Math 65, Exam 1 Review Name: Solutions

This review is meant to be a study guide but it does not mean that other kinds of problems from the term are off limits for me to put on the exam.

1. When graphing a system of equations, what does the intersection of the two lines represent? What does it mean if there is no intersection?

2. Determine if the given point is a solution to the system of equations.

a. Is $(1, 1)$ a solution to:

$$y = x$$
$$y = -x + 2$$

$1 = 1$ ✓

$1 \stackrel{?}{=} -1 + 2$

$1 = 1$ ✓

Yes, (1, 1) is a solution to the system.

b. Is $(3, 2)$ a solution to:

$$y = \frac{1}{3}x + 1$$
$$y = 3x - 7$$

$2 \stackrel{?}{=} \frac{1}{3}(3) + 1$

$2 \stackrel{?}{=} 1 + 1$

$2 = 2$ ✓

$2 \stackrel{?}{=} 3(3) - 7$

$2 \stackrel{?}{=} 9 - 7$

$2 = 2$ ✓

Yes, (3, 2) is a solution to the system.

3. Identify the slope and y-intercept of each of the following equaitons and then use them to solve the system of equations by graphing. Remember to write your solution in set notation.

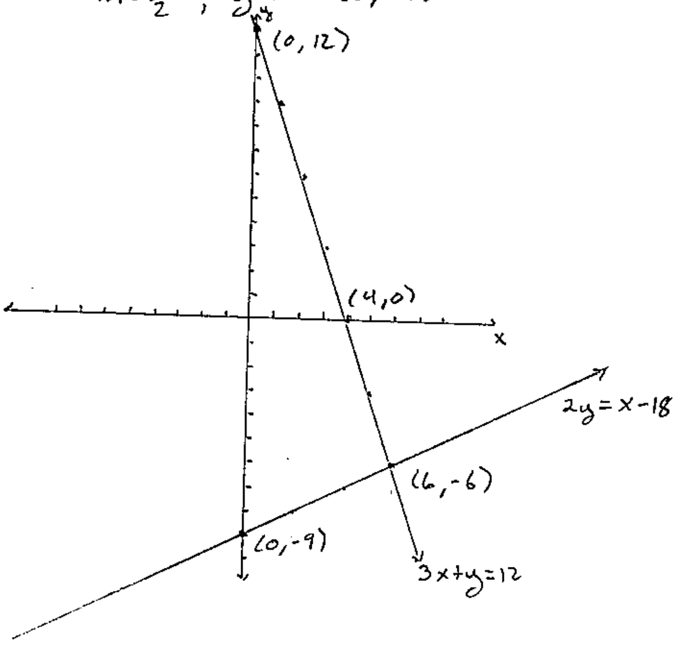
a. $3x + y = 12 \Rightarrow y = -3x + 12$

$m = -3$, y-int $= (0, 12)$

$$\frac{2y}{2} = \frac{x - 18}{2}$$

$y = \frac{1}{2}x - 9$

$m = \frac{1}{2}$, y-int $= (0, -9)$

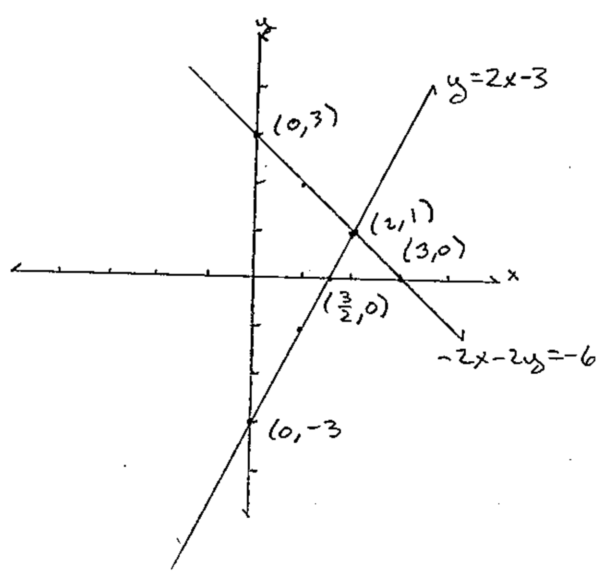


$\{(6, -6)\}$

b. $y = 2x - 3$ $m = 2$, y-int $= (0, -3)$

$$-2x - 2y = -6$$
$$+2x \qquad +2x$$
$$\frac{-2y}{-2} = \frac{2x - 6}{-2}$$

$y = -x + 3$ $m = -1$, y-int $= (0, 3)$



$\{(2, 1)\}$

4. Determine the solution to the system of equations by the *addition method.* Write the solution in set notation.

a. $x - 4y = -6$ → $x - 4y = -6$

$-2 \cdot (4x - 2y) = (4)(-2)$ → $-8x + 4y = -8$

$-7x = -14$

$\frac{-7x}{-7} = \frac{-14}{-7}$

$x = 2$

$(2) - 4y = -6$

$-2 \quad -2$

$\frac{-4y}{-4} = \frac{-8}{-4}$

$y = 2$

$\{(2, 2)\}$

b. $2 \cdot (2x + 5y = 10) \cdot 2$ → $4x + 10y = 20$

$-5 \cdot (-3x + 2y) = (4)(-5)$ → $15x - 10y = -20$

$19x = 0$

$\frac{19x}{19} = \frac{0}{19}$

$x = 0$

$-3(0) + 2y = 4$

$2y = 4$

$y = 2$

$\{(0, 2)\}$

5. Determine the solution to the system of equations by the *substitution method.* Write the solution in set notation.

a. $2x - 3y = 5$

$x = 7y - 2$

$2(7y - 2) - 3y = 5$

$14y - 4 - 3y = 5$

$11y - 4 = 5$

$11y = 9$

$y = 9/11$

$x = 7(9/11) - 2$

$x = \frac{63}{11} - \frac{22}{11}$

$= \frac{41}{11}$

b. $2x + 5y = 3$

$-4x - 10y = -6$

$+4x \qquad +4x$

$\frac{-10y}{-10} = \frac{4x - 6}{-10}$

$y = -\frac{2}{5}x + \frac{3}{5}$

$2x + 5\left(-\frac{2}{5}x + \frac{3}{5}\right) = 3$

$2x - 2x + 3 = 3$

$3 = 3$

Therefore these equation have the same solutions.

$\{(x, y) \mid 2x + 5y = 3\}$

6. Let x represent the first number and let y represent the second number. Suppose two times the first number, decreased by three times the second number is -1. Also, the first number increased by twice the second number is 7. Use the given conditions to write a system of equations and then solve that system using either method. Check your solution.

$2x - 3y = -1$

$x + 2y = 7$

$-2y \quad -2y$

$x = -2y + 7$

$2(-2y + 7) - 3y = -1$

$-4y + 14 - 3y = -1$

$-7y + 14 = -1$

$-14 \quad -14$

$-7y = -15$

$y = 15/7$

$x = -2\left(\frac{15}{7}\right) + 7$

$= \frac{-30}{7} + \frac{49}{7}$

$= \frac{19}{7}$

The numbers are $\frac{15}{7}$ & $\frac{19}{7}$.

Check:

$2\left(\frac{19}{7}\right) - 3\left(\frac{15}{7}\right) \stackrel{?}{=} -1$

$\frac{38}{7} - \frac{45}{7} \stackrel{?}{=} -1$

$-\frac{7}{7} \stackrel{?}{=} -1$

$-1 = -1$ ✓

$\frac{19}{7} + 2\left(\frac{15}{7}\right) \stackrel{?}{=} 7$

$\frac{19}{7} + \frac{30}{7} \stackrel{?}{=} 7$

$\frac{49}{7} \stackrel{?}{=} 7$

$7 = 7$ ✓

7. Identify each polynomial as a monomial, a binomial or a trinomial. Give the degree of the polynomial.

   a. $y^4 - 2y^2 + 1$

   Trinomial of degree 4.

   b. $6y - 5$

   Binomial of degree 1.

   c. 7

   Monomial of degree 0.

   d. $-9y^{192} + y$

   Binomial of degree 192.

8. Add or subtract the following polynomials:

   a. $(15y^3 - 6y + 2) + (10y^2 - y^3 + 3)$

   $= 14y^3 + 10y^2 - 6y + 5$

   b.

   $$\begin{array}{r} 5x^4 - x^3 - 3x^2 \\ -(2x^3 - 3x^2 + x - 7) \\ \hline 5x^4 - 3x^3 - x + 7 \end{array}$$

   c. $\left(\frac{1}{3}x^3 + \frac{2}{5}x - \frac{3}{4}\right) - \left(-\frac{2}{7}x^3 - \frac{3}{5}x + \frac{1}{4}\right)$

   $= \frac{1}{3}x^3 + \frac{2}{5}x - \frac{3}{4} + \frac{2}{7}x^3 + \frac{3}{5}x - \frac{1}{4}$

   $= \frac{13}{21}x^3 + x - 1$

   $\frac{1}{3} + \frac{2}{7} = \frac{7}{21} + \ldots$

   $= \frac{13}{21}$

   d.

   $$\begin{array}{r} 5y^4 - y^3 + 7y \\ -(-y^3 - 5y^2 + 7y - 5) \\ \hline 5y^4 + 5y^2 + 5 \end{array}$$

9. Multiply the following *by first distributing one polynomial into the other and* ***then*** *distributing a second time.* i.e. the long way.

   $\left(2x - \frac{1}{4}\right)\left(8x^3 - 12x^2 + 4x - 7\right) = 2x(8x^3 - 12x^2 + 4x - 7) - \frac{1}{4}(8x^3 - 12x^2 + 4x - 7)$

   $= 16x^4 - 24x^3 + 8x^2 - 14x - 2x^3 + 3x^2 - x + \frac{7}{4}$

   $= 16x^4 - 26x^3 + 11x^2 - 15x + \frac{7}{4}$

10. Multiply the following by using the correct form when appropriate and the FOIL method or the "fast" multiplication when no form is present.

a. $(-2x^5y^3)(7x^3y^{12}) = -14x^8y^{15}$

d. $(2x - y^2)(2x + y^2) = 4x^2 - y^4$

b. $-m(n^2 - mn + m^2) = -mn^2 - m^2n - m^3$

e. $(5x^2 - 3)(2x + 3) = 10x^3 + 15x^2 - 6x - 9$

c. $(x^3 - y^3)^2 = x^6 - 2x^3y^3 + y^6$

11. Determine whether each statement "makes sense" or "does not make sense" and explain your reasoning.

a. A linear system having graphs with the same y-intercepts must have infinitely many solutions.

This does not make sense. If the slopes are not the same then the system's only solution is the y-intercept.

b. When an inconsistent system is solved using the substitution method, a true statement, such as $1 = 1$, results.

This is the opposite of what happens, when an inconsistent system is being solved using the substitution method a false statement, such as $-3 = 2$ or $1 = 5$, results.

c. By first summing the exponents of each variable in each term of a polynomial in two variables, the degree of the polynomial can then be found by selecting the highest sum.

This makes sense. The degree of a polynomial in 2 variables is the degree of its highest term. The degree of a term is found by adding the exponents of its variables.

12. Simplify the following expressions by using the exponent rules gone over in class. Final forms should have only positive exponents.

a. $\frac{x^{100}y^{50}}{x^{25}y^{10}} = x^{75}y^{40}$

d. $\frac{8x^3+6x^2-2x}{2x} = 4x^2+3x-1$

b. $(100y)^0 = 1$

e. $\left(\frac{4x^5}{2x^2}\right)^{-4} = (2x^3)^{-4} = \left(\frac{1}{2x^3}\right)^4 = \frac{1}{16x^{12}}$

c. $\frac{-5x^{10}y^{12}z^6}{50x^2y^3z^2} = -\frac{x^8y^9z^4}{10}$

$\underline{\underline{\text{or}}}$ $= -\frac{1}{10}x^8y^9z^4$

f. $\left(\frac{x^2}{y^3}\right)^{-3} = \left(\frac{y^3}{x^2}\right)^3 = \frac{y^9}{x^6}$

13. Simplify the following expressions using scientific notation. Write the simplified form in BOTH scientific AND decimal notation.

a. $(3 \times 10^4)(3 \times 10^2) = 9 \times 10^6 = 9{,}000{,}000$

b. $\frac{180\times10^6}{2\times10^3} = 90 \times 10^3 = \underbrace{9.0 \times 10^4}_{\text{scientific}} = 90{,}000$

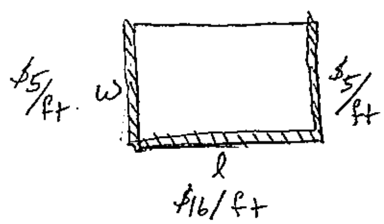
c. $(5 \times 10^2)^3 = 125 \times 10^6 = \underbrace{1.25 \times 10^8}_{\text{scientific}} = 125{,}000{,}000$

14. *This problem will be in the calculator portion of the exam.* The Andromeda Galaxy is approximately 2.54×10^6 light years from the Milky Way Galaxy that we live in. Now, a light years is NOT a measure of time, it is a measure of DISTANCE. In fact, 1 light year is approximately 9.46×10^15 meters. Use this information to determine approximately how many meters the Andromeda Galaxy is from the Milky Way Galaxy. Do your calculations and write your answer in scientific notation.

$$2.54 \times 10^6\ ly \cdot \frac{9.46 \times 10^{15}\ m}{1\ ly} = 24.0284 \times 10^{21}\ m \approx 2.40 \times 10^{22}\ m.$$

The Andrometa Galaxy is approximately 2.40×10^{22} m from the Milky Way Galaxy.

15. A rectangular lot whose perimeter is 320 feet is fenced along three sides. An expensive fencing along the lot's length costs \$16 per foot. An inexpensive fencing along the two side widths costs only \$5 per foot. The total cost of the fencing along the three sides comes to \$2140. What are the lot's dimensions?



$P = 320$ ft

$(-5)(2l + 2w) = 320(-5) \rightarrow -10l - 10w = -1600$

$16l + 10w = 2140$

$\frac{6l}{6} = \frac{540}{6}$

$l = 90$

$16l + 5w + 5w = 2140$

$16l + 10w = 2140$

$16(90) + 10w = 2140$

$1440 + 10w = 2140$

$-1440 \qquad -1440$

$\frac{10w}{10} = \frac{700}{10}$

$w = 70$

The length is 90 ft & the width is 70 ft.

16. You are choosing between two long distance telephone plans. Plan A has a monthly fee of \$20 with a charge of \$0.05 per minute for all long-distance calls. Plan B has a monthly fee of \$5 with a charge of \$0.10 per minute for all long-distance calls.

Let y = cost of a long distance plan for 1 month
& x = minutes used

Plan A: $y = 0.05x + 20$

Plan B: $y = 0.10x + 5$

$$0.05x + 20 = 0.10x + 5$$
$$-0.05x \qquad -0.05x$$
$$20 = 0.05x + 5$$
$$-5 \qquad -5$$
$$\frac{15}{0.05} = \frac{0.05x}{0.05}$$
$$300 = x$$

$$y = 0.10(300) + 5$$
$$= 30 + 5 = 35$$

If you use 300 minutes then either plan will cost \$35.00.

17. How many ounces of a 15% alcohol solution must be mixed with 4 ounces of a 20% alcohol solution to make a 17% alcohol solution?

Let x = oz of 15% solution & y = oz of final 17% mixture

$$x + 4 = y$$
$$.15x + .20(4) = .17y$$
$$.15x + .8 = .17y$$
$$.15x + .8 = .17(x+4)$$
$$.15x + .8 = .17x + .68$$
$$-.68 \qquad -.68$$
$$.15x + .12 = .17x$$
$$-.15x \qquad -.15x$$
$$\frac{.12}{.02} = \frac{.02x}{.02}$$
$$6 = x$$
$$6 + 4 = y$$
$$10 = y$$

We need to mix 6oz of 15% alcohol solution with 4oz of 20% alcohol solution to make 10 oz of 17% alcohol solution.