

Section 5.7 Negative Exponents

$$\begin{array}{l}
 3^3 = 27 \\
 3^2 = 9 \\
 3^1 = 3 \\
 3^0 = 1 \\
 3^{-1} = \frac{1}{3} \\
 3^{-2} = \frac{1}{9} \\
 3^{-3} = \frac{1}{27}
 \end{array}$$

$\downarrow \div 3$
 $\downarrow \div 3$
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 $\downarrow \div 3$
 $\downarrow \div 3$

$$1 \div 3 = \frac{1}{3}$$

Negative exponents are fractions

$$3^{-3} = \frac{1}{3^3}$$

$$x^{-2} = \frac{1}{x^2}$$

A negative exponent moves a base from the bottom to the top or the top to bottom and the sign of the exponent changes

$$\frac{1}{x^2} = x^2$$

$$\frac{1}{3^{-3}} = 3^3 = 27$$

cancel

$$\begin{aligned}
 \frac{x^2}{x^{14}} &= \frac{\cancel{1} \cdot \cancel{x} \cdot \cancel{x}}{\cancel{1} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x}} \\
 &= \frac{1}{x^{12}} \quad \frac{x^0}{x^{12}} = \frac{1}{x^{12}}
 \end{aligned}$$

$$\frac{X^{14}}{X^2} = X^{14-2} = X^{12}$$

$$\frac{X^2}{X^{14}} = \frac{1}{X^{14-2}} = \frac{1}{X^{12}}$$

Simplify so that
all exponents
are positive

$$11. \quad 2^{-1} + 3^{-1}$$

$$= \frac{1 \cdot 3}{2 \cdot 3} + \frac{1 \cdot 2}{3 \cdot 2}$$

$$= \frac{3}{6} + \frac{2}{6}$$

$$= \frac{5}{6}$$

Negative exponents
take the reciprocal

$$18. \quad \frac{4^{-3}}{2^{-2}} = \frac{2^2}{4^3}$$

$$= \frac{4}{64}$$

$$= \frac{1}{16}$$

4.4.4

22.

$$\left(\frac{3}{4}\right)^{-3} = \left(\frac{4}{3}\right)^3$$

$$= \frac{64}{27}$$

$$52. \frac{(4y^5)^3}{y^{-4}} = \frac{4^3 y^{15}}{y^{-4}} = 64 y^{15-(-4)} \\ = 64 y^{19}$$

or $64 y^{15} y^4 = 64 y^{19}$

$$62. \frac{3x^3 \cdot \cancel{5x^{14}}}{\cancel{20x^{14}}_4} = \frac{3}{4} x^3$$

or $\frac{\cancel{15x^{17-14}}}{\cancel{20x^{14}}_4} = \frac{3}{4} x^3$

or $\frac{3x^3}{4}$

10:19

$$*128. \frac{(xy^{-2})^{-2}}{(x^{-2}y)^{-3}} = \frac{x^{-2} y^4}{x^6 y^{-3}} = \frac{y^7}{x^8}$$

or

$$\frac{(x^{-2}y)^3}{(xy^{-2})^2} = \frac{x^{-6} y^3}{x^2 y^{-4}} = \frac{y^7}{x^8}$$

$$\frac{x^{-2} y^4}{x^6 y^{-3}} = \frac{1}{x^{6-(-2)}} y^{4-(-3)} \\ = \frac{y^7}{x^8}$$

128.

with every step shown

$$\frac{(X^1 y^{-2})^{-2}}{(X^{-2} y^1)^{-3}} = \frac{X^{1(-2)} y^{-2(-2)}}{X^{-2(-3)} y^{1(-3)}}$$

$$= \frac{X^{-2} y^4}{X^6 y^{-3}}$$

$$= \frac{y^4 y^3}{X^6 X^2}$$

$$= \frac{y^{4+3}}{X^{6+2}}$$

$$= \frac{y^7}{X^8}$$

Put in the |
exponents and
multiply the
exponents

bases with
negative
exponents
move to make
positive
exponents

add the
exponents

132.

$$\begin{aligned}
 & \left(\frac{x^4 y^5 z^6}{x^{-4} y^{-5} z^{-6}} \right)^{-4} = \frac{1x^{-16} y^{-20} z^{-24}}{x^{16} y^{20} z^{24}} \\
 & = \frac{1}{x^{16} \cdot x^{16} y^{20} y^{20} z^{24} z^{24}} \\
 & = \frac{1}{x^{32} y^{40} z^{48}} \\
 & = \frac{1}{x^{32} y^{40} z^{48}}
 \end{aligned}$$

Scientific Notation

$$2^1 = 2$$

$$2^2 = 4$$

$$2^3 = 8$$

$$\vdots$$

$$2^? = 1,000,000$$

$$2^{20} > 1,000,000$$

$$2^{30}$$

$$2^{34} \approx 1.72 \times 10^{10} \text{ or } 1.72 \text{E}10$$

$$2^{15}$$

$$2^{20}$$

$$2^{60}$$

1. Complete the table below:

Scientific Notation

Decimal Notation

Calculator Notation

(What your calculator shows)

$$2.42 \times 10^7$$

→ right

$$3.203 \times 10^{-5}$$

← left

$$-5.11 \times 10^{-9}$$

$$3.902 \times 10^6$$

$$2.34 \times 10^{-8}$$

$$-1.4001 \times 10^0$$

Decimal goes after the 1st digit

$$24,200,000$$

$$.00003203$$

$$-.00000000511$$

8 zeros

$$3,902,000$$

$$0.0000000234$$

$$-1.4001$$

small # neg exp

$$2.42 \times 10^7 \text{ or } 2.42E7$$

$$3.203 \times 10^{-5} \text{ or } 3.203E-5$$

$$-5.11 \times 10^{-9} \text{ or } -5.11E-9$$

$$3.902 \times 10^6 \text{ or } 3.902E6$$

$$2.34 \times 10^{-8} \text{ or } 2.34E-8$$

$$-1.4001 \text{ or } -1.4001E0$$

2. Perform the indicated operations using your calculator. Write your answers in scientific notation.

a. $(3.21 \times 10^6)(1.2 \times 10^4)$

$$= 3.852 \times 10^{10}$$

$$(3.21 \cdot 1.2) 10^6 \cdot 10^4$$

$$= 3.852 \times 10^{10}$$

b. $\left(\frac{9.5 \times 10^{-7}}{4.3 \times 10^{-4}} \right)$

$$= .0022$$

$$= 2.2 \times 10^{-3}$$

3. The United States has a land area of 3.54×10^6 square miles. In 1990, the population of the United States was 2.5×10^8 . What was the population density (in people per square mile) of the United States in 1990?

word problems with scientific notation will not be on the midterm

1. Complete the table below:

| Scientific Notation | Decimal Notation | Calculator Notation (What your calculator shows) |
|------------------------|------------------|---|
| 2.42×10^7 | _____ | _____ |
| 3.203×10^{-5} | _____ | _____ |
| -5.11×10^{-9} | _____ | _____ |
| _____ | 3,902,000 | _____ |
| _____ | 0.0000000234 | _____ |
| _____ | -1.4001 | _____ |

2. Perform the indicated operations using your calculator. Write your answers in scientific notation.

a. $(3.21 \times 10^6)(1.2 \times 10^4)$

b. $\frac{9.5 \times 10^{-7}}{4.3 \times 10^{-4}}$

3. The United States has a land area of 3.54×10^6 square miles. In 1990, the population of the United States was 2.5×10^8 . What was the population density (in people per square mile) of the United States in 1990?

4. Challenge Problem:

Approximately 44,000 thunderstorms and 8 million lightning flashes take place daily around the world. In the United States, lightning annually causes about 150 deaths and \$20 million in property damage. It sets 10,000 forest fires a year, which destroy \$30 million worth of timber. (Source: Scientific American)

a. Approximate the number of lightning flashes that occur each year on Earth.

b. Approximate the number of thunderstorms that occur each year on Earth.

c. What is the average number of lightning flashes in a thunderstorm?