Math 65, Exam 1 Review Name: Solutions

This review is meant to be a study guide but it does not mean that other kinds of problems from the term are off limits for me to put on the exam.

1. When graphing a system of equations, what does the intersection of the two lines represent? What does it mean if there is no intersection?

2. Determine if the given point is a solution to the system of equations.

a. Is $(1,1)$ a solution to:

$$y = x$$
$$y = -x + 2$$

$1 = 1$ ✓

$1 \stackrel{?}{=} -1 + 2$

$1 = 1$ ✓

Yes, (1, 1) is a solution to the system.

b. Is $(3,2)$ a solution to:

$$y = \frac{1}{3}x + 1$$
$$y = 3x - 7$$

$2 \stackrel{?}{=} \frac{1}{3}(3) + 1$

$2 \stackrel{?}{=} 1 + 1$

$2 = 2$ ✓

$2 \stackrel{?}{=} 3(3) - 7$

$2 \stackrel{?}{=} 9 - 7$

$2 = 2$ ✓

Yes, (3, 2) is a solution to the system.

3. Identify the slope and y-intercept of each of the following equaitons and then use them to solve the system of equations by graphing. Remember to write your solution in set notation.

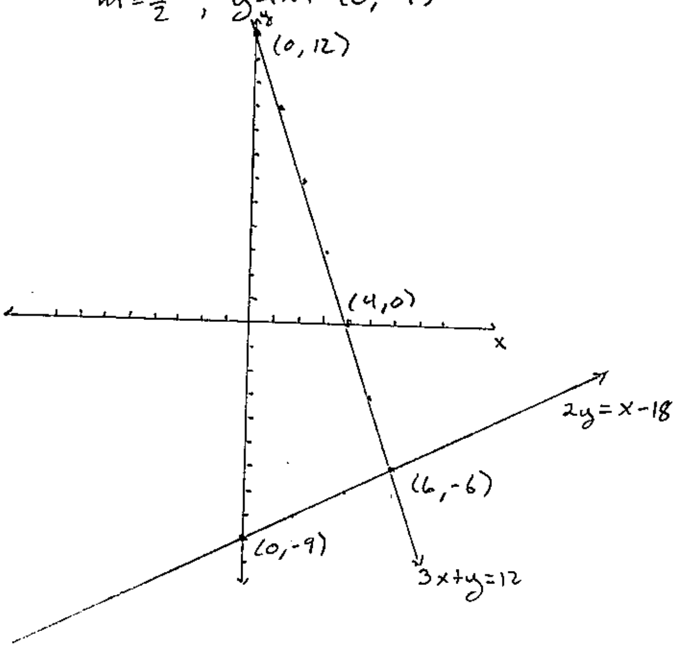
a. $3x + y = 12 \Rightarrow y = -3x + 12$

$m = -3$, y-int $= (0, 12)$

$$\frac{2y}{2} = \frac{x - 18}{2}$$

$y = \frac{1}{2}x - 9$

$m = \frac{1}{2}$, y-int $= (0, -9)$



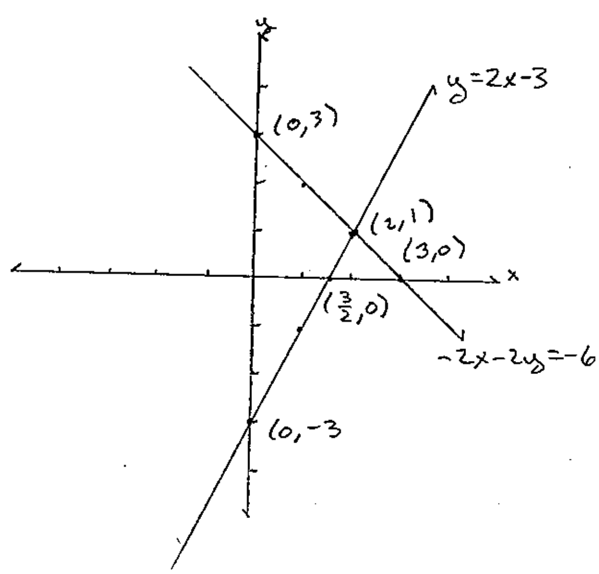
$\{(6, -6)\}$

b. $y = 2x - 3$ $m = 2$, y-int $= (0, -3)$

$$-2x - 2y = -6$$
$$+2x \qquad +2x$$

$$\frac{-2y}{-2} = \frac{2x - 6}{-2}$$

$y = -x + 3$ $m = -1$, y-int $= (0, 3)$



$\{(2, 1)\}$

4. Determine the solution to the system of equations by the *addition method.* Write the solution in set notation.

a. $x - 4y = -6 \rightarrow x - 4y = -6$
$-2 \cdot (4x - 2y) = (4)(-2)$ $\quad -8x + 4y = -8$

$-7x = -14$

$\frac{-7x}{-7} = \frac{-14}{-7}$

$x = 2$

$(2) - 4y = -6$

$-2 \qquad -2$

$-4y = -8$

$\frac{-4y}{-4} = \frac{-8}{-4}$

$y = 2$

$\{(2,2)\}$

b. $2 \cdot (2x + 5y = 10) \cdot 2 \rightarrow 4x + 10y = 20$
$-5 \cdot (-3x + 2y) = (4)(-5)$ $\quad 15x - 10y = -20$

$19x = 0$

$\frac{19x}{19} = \frac{0}{19}$

$x = 0$

$-3(0) + 2y = 4$

$2y = 4$

$y = 2$

$\{(0,2)\}$

5. Determine the solution to the system of equations by the *substitution method.* Write the solution in set notation.

a. $2x - 3y = 5$
$x = 7y - 2$

$2(7y - 2) - 3y = 5$

$14y - 4 - 3y = 5$

$11y - 4 = 5$

$11y = 9$

$y = 9/11$

$x = 7(9/11) - 2$

$x = \frac{63}{11} - \frac{22}{11}$

$= \frac{41}{11}$

b. $2x + 5y = 3$
$-4x - 10y = -6$
$+4x \qquad +4x$

$-10y = 4x - 6$

$\frac{-10y}{-10} = \frac{4x - 6}{-10}$

$y = -\frac{2}{5}x + \frac{3}{5}$

$2x + 5(-\frac{2}{5}x + \frac{3}{5}) = 3$

$2x - 2x + 3 = 3$

$3 = 3$

Therefore these equation have the same solutions.

$\{(x,y) \mid 2x + 5y = 3\}$

6. Let x represent the first number and let y represent the second number. Suppose two times the first number, decreased by three times the second number is -1. Also, the first number increased by twice the second number is 7. Use the given conditions to write a system of equations and then solve that system using either method. Check your solution.

$2x - 3y = -1$

$x + 2y = 7$
$-2y \quad -2y$

$x = -2y + 7$

$2(-2y + 7) - 3y = -1$

$-4y + 14 - 3y = -1$

$-7y + 14 = -1$
$-14 \quad -14$

$-7y = -15$

$y = 15/7$

$x = -2(\frac{15}{7}) + 7$

$= \frac{-30}{7} + \frac{49}{7}$

$= \frac{19}{7}$

The numbers are $\frac{15}{7}$ & $\frac{19}{7}$.

Check:

$2(\frac{19}{7}) - 3(\frac{15}{7}) \stackrel{?}{=} -1$

$\frac{38}{7} - \frac{45}{7} \stackrel{?}{=} -1$

$-\frac{7}{7} \stackrel{?}{=} -1$

$-1 = -1$ ✓

$\frac{19}{7} + 2(\frac{15}{7}) \stackrel{?}{=} 7$

$\frac{19}{7} + \frac{30}{7} \stackrel{?}{=} 7$

$\frac{49}{7} \stackrel{?}{=} 7$

$7 = 7$ ✓

7. Identify each polynomial as a monomial, a binomial or a trinomial. Give the degree of the polynomial.

a. $y^4 - 2y^2 + 1$

Trinomial of degree 4.

b. $6y - 5$

Binomial of degree 1.

c. 7

Monomial of degree 0.

d. $-9y^{192} + y$

Binomial of degree 192.

8. Add or subtract the following polynomials:

a. $(15y^3 - 6y + 2) + (10y^2 - y^3 + 3)$

$= 14y^3 + 10y^2 - 6y + 5$

c. $\left(\frac{1}{3}x^3 + \frac{2}{5}x - \frac{3}{4}\right) - \left(-\frac{2}{7}x^3 - \frac{3}{5}x + \frac{1}{4}\right)$

$= \frac{1}{3}x^3 + \frac{2}{5}x - \frac{3}{4} + \frac{2}{7}x^3 + \frac{3}{5}x - \frac{1}{4}$

$= \frac{13}{21}x^3 + x - 1$

$\frac{1}{3} + \frac{2}{7} = \frac{7}{21} + \frac{6}{21}$

$= \frac{13}{21}$

b.

$$\begin{array}{r} 5x^4 - x^3 - 3x^2 \\ -(2x^3 - 3x^2 + x - 7) \\ \hline 5x^4 - 3x^3 - x + 7 \end{array}$$

d.

$$\begin{array}{r} 5y^4 - y^3 + 7y \\ -(-y^3 - 5y^2 + 7y - 5) \\ \hline 5y^4 + 5y^2 + 5 \end{array}$$

9. Multiply the following *by first distributing one polynomial into the other and* ***then*** *distributing a second time.* i.e. the long way.

$\left(2x - \frac{1}{4}\right)\left(8x^3 - 12x^2 + 4x - 7\right) = 2x(8x^3 - 12x^2 + 4x - 7) - \frac{1}{4}(8x^3 - 12x^2 + 4x - 7)$

$= 16x^4 - 24x^3 + 8x^2 - 14x - 2x^3 + 3x^2 - x + \frac{7}{4}$

$= 16x^4 - 26x^3 + 11x^2 - 15x + \frac{7}{4}$

10. Multiply the following by using the correct form when appropriate and the FOIL method or the "fast" multiplication when no form is present.

a. $(-2x^5y^3)(7x^3y^{12}) = -14x^8y^{15}$

d. $(2x - y^2)(2x + y^2) = 4x^2 - y^4$

b. $-m(n^2 - mn + m^2) = -mn^2 - m^2n - m^3$

e. $(5x^2 - 3)(2x + 3) = 10x^3 + 15x^2 - 6x - 9$

c. $(x^3 - y^3)^2 = x^6 - 2x^3y^3 + y^6$

11. Determine whether each statement "makes sense" or "does not make sense" and explain your reasoning.

a. A linear system having graphs with the same y-intercepts must have infinitely many solutions.

This does not make sense. If the slopes are not the same then the system's only solution is the y-intercept.

b. When an inconsistent system is solved using the substitution method, a true statement, such as $1 = 1$, results.

This is the opposite of what happens, when an inconsistent system is being solved using the substitution method a false statement, such as $-3 = 2$ or $1 = 5$, results.

c. By first summing the exponents of each variable in each term of a polynomial in two variables, the degree of the polynomial can then be found by selecting the highest sum.

This makes sense. The degree of a polynomial in 2 variables is the degree of its highest term. The degree of a term is found by adding the exponents of its variables.

12. Simplify the following expressions by using the exponent rules gone over in class. Final forms should have only positive exponents.

a. $\frac{x^{100}y^{50}}{x^{25}y^{10}} = x^{75}y^{40}$

d. $\frac{8x^3+6x^2-2x}{2x} = 4x^2+3x-1$

b. $(100y)^0 = 1$

e. $\left(\frac{4x^5}{2x^2}\right)^{-4} = (2x^3)^{-4} = \left(\frac{1}{2x^3}\right)^4 = \frac{1}{16x^{12}}$

c. $\frac{-5x^{10}y^{12}z^6}{50x^2y^3z^2} = -\frac{x^8y^9z^4}{10}$

or $= -\frac{1}{10}x^8y^9z^4$

f. $\left(\frac{x^2}{y^3}\right)^{-3} = \left(\frac{y^3}{x^2}\right)^3 = \frac{y^9}{x^6}$

13. Simplify the following expressions using scientific notation. Write the simplified form in BOTH scientific AND decimal notation.

a. $(3 \times 10^4)(3 \times 10^2) = 9 \times 10^6 = 9{,}000{,}000$

b. $\frac{180\times10^6}{2\times10^3} = 90 \times 10^3 = \underbrace{9.0 \times 10^4}_{\text{scientific}} = 90{,}000$

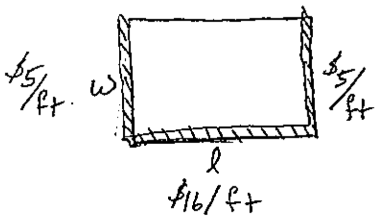
c. $(5 \times 10^2)^3 = 125 \times 10^6 = \underbrace{1.25 \times 10^8}_{\text{scientific}} = 125{,}000{,}000$

14. *This problem will be in the calculator portion of the exam.* The Andromeda Galaxy is approximately 2.54×10^6 light years from the Milky Way Galaxy that we live in. Now, a light years is NOT a measure of time, it is a measure of DISTANCE. In fact, 1 light year is approximately 9.46×10^15 meters. Use this information to determine approximately how many meters the Andromeda Galaxy is from the Milky Way Galaxy. Do your calculations and write your answer in scientific notation.

$$2.54 \times 10^6\, ly \cdot \frac{9.46 \times 10^{15}\, m}{1\, ly} = 24.0284 \times 10^{21}\, m \approx 2.40 \times 10^{22}\, m.$$

The Andrometa Galaxy is approximately 2.40×10^{22} m from the Milky Way Galaxy.

15. A rectangular lot whose perimeter is 320 feet is fenced along three sides. An expensive fencing along the lot's length costs \$16 per foot. An inexpensive fencing along the two side widths costs only \$5 per foot. The total cost of the fencing along the three sides comes to \$2140. What are the lot's dimensions?



$P = 320$ ft

$(-5)(2l + 2w) = 320(-5) \rightarrow -10l - 10w = -1600$

$16l + 5w + 5w = 2140$

$16l + 10w = 2140$

$$\begin{aligned} -10l - 10w &= -1600 \\ 16l + 10w &= 2140 \\ \hline \frac{6l}{6} &= \frac{540}{6} \\ l &= 90 \end{aligned}$$

$$\begin{aligned} 16(90) + 10w &= 2140 \\ 1440 + 10w &= 2140 \\ -1440 \qquad &\quad -1440 \\ \hline \frac{10w}{10} &= \frac{700}{10} \\ w &= 70 \end{aligned}$$

The length is 90 ft & the width is 70 ft.

16. You are choosing between two long distance telephone plans. Plan A has a monthly fee of \$20 with a charge of \$0.05 per minute for all long-distance calls. Plan B has a monthly fee of \$5 with a charge of \$0.10 per minute for all long-distance calls.

Let y = cost of a long distance plan for 1 month

& x = minutes used

Plan A: $y = 0.05x + 20$

Plan B: $y = 0.10x + 5$

$$0.05x + 20 = 0.10x + 5$$
$$-0.05x \qquad -0.05x$$
$$20 = 0.05x + 5$$
$$-5 \qquad -5$$
$$\frac{15}{0.05} = \frac{0.05x}{0.05}$$
$$300 = x$$

$$y = 0.10(300) + 5$$
$$= 30 + 5 = 35$$

If you use 300 minutes then either plan will cost \$35.00.

17. How many ounces of a 15% alcohol solution must be mixed with 4 ounces of a 20% alcohol solution to make a 17% alcohol solution?

Let x = oz of 15% solution & y = oz of final 17% mixture

$$x + 4 = y$$
$$.15x + .20(4) = .17y$$
$$.15x + .8 = .17y$$
$$.15x + .8 = .17(x+4)$$
$$.15x + .8 = .17x + .68$$
$$-.68 \qquad -.68$$
$$.15x + .12 = .17x$$
$$-.15x \qquad -.15x$$
$$\frac{.12}{.02} = \frac{.02x}{.02}$$
$$6 = x$$
$$6 + 4 = y$$
$$10 = y$$

We need to mix 6 oz of 15% alcohol solution with 4 oz of 20% alcohol solution to make 10 oz of 17% alcohol solution.

11. The functions f and g are given below in a table and a graph. Answer the questions below using these functions.

x	-4	-2	0	1	3	5
$f(x)$	8	5	3	0	-2	-6

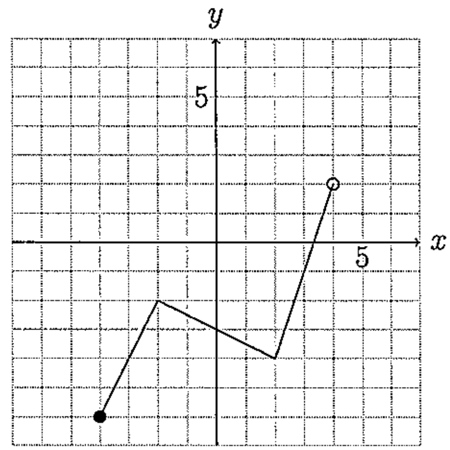


Figure 13: $g(x)$

a. Find the following

i. $f(3) =$ -2

ii. $f(-2) =$ 5

iii. $g(0) =$ -3

iv. $g(2) =$ -4

v. When $f(x) = 5$, then $x =$ -2.

vi. When $g(x) = -1$, then $x =$ 3.

vii. When $g(x) = -4$, then $x =$ 2 or $x=-3$.

b. What is the domain of g?

$[-4, 4) = \{x \mid -4 \le x < 4\}$

c. What is the range of g?

$[-6, 2) = \{y \mid -6 \le y < 2\}$

d. Find all x for which $g(x) > -1$

$g(x) > -1$ when

$3 < x < 4$.

The set of solutions is $\{x \mid 3 < x < 4\}$

$= \underbrace{(3,4)}_{\text{interval notation}}$

2. Given $g(x) = \frac{x}{x+2}$, find each of the following and write the corresponding ordered pair:

a. $g(2) = \frac{2}{2+2} = \frac{1}{2}$

c. $g(0) = 0$

b. $g(-3) = 3$

d. $g(-1) = -1$

3. Determine whether the following relationships between inputs and outputs can be categorized as a function and justify your response. State the domain and range of both relations.

a. $\{(-2,7),(3,4),(5,9),(2,4),(3,-7)\}$

This is not a function since the input of 3 has 2 outputs.

$D = \{-2, 2, 3, 5\}$

$R = \{-7, 4, 7, 9\}$

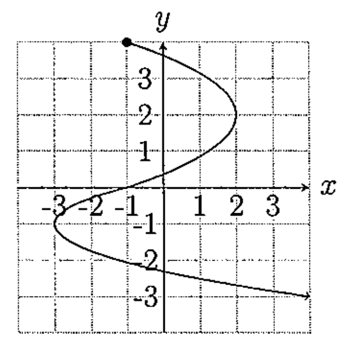
b. $\{(2,.5),(-7,.5),(-1,.5),(3,2),(4,2)\}$

This is a function since each input has a unique output.

$D = \{-7, -1, 2, 3, 4\}$

$R = \{0.5, 2\}$

c.



This is not a function since 0 (along with many other inputs) has more than one output.

$D = [-3, \infty)$

$R = (-\infty, 4]$

4. What is the one requirement for a relationship between inputs and outputs to be considered a function?

Each input in the domain of the function has just one output.

5. What is the definition of a solution to an equation in one variable?

A number which, when plugged in for the variable, results in the expression on the left holding the same value as the expression on the right.

6. For the next two problems, set up two functions, one to represent each side of the equation then solve the equation numerically (using the function name in your header), symbolically, and graphically (labeling both graphs using the correct function name). State a conclusion using a full sentence and proper set notation.

a. $2x + 3 = 5$ Let $f(x) = 2x+3$ & $g(x) = 5$

i. Numerically:

x	$f(x)$	$g(x)$
0	3	5
3	9	5
2	7	5
1	5	5

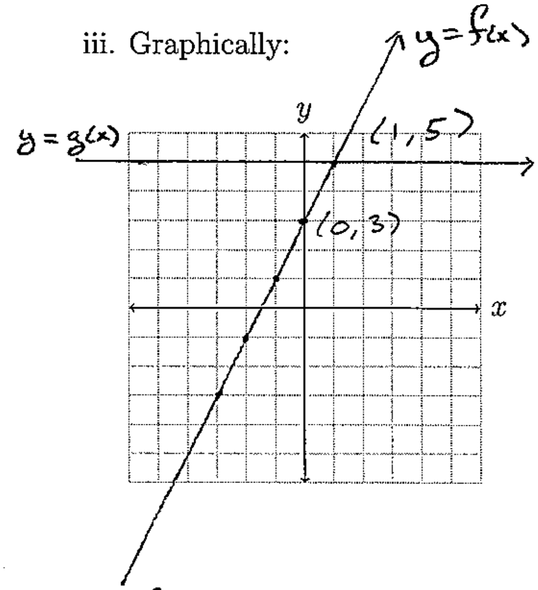
ii. Symbolically:

$2x + 3 = 5$

$\frac{2x}{2} = \frac{2}{2}$

$x = 1$

iii. Graphically:



The set of solutions is $\{1\}$.

b. $x - 2 = -2x + 1$ Let $f(x) = x-2$ & $g(x) = -2x+1$

i. Numerically:

x	$f(x)$	$g(x)$
0	−2	1
5	3	−9
3	1	−5
1	−1	−1

ii. Symbolically:

$x - 2 = -2x + 1$

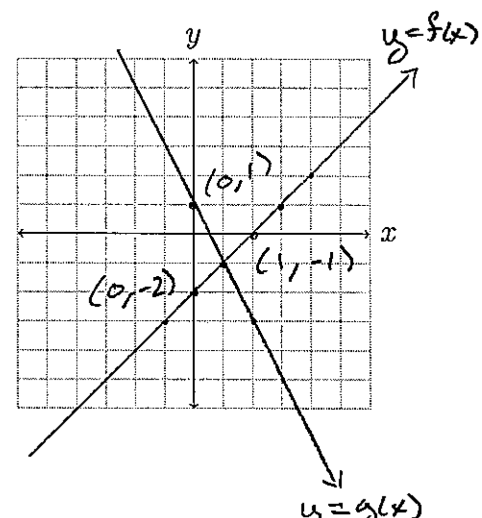
$+2x \qquad +2x$

$3x - 2 = 1$

$\frac{3x}{3} = \frac{3}{3}$

$x = 1$

iii. Graphically:



The set of solutions is $\{1\}$.

5. For the next two problems, set up two or three functions, one to represent each side of the equation then solve the equation numerically (using the function name in your header), symbolically, and graphically (labeling both graphs using the correct function name). State a conclusion using a full sentence and proper set notation.

a. $2x - 4 < -2$ or $2x - 4 \geq 6$

Let $f(x) = -2$, $g(x) = 2x-4$ & $h(x) = 6$

a. Numerically:

x	f(x)	g(x)	h(x)
0	-2	-4	6
* 1	-2	-2	6
3	-2	2	6
7	-2	10	6
6	-2	8	6
* 5	-2	6	6

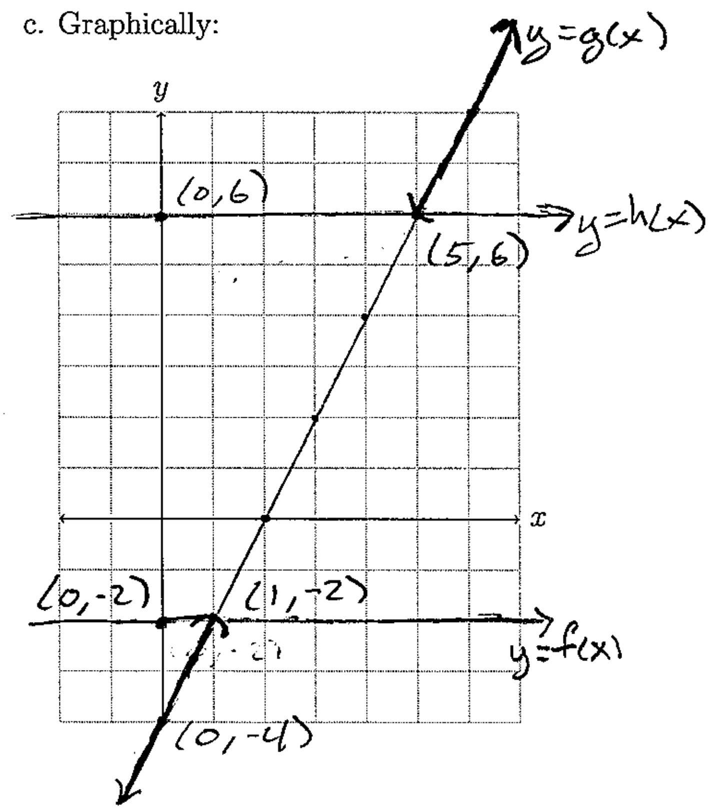
b. Symbolically:

$2x-4 < -2$ or $2x-4 \geq 6$

$2x < 2$ $2x \geq 10$

$x < 1$ or $x \geq 5$

c. Graphically:



The solutions are in the interval $(-\infty, 1) \cup (5, \infty)$. The set of solutions is $\{x \mid x < 1 \text{ or } x > 5\}$.

b. $-2 \leq -\frac{1}{2}x + 2 < 6$

Let $f(x) = -2$, $g(x) = -\frac{1}{2}x + 2$, $h(x) = 6$

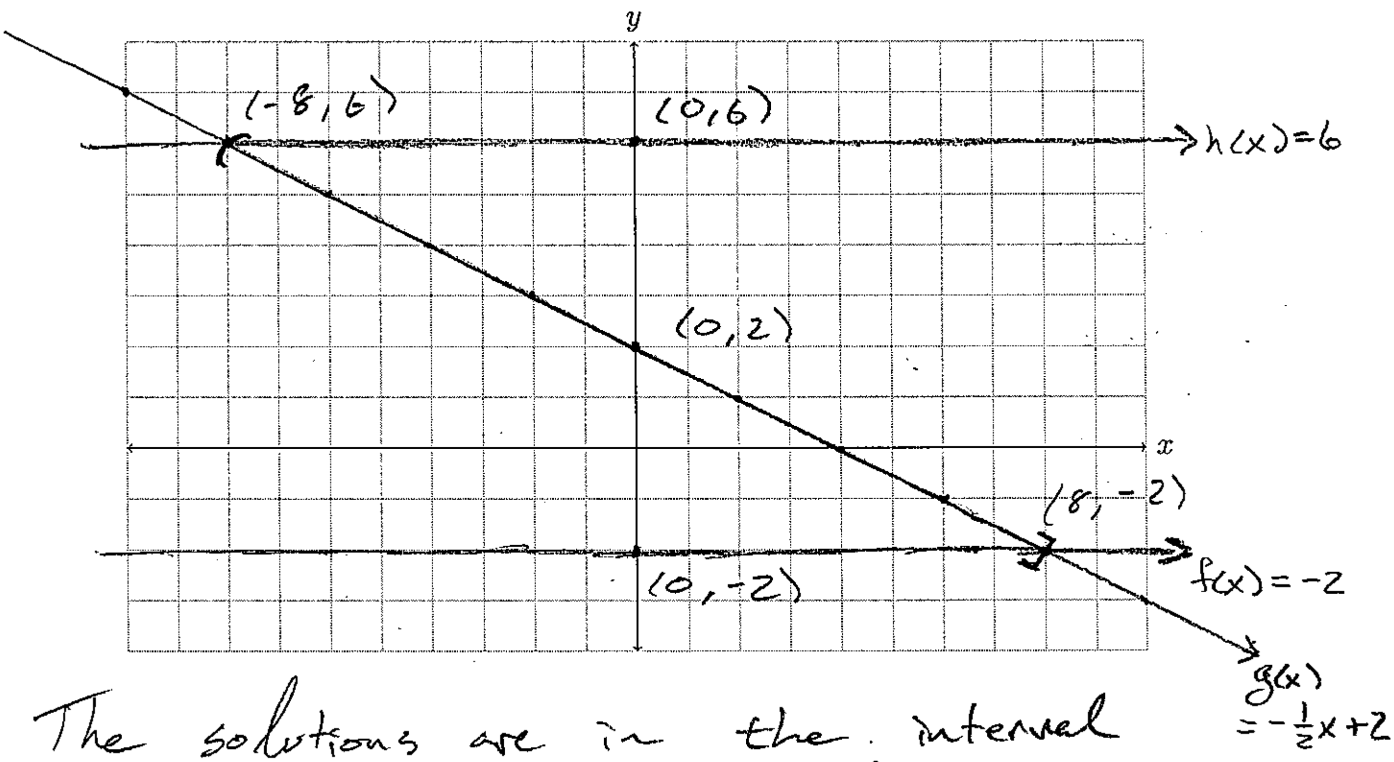
a. Numerically:

x	f(x)	g(x)	h(x)
0	-2	2	6
-4	-2	4	6
* -8	-2	6	6
4	-2	0	6
* 8	-2	-2	6

b. Symbolically:

$$-2 \leq -\frac{1}{2}x + 2 < 6$$
$$-4 \leq -\frac{1}{2}x < 4$$
$$8 \geq x > -8$$
$$-8 < x \leq 8$$

c. Graphically:



The solutions are in the interval $(-8, 8]$. The set of solutions is $\{x \mid -8 < x \leq 8\}$.

2. Solve the following equations and inequalities graphically.

a. $|5 - x| = 3$

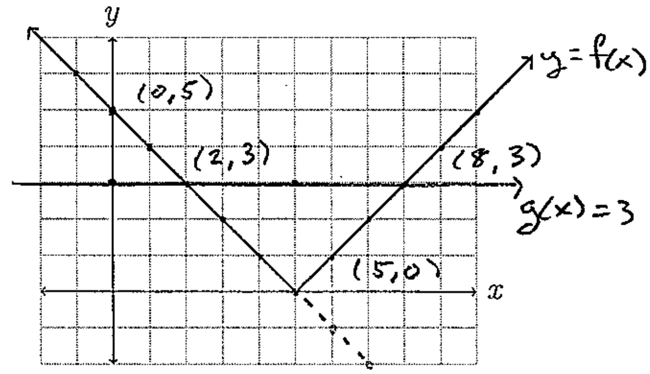
Let $f(x) = |5-x| = |-x+5|$

$m = -1$

$y\text{-int} = (0,5)$

& $g(x) = 3$

The set of solutions is $\{2, 8\}$



b. $|-2x+2| \geq 6$

Let $f(x) = |-2x+2|$

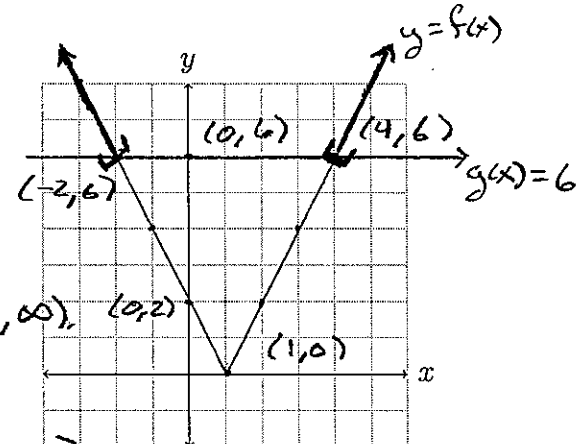
$m = -2$

$y\text{-int} = (0,2)$

& $g(x) = 6$

The interval of solutions is $(-\infty, -2] \cup [4, \infty)$.

The set of solutions is $\{x \mid x \leq -2 \text{ or } x \geq 4\}$.

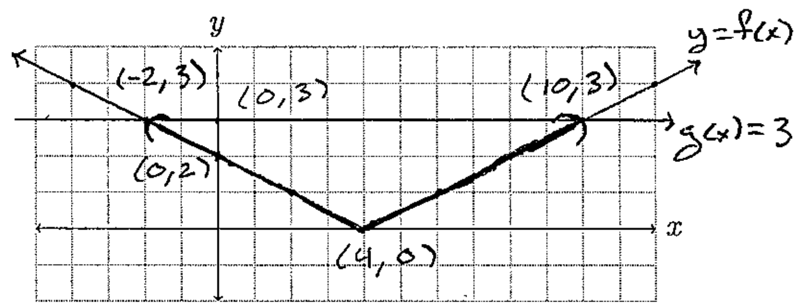


c. $\left|\frac{-1}{2}x + 2\right| < 3$

Let $f(x) = |-\frac{1}{2}x + 2|$

$m = -\frac{1}{2}$ $y\text{-int} = (0,2)$

& $g(x) = 3$



The interval of solutions is $(-2, 10)$.

The set of solutions is $\{x \mid -2 < x < 10\}$

9. Solve the following equations and inequalities numerically.

a. $\left|\frac{2}{3}x - 5\right| < 3$

Let $f(x) = \left|\frac{2}{3}x - 5\right|$

& $g(x) = 3$

$\left|\frac{2}{3}x - 5\right| < 3$ when $3 < x < 12$.

The set of solutions is $\{x \mid 3 < x < 12\}$.

The interval of solutions is $(3, 12)$.

	x	$f(x) <$	$g(x)$
	-3	7	3
	0	5	3
*	3	3	3
✓	6	1	3
✓	9	1	3
*	12	3	3

10. Solve the following equations symbolically.

a. $|5 - 3x| - 3 = 1$

$|5 - 3x| = 4$

If $5 - 3x \geq 0$

Then $|5-3x| = 4$

becomes $5 - 3x = 4$

$\Rightarrow -3x = -1$

$\Rightarrow x = 1/3$

If $5 - 3x < 0$

Then $|5-3x| = 4$

becomes $-(5-3x) = 4$

$\Rightarrow -5 + 3x = 4$

$\Rightarrow 3x = 9$

$\Rightarrow x = 3$

The set of solutions is $\{1/3, 3\}$.

b. $|2x| = |x - 3|$

If $2x$ & $x-3$ are <u>both</u> positive or <u>both</u> negative

The $|2x| = |x-3|$

becomes $2x = x - 3$

$\Rightarrow x = -3$

If one of $2x$ & $x-3$ is positive & one negative

then $|2x| = |x-3|$

becomes $-2x = x - 3$

$\Rightarrow -3x = -3$

$\Rightarrow x = 1$

The set of solutions is $\{-3, 1\}$.

c. $|2x-5| > -1$ OR

If $2x-5 \geq 0$

Then $|2x-5| > -1$

becomes $2x-5 > -1$

$\Rightarrow 2x > 4$

$\Rightarrow x > 2$

OR

If $2x-5 < 0$

Then $|2x-5| > -1$

becomes $-(2x-5) > -1$

$\Rightarrow -2x+5 > -1$

$\Rightarrow -2x > -6$

$\Rightarrow x < 3$

All real #s are either greater than 2 OR less than 3 so the interval of solutions is $(-\infty, \infty)$.

d. $-2|5x-1| \geq 5$

$$\frac{-2|5x-1|}{-2} \geq \frac{5}{-2}$$

$|5x-1| \leq -\frac{5}{2}$ AND

If $5x-1 \geq 0$ then

$|5x-1| \leq -\frac{5}{2}$

becomes $5x-1 \leq -\frac{5}{2}$

$\Rightarrow 5x \leq \frac{-3}{2}$

$\Rightarrow x \leq \frac{-3}{10}$

AND

If $5x-1 < 0$ then

$|5x-1| \leq -5/2$

becomes $-(5x-1) \leq -5/2$

$\Rightarrow -5x+1 \leq -5/2$

$\Rightarrow -5x \leq -7/2$

$\Rightarrow x \geq \frac{7}{10}$

There are no real numbers which are both less than (or = to) $\frac{-3}{10}$ AND bigger than (or = to) $\frac{7}{10}$.

The set of solutions is $\{\}$.