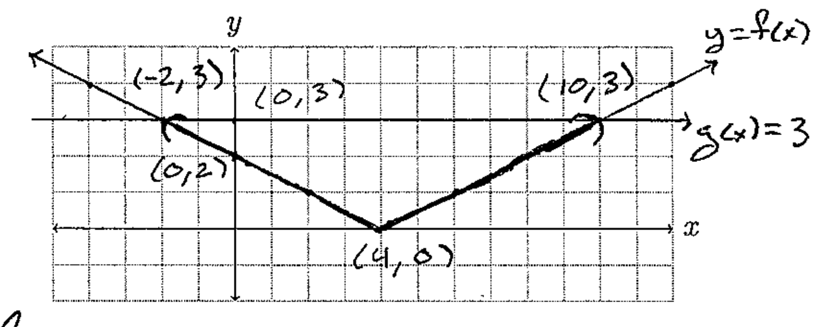
The set of solutions is $\{x \mid x \leq -2 \text{ or } x \geq 4\}$.

c. $\left|\frac{-1}{2}x + 2\right| < 3$

Let $f(x) = |-\frac{1}{2}x + 2|$

$m = -\frac{1}{2}$ y-int $= (0,2)$

& $g(x) = 3$



The interval of solutions is $(-2, 10)$.

The set of solutions is $\{x \mid -2 < x < 10\}$

9. Solve the following equations and inequalities numerically.

a. $\left|\frac{2}{3}x - 5\right| < 3$

Let $f(x) = \left|\frac{2}{3}x - 5\right|$

& $g(x) = 3$

$\left|\frac{2}{3}x - 5\right| < 3$ when $3 < x < 12$.

The set of solutions is $\{x \mid 3 < x < 12\}$.

The interval of solutions is $(3, 12)$.

	x	$f(x)$ <	$g(x)$
	-3	7	3
	0	5	3
*	3	3	3
✓	6	1	3
✓	9	1	3
*	12	3	3

10. Solve the following equations symbolically.

a. $|5 - 3x| - 3 = 1$ First isolate the absolute value:

$$\begin{aligned} +3 \quad +3 \\ |5-3x| = 4 \end{aligned}$$

So we want the size of $5-3x$ to be 4, hence:

$5 - 3x = 4$ or $5 - 3x = -4$

$-3x = -1$ $\quad$ $-3x = -9$

$x = 1/3$ $\quad$ $x = 3$

The sol set is $\{1/3, 3\}$.

b. $|2x| = |x - 3|$ We want the sizes to be the same so either:

$2x = x - 3$ or $-(2x) = x - 3$

$x = -3$ $\quad$ $-2x = x - 3$

$-3x = -3$

$x = 1$

The sol. set is $\{-3, 1\}$.

c. $|2x-5| > -1$

We want the <u>size</u> of $2x-5$ to be bigger than -1.

But, every <u>size</u> is positive (or zero)

so $|2x-5|$ is bigger than -1 no matter x's value

hence the sol. set is $\mathbb{R}$.

d. $\dfrac{-2|5x-1|}{-2} \geq \dfrac{5}{-2}$ <u>First</u> isolate the absolute value:

$|5x-1| \leq -\frac{5}{2}$ So we want the <u>size</u> of $5x-1$ to be less than $-5/2$.

But <u>sizes</u> are positive (or zero)

so this isn't possible so there are <u>no</u> solutions.

The sol. set is $\{\}$.

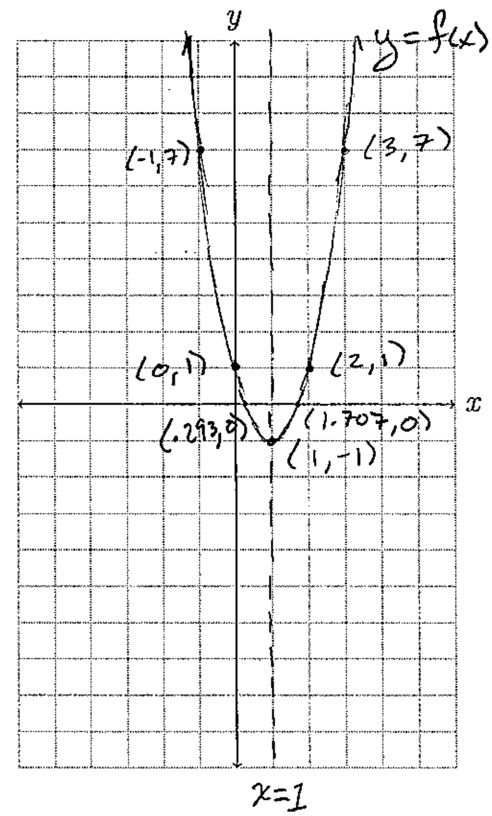
1. Follow the given steps to graph the parabola which corresponds to the given quadratic function. Remember to label all of the points shown in the table and also the equation of the axis of symmetry when you graph the function. Then evaluate the function for the specified values of x.

a. $f(x) = 2x^2 - 4x + 1$

a. Complete the table with the points needed to sketch a graph of $f(x)$. Show your work in the space below.

Name of point	x	$f(x)$
Vertex	1	-1
y-intercept	0	1
y-int mirrow	2	1
x-intercept	0.293	0
x-intercept	1.707	0
Extra Pnts	-1	7
	3	7

b. Graph the function.



$X_v = \frac{4}{2(2)} = 1$

$f(1) = -1$

If $f(x)=0$ then $x = \frac{4 \pm \sqrt{16 - 4(2)(1)}}{2(2)}$

$= \frac{4 \pm \sqrt{8}}{4}$

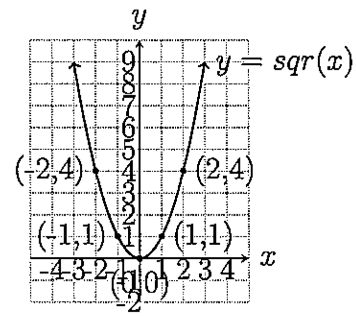
$= \frac{4}{4} \pm \frac{2\sqrt{2}}{4}$

$= 1 \pm \frac{1}{2}\sqrt{2} \approx 0.293, 1.707$

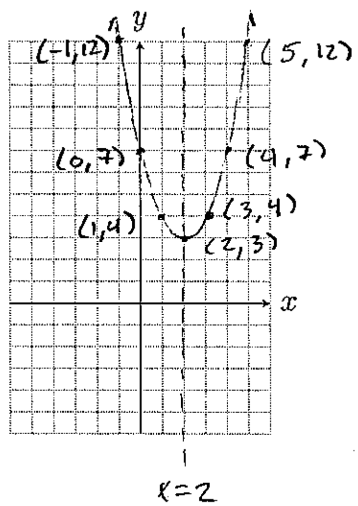
$f(-1) = 7$

12. Graph the following functions by translating the graph of $sqr(x) = x^2$. Use the 5 key points shown below.

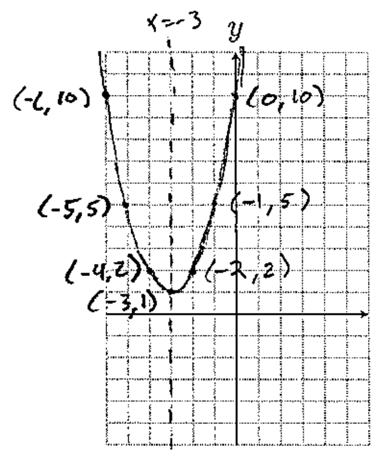
x	$sqr(x)$
-2	4
-1	1
0	0
1	1
2	4



a. $f(x) = (x-2)^2 + 3$

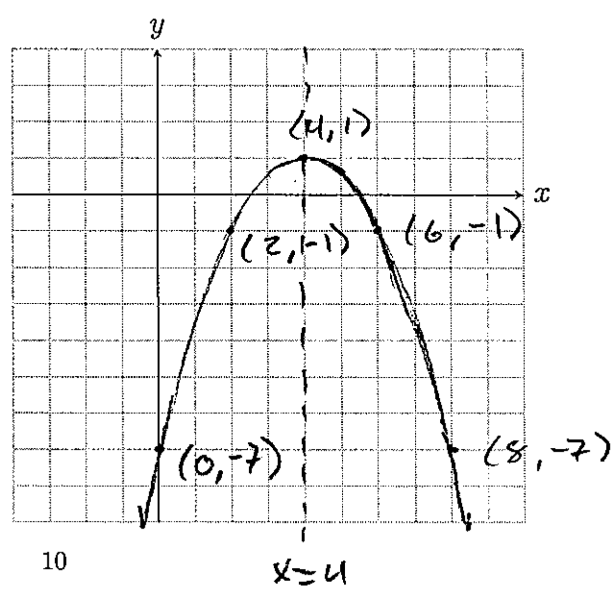


b. $f(x) = (x+3)^2 + 1$



13. Fill in the following table and use the data to graph the function $f(x) = -\frac{1}{2}(x-4)^2 + 1$.

Name of point	x	$f(x)$
y-int Mirror	8	-7
Extra Point	6	-1
Vertex	4	1
Extra Point	2	-1
y-int	0	-7



14. Solve $-\frac{1}{2}(x-3)^2+2=0$ numerically, symbolically, and graphically.

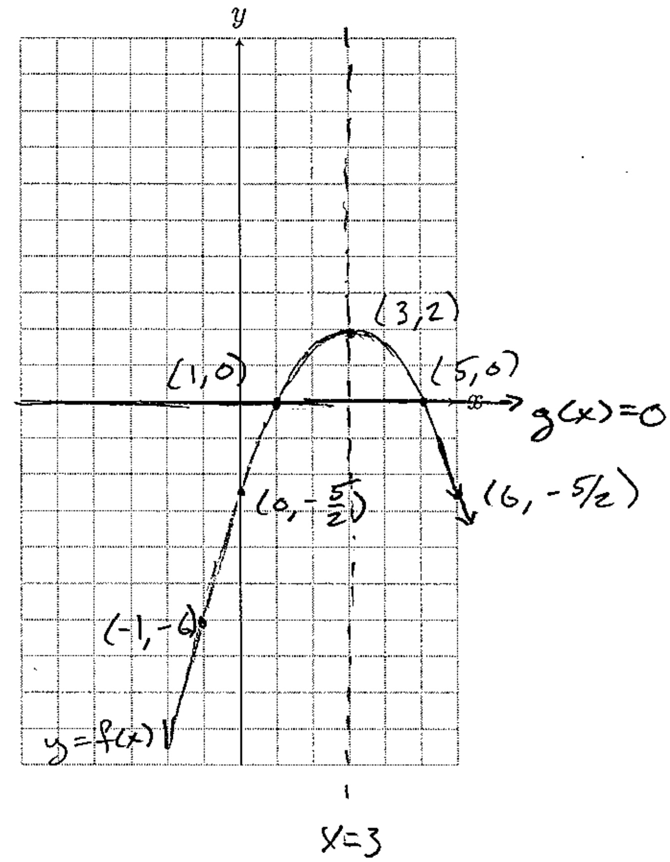
Let $f(x) = -\frac{1}{2}(x-3)^2+2$ & $g(x)=0$

a. Numerically:

	x	$f(x)$	$g(x)$
			0
	-1	-6	0
*	1	0	0
vertex →	3	2	0
*	5	0	0
	7	-6	0

I started at the vertex & moved out

b. Graphically:



c. Symbolically:

$$(-2)\cdot\left[-\frac{1}{2}(x-3)^2+2\right]=0\,(-2)$$

$$(x-3)^2-4=0$$

$$(x-3)^2=4$$

$$x-3=\pm 2$$

$$x=3\pm 2$$

$$x=5 \quad \text{or} \quad x=1$$

The set of solutions is $\{1,5\}$.

15. Complete the square of the following functions to put them into vertex form. Then state the vertex.

a. $f(x) = x^2 - 4x$ $\left(\frac{-4}{2}\right)^2 = 4$

$= x^2 - 4x + 4 - 4$

$= (x-2)^2 - 4$

Vertex $= (2, -4)$

c. $f(x) = 2x^2 - 4x + 1$ $\left(\frac{-2}{2}\right)^2 = 1$

$= 2(x^2 - 2x) + 1$

$= 2(x^2 - 2x + 1 - 1) + 1$

$= 2(x-1)^2 - 2 + 1$

$= 2(x-1)^2 - 1$

Vertex $= (1, -1)$

b. $f(x) = x^2 - 6x + 5$ $\left(\frac{-6}{2}\right)^2 = 9$

$= x^2 - 6x + 9 - 9 + 5$

$= (x-3)^2 - 4$

vertex $= (3, -4)$

d. $f(x) = -3x^2 - 4x + 1$ $\left(\frac{4}{3} \cdot \frac{1}{2}\right)^2 = \frac{4}{9}$

$= -3(x^2 + \frac{4}{3}x) + 1$

$= -3(x^2 + \frac{4}{3}x + \frac{4}{9} - \frac{4}{9}) + 1$

$= -3(x + \frac{2}{3})^2 + \frac{4}{3} + 1$

$= -3(x + \frac{2}{3})^2 + \frac{7}{3}$

vertex $= (-\frac{2}{3}, \frac{7}{3})$

16. Solve the following equations symbolically using the completing square method.

a. $x^2 - 4x + 2 = 0$ $\left(\frac{-4}{2}\right)^2 = 4$

$x^2 - 4x = -2$

$x^2 - 4x + 4 = -2 + 4$

$(x-2)^2 = 2$

$x - 2 = \pm\sqrt{2}$

$x = 2 \pm \sqrt{2}$

The set of solutions is $\{2+\sqrt{2}, 2-\sqrt{2}\}$.

b. $2x^2 + 7x - 5 = 0$ $\left(\frac{7}{2} \cdot \frac{1}{2}\right)^2 = \frac{49}{16}$

$\frac{2x^2 + 7x}{2} = \frac{5}{2}$

$x^2 + \frac{7}{2}x = \frac{5}{2}$

$x^2 + \frac{7}{2}x + \frac{49}{16} = \frac{5}{2} + \frac{49}{16}$

$(x + \frac{7}{4})^2 = \frac{89}{16}$

$x + \frac{7}{4} = \pm\frac{\sqrt{89}}{4}$

$x = -\frac{7}{4} \pm \frac{\sqrt{89}}{4}$

The set of solutions is $\{-\frac{7}{4} + \frac{\sqrt{89}}{4}, -\frac{7}{4} - \frac{\sqrt{89}}{4}\}$

17. Evaluate the discriminant and the state how many real solutions to the equation there must be based upon what number you found when evaluating the discriminant.

b^2-4ac is the discriminant.

a. $\frac{1}{2}x^2 + \frac{3}{2}x + 2 = 0$

$\left(\frac{3}{2}\right)^2 - 4\left(\frac{1}{2}\right)2 = \frac{9}{4} - 4 = \frac{-7}{4}$

There are no solutions since the discriminant is negative.

b. $2x^2 - x + 3 = 0$

$(-1)^2 - 4(2)(3) = 1 - 24 = -23$

There are no solutions since the discriminant is negative.

18. Solve the following quadratic equations using the quadratic formula. If your answer is complex, write it in standard complex number form.

a. $2x^2 + 11x - 6 = 0$

$$x = \frac{-11 \pm \sqrt{(11)^2 - 4(2)(-6)}}{2(2)}$$
$$= \frac{-11 \pm \sqrt{121+48}}{4}$$
$$= \frac{-11 \pm \sqrt{169}}{4}$$
$$= \frac{-11 \pm 13}{4}$$

$x = 1/2$ or $x = -6$

The set of solutions is $\{-6, 1/2\}$.

b. $-3x^2 + 10x - 5 = 0$

$$x = \frac{-10 \pm \sqrt{100 - 4(-3)(-5)}}{2(-3)}$$
$$= \frac{-10 \pm \sqrt{40}}{-6}$$
$$= \frac{-10}{-6} \pm \frac{2\sqrt{10}}{-6}$$
$$= \frac{5}{3} \pm \frac{\sqrt{10}}{3}$$

The set of solutions is $\left\{\frac{5}{3} + \frac{\sqrt{10}}{3}, \frac{5}{3} - \frac{\sqrt{10}}{3}\right\}$.

c. $x^2 + 2x + 3 = 0$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(3)}}{2(1)}$$
$$= \frac{-2 \pm \sqrt{4-12}}{2}$$
$$= -\frac{2}{2} \pm \frac{\sqrt{-8}}{2}$$
$$= -1 \pm \frac{2\sqrt{2} \cdot i}{2}$$
$$= -1 \pm \sqrt{2} \cdot i$$

The set of solutions is $\{-1+\sqrt{2}\,i, -1-\sqrt{2}\,i\}$.

d. $-\frac{1}{3}x^2 + x = 2$ $\rightarrow$ $-\frac{1}{3}x^2 + x - 2 = 0$

$$x = \frac{-1 \pm \sqrt{1 - 4(-\frac{1}{3})(-2)}}{2(-\frac{1}{3})}$$

$1 - \frac{8}{3} = -\frac{5}{3}$

$$= \frac{-1 \pm \sqrt{-5/3}}{-2/3}$$
$$= \left(-1 \pm \sqrt{\tfrac{5}{3}} \cdot i\right)\left(-\tfrac{3}{2}\right)$$
$$= \frac{3}{2} \pm \frac{3}{2}\sqrt{\frac{5}{3}}\, i$$

The set of solutions is $\left\{\frac{3}{2} + \frac{3}{2}\sqrt{\frac{5}{3}}\,i, \frac{3}{2} - \frac{3}{2}\sqrt{\frac{5}{3}}\,i\right\}$.

19. In a study of the effect of temperature on the growth of melon seedlings, the seedlings were grown at different temperatures, and their heights were measured after a fixed period of time. The findings of this study can be modeled by

$$f(x) = -0.095x^2 + 5.4x - 52.2,$$

where x is the temperature in degrees Celsius and the output $f(x)$ gives the resulting average height in centimeters.

a. Use your calculator to graph $y = f(x)$ and then use the **max** feature of your calculator to determine the maximum height for the melon seedlings and the temperature which results in this greatest height.

The seedling grow to a maximum height of about 24.54 cm when they are grown at a temperture of about 28.4211°C.

b. Solve part (a) symbolically.

$x_v = \frac{-5.4}{2(-0.095)} \approx 28.4211$

$f(x_v) \approx 24.5368$

20. A camper paddles a canoe 2 miles downstream in a river that has a 2-mile-per-hour current. To return to camp, the canoeist travels upstream on a different branch of the river. It is 4 miles long and has a 1-mile-per-hour current. The total trip (both ways) takes 3 hours. Find the average speed of the canoe in still water. (*Hint:* Time equals distance divided by rate.)

D ÷ R = T

Let x = ave. speed of canoe in still water (mph).

down-stream: $2 \div x+2 = 2/(x+2)$

up-stream: $2 \div x-2 = 2/(x-2)$

$(x+2)(x-2)\left(\frac{2}{x+2} + \frac{2}{x-2}\right) = (3)(x+2)(x-2)$

$2(x-2) + 2(x+2) = 3(x^2-4)$

$4x = 3x^2 - 12$

$0 = 3x^2 - 4x - 12$

-36
1 36
2 18
3 12
4 9
6 6

$x = \frac{4 \pm \sqrt{16 + 4(3)(+12)}}{2(3)}$

$= \frac{4 \pm \sqrt{160}}{6}$

$= \frac{4 \pm 4\sqrt{10}}{6}$

$= \frac{2}{3} \pm \frac{2}{3}\sqrt{10}$

$x \approx 2.77$ (the negative value isn't applicable.)

The average speed of the canoe in still water is about 2.77 mph.